THE ORIGIN OF THE LOGIC OF SYMBOLIC MATHEMATICS

EDMUND HUSSERL AND JACOB KLEIN

BURT C. HOPKINS

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STUDIES IN CONTINENTAL THOUGHT

John Sallis, editor

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Die Zeiten, in welche die Entstehung der Zahl- und Zahlzeichensysteme fällt, kannten keine historische Überlieferung, und so ist den an eine Reproduktion der historischen Entwicklung nicht zu denken. Gleichwohl besitzen wir Anhaltspunkte genug . . . , um die psychologische Entwicklung derartiger Systembildungen a posteriori und doch in allen wesentlichen Punkten zutreffend zu rekonstruieren.

The periods within which the origination of number systems and number sign systems falls are unknown to any historical tradition. Therefore there can be no thought of a reproduction of the historical development. We nevertheless possess sufficient clues . . . in order to reconstruct the psychological development of such systematic formations in an *a posteriori* fashion that is still correct in all essential points.

— Edmund Husserl, *Philosophy of Arithmetic* (1891)

Es kommt also darauf an, die Rezeption der griechischen Mathematik im 16. Jahrh. nicht von ihren Ergebnissen aus zu beurteilen, sondern sie sich in ihrem faktischen Vollzuge zu vergegenwärtigen.

Hence our object is not to evaluate the revival of Greek mathematics in the sixteenth century in terms of its results retrospectively, but to rehearse the actual course of its genesis prospectively.

— Jacob Klein, *Greek Mathematical Thought and the Origin of Algebra* (1934)

Gewiß ist die historische Rückbeziehung niemandem eingefallen; und gewiß ist die Erkenntnistheorie nie als eine eigentümlich historische Aufgabe angesehen worden.

Certainly the historical backward reference [from present-day geometrical knowledge to its genesis] has not occurred to anyone; certainly theory of knowledge has never been seen as a peculiarly historical task.

— Edmund Husserl, "The Origin of Geometry" (1936)

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Preface

Jacob Klein's foundational work on the roots of modernity could not have been more fortunate in its first comprehensive expositor and interpreter. Burt Hopkins lays out its argument, which is as intricate as it is bold, with discerning precision and sympathetic acuity. It is actuated by the judgment that he is explicating one of the philosophical masterpieces of the twentieth century (§ 208).

At the time I translated *Greek Mathematical Thought and the Origin of Algebra* some forty years ago, I had felt its force and appreciated its complexity sufficiently to achieve a passable rendering. But it was not until I read and reread the book which is before you that I began to apprehend Klein's faithful originality. I mean his ingenious yet unforced use of sources. The translation has had a favorable publication history. The MIT Press first brought it out in 1968, and Dover reprinted it in 1992, effectively giving Klein's work the status of a classic.

Nevertheless, though mentioned increasingly often in the scholarly literature on Plato, Descartes, and the philosophy of mathematics, it was not altogether well understood, as Hopkins's detailed critical footnotes show. Not that the various treatments were grossly mistaken, but that the fine points—and in this undertaking the devil *is* in the details—had not been carefully enough considered. Moreover, the significant relation to Husserl's work on arithmetic had not been worked out.

Hopkins, following up the "scholarly curiosity" of Klein's silence about Husserl's theories of intentionality and symbolic thinking, shows that Klein had indeed in some respects anticipated and corrected before the fact, as it were, Husserl's analysis of the concept of number in particular and of the conceptual presuppositions of modern science in general. What Hopkins does is to set out in detail how Klein's analysis undercuts Husserl's basic assumption that modern mathematical thinking must be understood from the perspective of direct experience. Yet he also shows that Klein's investigation is in fact a large-scale actualization of Husserl's historical method (§ 28).

This demonstration does far more than establish priorities (which would have been of the least imaginable interest to Klein himself). It is, in its penetrating meticulousness, an invaluable introduction to Husserl's defining works. Moreover, the very closeness of Hopkins's critical treatment serves to bring out Husserl's human probity and philosophical depth, the fundamental well-directedness of his search for the nature of the modern scientific achievement, and the cognitive crisis it has brought in its train.

Certainly in 1966 I had no clear notion of the explanatory force residing in the kind of history sketched out by one man and actually pursued by the other, an enterprise totally different from the crude historicism then quite prevalent in the academy, according to which all the works of thought were simply attributed to the spirit of the times.

I will not preempt Hopkins's careful delineations of Husserl's "intentional history," except to say that it enjoins on the historian one chief task, that of "desedimentation" and, as Hopkins says, "reactivation." These terms refer to the scraping away of the accumulated strata of tradition and the reanimating recollection of the thinking that had been skewed, superseded, and "ruptured" at crucial moments, but had remained embalmed—one might say, semi-consciously preserved—within those modern concepts that are so effective precisely because the burden of their origin is ignored.

It seems to me now a major finding that the revolution in the conceptuality of "number" both marked and characterized the inception of modernity. I say 'conceptuality'—as opposed to 'concept'—advisedly. Such abstract "second-intentional" terminology (see below) was deeply, even passionately repugnant to Klein; it was the personal consequence of his studies. Yet the revolution in question was not about this or that individual notion, but about the mode of concept-formation itself, which *is* properly called 'conceptuality'. The much abused word 'revolution' is also in order: Although long in preparation, the new number concept came abruptly upon the world, its prime agent being Vieta (§ 101), its clarifier Descartes (§ 111), and its finalizer Wallis (§ 117). Moreover, number was not only an emblem of the new order of the age, meaning that the way of conceiving that produced it showed up in non-mathematical theorizing, but also perhaps the moderns' most powerful tool. For it was algebra, the first, powerful fruit of the new conceptualizing, that made analytic natural science possible.

The desedimentation of our concept of number, while a work of philosophical inquiry, required large scholarly means: the ability to deal directly with sources in ancient and modern languages, because translations tend to the heedless use of merely apparent modern equivalents; some competence in ancient and early modern mathematics, which, though elemen-

tary, uses notation that is at once obsolete and significant; and a willingness to let the sources speak for themselves.

What this desedimenation, this philosophical archaeology, recovered, then, were the foundation stones of our modernity, and with them a proper estimation of the huge gain in efficacy and the great loss in immediacy that comes—to state the prime thesis summarily—with ceasing to think of the objects of mathematics and nature in the ancient mode—namely, directly—as things that come to us through the senses or as ideal beings discerned by intellect, and endeavoring instead, in the mode definitive of modernity, to operate with "symbols," understood as signs designating the derivative products of abstract reasoning but now deliberately used as if they themselves were primary things.[1] This seems to be an interpretation, at once potent and plausible, of our condition, our life as it is affected by modern science, which takes nature, including human nature, as mathematicizable, not primarily by means of the beautifully figurative geometry of the ancient astronomers but by the powerfully symbolic non-figurative algebra of the modern physicists.

It was not until I read Hopkins's book, however, that Klein's reach in delineating the moderns' renunciation of direct apprehension and imaginative concreteness in favor of manipulative facility and lawful generality came home to me with much distinctness.

Those just embarking on this study might be interested in a brief account of the situation in which I produced the translation that made Klein's work accessible in America under the title of *Greek Mathematical Thought and the Origin of Algebra*. I had been told by Seth Benardete, whom I succeeded at St. John's College in Annapolis in 1957, that, if I wanted to understand the intellectual basis of the Program then and now pursued at the school I was about to join, I must read Klein's "Die Griechische Logistik und die Entstehung der Algebra," published in two parts in 1934 and 1936, but even after the war scarcely available in this country. His advice was to the point. The St. John's Program is, as it were, an elementary preparation for the kind of "intentional history" realized in Klein's book. All students learn some Greek and French, study first ancient and then analytical geometry, read in

1. The new "analytical art," algebra, used these symbols in equations that were based on an adaptation of the ancient proof-method called "analysis," that is to say, "backward solution." The proof began by assuming the solutions sought and working back through the consequences to something admitted as true. In the modern adaptation, the unknowns (now symbolized as x, y, . . .) are involved in an equation on an equal footing with the knowns (a, b, . . .); thus, a manipulation that isolates the unknown on one side easily yields the solution in terms of the knowns on the other. The above renders, in simplified form, the outcome of a complex development in which by intricate stages the symbolic terms and the analytic method grow together and in which Vieta played the decisive role (§§ 97–106).

ancient and modern physics, replicate experiments—and do all this mostly through original texts: Euclid, Apollonius, Descartes, Newton, Einstein. All these studies are to be in aid of seminar discussions based on wide, chronologically arranged readings in the philosophical and literary tradition of the West. Moreover, the Program clearly has as one crux the early modern "Quarrel between the Ancients and the Moderns"; its problematic is welcomed as an issue for the students.

As for myself, I was German-speaking by birth, had studied classics and archaeology before I came to St. John's, and had worked my way through the mathematics part of the Program, from Euclid's *Elements* through Descartes's analytical geometry to Lobachevsky's *Theory of Parallels*, since teachers are expected to teach everything that students study in this all-required Program. So I was at least minimally prepared to tackle so immensely learned a book.

The rub was in Klein's willingness. The finding of the college had been doubly serendipitous for him: as a haven from the Nazis and as a place seemingly ready-made for his preoccupations. The Program indeed antedated his coming and what hand he had in it was mostly that of providing, *ex post facto*, its philosophical rationale. What really won his heart about the school was that it made the perfect venue for his pedagogic genius, whose exercise in Europe, where he had been a private scholar, was limited. The unabashed do-it-yourself directness encouraged in these naively intelligent American youths and their forthright, unmediated confrontation with the original texts were deeply congenial to him, as were they themselves. He began to regard the eliciting of thought from the young as more valuable than the production of scholarly books. This distancing of his own activity from the scholarly mode shows up in the "Author's Note" to the translation. Here he says that he would now have written in a less scholarly vocabulary and have taken a wider perspective on the change from the ancient to the modern mode of thinking.

He went further: he had a positive aversion to publishing. Since bringing alive texts, and particularly the Platonic dialogs, was a mission for him and for the school, I am persuaded that he was simply living out the attack on writing mounted in Plato's *Phaedrus*: "because those who put trust in writing recollect from the outside with foreign signs rather than themselves recollecting from within by themselves" (275a). This frame of mind may partly account for the second "scholarly curiosity" Hopkins observes: that Klein never explicitly refers to *Greek Mathematical Thought and the Origin of Algebra* in his American lectures.[2]

2. Perhaps his reluctance also had a little to do with an unfortunate earlier attempt at translation. The clueless young man had produced a title page—I saw it with my own eyes—

At any rate, my proposal to translate the work found so little encouragement that I set about the task surreptitiously, over the course of a year. When the thing was done, I had, of course, to bring my midnight labors to his notice so that I could ask the dozens of questions I had accumulated.[3] He quickly forgave my unauthorized activity and became, in fact, quite interested in the book's publication.

It is not entirely implausible that Klein's *Origin of Algebra* might, for all its scholarship, eventually play a role in the ever-current effort to revivify the liberal arts in American higher education. It is, after all, itself an example of the kind of inquiry that can be carried out only on the basis of broad liberal learning that unites language studies, ancient and modern, as well as mathematics and science with the discipline of letting the textual sources speak for themselves. Moreover, it is a model of the value of such learning to us, now, in achieving self-knowledge and self-orientation in our own speech.

If Klein had written an American version purged of scholarly terminology and the book's first expositor had simplified its argument, the influence of the work might have been more immediate and wider. Yet the illumination and conviction that arise from the meticulous detail of the argument would have been lost. It seems fortunate that Klein desisted and that Hopkins explicates *Greek Mathematical Thought and the Origin of Algebra* paragraph by paragraph, preserving the cumulative continuum that constitutes the stream of intentional history.

Since it is the history of a rupture—the transformation of *arithmos*, a counted assemblage of definite objects, be they sensible things or pure monads, into "number," a symbol signifying magnitude in general and designed to be involved in an equation (§§ 99–100)—the work has its high point. It occurs, or so it seems to me, in its second part, where the transition to the modern mode of concept-formation is treated.[4] Klein borrows the scholastic notion of intentionality to characterize the two-step operation by which Descartes

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to a book whose author was one "Jake Little." (Jacob Klein, a Russian Jew, was, incidentally, universally known as "Jasha.")

3. There is only one change of interest in the English version. 'Symbol-generating abstraction' is, of course, a key term. The German phrase was 'symbolic abstraction'. This, I argued, was at best ambiguous. "Abstraction" is a mode of conceptualization, the production by the intending mind of a derivative new object, the symbol, but "abstraction" is obviously not itself symbolic; Klein welcomed the new term.

4. There is, I think, a high point in the first, ancient part as well. It is Klein's interpretation, by way of establishing the concept of *arithmos*, of Aristotle's brief report that the Platonic forms (*eide*) were organized in arithmological assemblages (eidetic *arithmoi*) and were thus themselves *number-like:* unique assemblages of incomparable monads—a bold contradiction in ordinary terms (§§ 69–83). Klein's theory gives substantial clues to the structure of the Platonic forms individually and as a realm.

finally establishes the new concept of symbolic number with full clarity. The intellect first operates on an object given directly and primarily, an object of "first intention" (in this instance, a definite amount of definite units), to convert it into an abstract concept, a being of thought or of "second intention" (now a "mere" indeterminate multitude). In a second operation, performed not by the intellect but by the imagination, this derivative abstraction is in turn represented as primary in the form of an algebraic *symbol* (§§ 106, 111–15).

The recovery of the conceptual history of symbolic number, which leads to a coherent understanding of the mainstay of modern mathematics and mathematical science, is, as Hopkins shows, absent in Husserl's work, even from his later self-corrections. The last part of Hopkins's book is, consequently, largely devoted to establishing this case. Here, too, it is the detail that leads to depth, but I might venture a summary: While for Husserl the logical operations of symbolic mathematics work on an ultimately alien object opaquely abstracted or generalized from a perception-based counted collection (*Anzahl*), for Klein this object, symbolic number (*Zahl*), is intimately related to its method of calculation by that "symbol-generating abstraction" which produces a symbol that is then conceived as an actual object.

Jacob Klein often indicated that this result, the recovery of a new type of conceptualization, had the widest significance for the understanding of modernity, one of whose deepest features was the sometimes deliberate, more often thoughtless, choice of subjective construction over given object.[5] One might think that these arcana could scarcely bear so much real-life significance, but Husserl and Klein clearly agree on this matter, and Hopkins is working entirely in their spirit: Not impressionistic, broad strokes can explain us to ourselves but closely reasoned conceptual analysis of our terms and the patient desedimentation of the strata of occulted meanings that directed their genesis.

Hopkins, then, presents these conceptual events, whose topical exploitation might easily be oversimplified, with all care for detail and nuance. Thus, he positions *Greek Mathematical Thought and the Origin of Algebra* for playing its role in facing our epochal predicaments.

Eva Brann
Annapolis, 2008

5. As exemplified in the privileging, e.g., of method over matter, process over product, mentation over immediacy, formalism over content, experiment over experience, and, in general, in a preference for abstract terms over concrete words.

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I dedicate this book to my teachers, James Sheridan (in memoriam), Algis Mickunas, and Parvis Emad, and to my wife and soul mate, Olga Vishynakova.

Burt C. Hopkins
Seattle, 2010

Abbreviations

Caton	Caton, “Review of Jacob Klein’s *Greek Mathematical Thought and the Origin of Algebra*”
CM	Husserl, *Cartesian Meditations*
Crisis	Husserl, *The Crisis of European Sciences and Transcendental Phenomenology*
Early Writings	Husserl, *Early Writings in the Philosophy of Logic and Mathematics*
EJ	Husserl, *Experience and Judgment*
Frege	Frege, “Review of Dr. E. Husserl’s *Philosophy of Arithmetic*”
FTL	Husserl, *Formal and Transcendental Logic*
GMTOA	Klein, *Greek Mathematical Thought and the Origin of Algebra*
Ideas I	Husserl, *Ideas Pertaining to a Pure Phenomenology and to a Phenomenological Philosophy. First Book: General Introduction to a Pure Phenomenology*
ILI	Husserl, “Introduction to the *Logical Investigations*” (1913)
LI	Husserl, *Logical Investigations*
Origin	Husserl, “The Origin of Geometry” (F = Fink’s edition; K = Biemel’s edition; C = Carr’s translation)
PA	Husserl, *Philosophy of Arithmetic*
PHS	Klein, “Phenomenology and the History of Science”
Prolegomena	Husserl, *Prolegomena to Pure Logic*, in *LI*, vol. I
PRS	Husserl, “Philosophy as Rigorous Science”
Schröder	Husserl, “Review of Ernst Schröder’s *Vorlesungen über die Algebra der Logik*”
Semiotic	Husserl, “On the Logic of Signs (Semiotic)”
Stumpf Letter	Husserl, “Letter from Edmund Husserl to Carl Stumpf”
WP	Klein, “The World of Physics and the ‘Natural’ World”

The Origin of the Logic of Symbolic Mathematics

Introduction

The Subject Matter, Thesis, and Structure of This Study

This study is concerned with the origination of the logic of symbolic mathematics as investigated by Edmund Husserl and Jacob Klein. The 'logic' of symbolic mathematics at issue here is that which allows everyone—from barely literate school children to master mathematicians—to employ sense-perceptible letter signs, without a second thought, in a "mathematical" manner. The content of mathematics, like the content of its logic, is immaterial to its topic, which is how it has come about that such signs are self-evidently perceived to represent an "indeterminate" conceptual content as readily and unproblematically as, for example, the perception of the color and shape of this book.

What is responsible for this topic is uncontroversially referred to as 'formalization'. What formalization is, however, is controversial. At one extreme, formalization is understood as the employment of letter signs or other marks to, at the very least, "stand for" or "symbolize" any arbitrary object or content—"whatever"—belonging to a certain "domain." Let '3' stand for the number of any arbitrary objects whatever; let 'X' stand for any arbitrary number whatever; let 'S' stand for any arbitrary subject member of any proposition whatever—all these expressions are examples of formalization, and when "interpreted" in a manner that finds nothing especially problematic to speak of here, these examples illustrate pretty much all that is needed—or the minimum needed—to begin formalization. At the other extreme is the view that formalization is the fulcrum for an unprecedented transformation in how the science of the so-called West forms its concepts, a transformation that is as all-encompassing as it is invisible to this day—especially to those who study the history of this science or are engaged in scientific inquiry.

The present study has as its subject matter the latter understanding of formalization. In it we shall investigate the major work of its first proponent, the twentieth-century historian and philosopher of mathematics, Jacob Klein. The work in question is entitled "Die griechische Logistik und die Entstehung

der Algebra," which was originally published in two parts in 1934 and 1936 and then in English translation as *Greek Mathematical Thought and the Origin of Algebra* in 1968.[1] Klein's major thesis there is that the history of the transformation of what he calls the "conceptuality" of the most basic concept employed by science, that of number, from a non-conceptual and non-linguistic multitude of determinate things to a concept that is identical with a symbolic language, is inseparable from the meaning of the symbolic employment of letter signs—from the most elementary, such as '2', to the most universal, say, 'X'.

This history is important for Klein because it discloses that the conceptuality of the symbolically transformed concept of number represents, in a paradigmatic way, a radically different (and philosophically significant) apprehension of things from how they were apprehended before that transformation. Prior to it, things were apprehended "directly," first through the senses and then through the employment of concepts that were apprehended as different from both the things whose apprehension they permitted and from the language that made use of concepts in order to bring about this apprehension. Subsequent to that transformation, things are apprehended "indirectly," through the mediation of both the concepts and language that now "define" them. In other words, Klein's thesis is that before that historical transformation, knowledge of the "being"—however mysterious or unknown—of things was incapable, in principle, of being identified with the concepts and language employed to apprehend them, while, subsequent to it, knowledge of the "being" of things—again, however mysterious or unknown—is approached only through the concepts and language used to apprehend them. Or put more succinctly: Klein's thesis is that prior to this transformation what things are was not understood to be conceptual and linguistic, while now it is.

Connected with this thesis is Klein's sub-thesis that this transformation in conceptuality has gone unnoticed because, simultaneously with its occurrence, the conceptuality it superseded was, unwittingly, apprehended from the conceptual level engendered by its transformation. On his view, the conceptuality of number both illustrates this and is the paradigm for the transformation. Prior to its symbolic transformation, 'number' was identical with a determinate amount of a multitude of determinate things. The concepts of number, such as the 'odd' and the 'even', were employed to apprehend numbers, non-conceptual beings that are both one and many, but these concepts were not understood to be numbers, that is, to be determinate

1. See the Bibliography for complete bibliographic information on works referred to here, as well as the conventions followed in citing texts throughout this study.

amounts of a multitude of determinate things. Subsequent to its symbolic transformation, number is no longer identical with a determinate amount of a multitude of determinate things, but with the concept of a determinate amount of a multitude of things or, more concisely, with the concept of a multiplicity. As a concept, 'number' is no longer identical with non-conceptual things in their multiplicity but rather with the letter sign that now "represents" its concept. However, because this concept and its representation are understood, simultaneously, to refer (indirectly, to be sure) to the multitude of things identical with the pre-symbolic number, the "being" of the latter is now taken to be, as a matter of course, the symbol that represents it.

The stated purpose of Klein's study of Greek mathematical thought and the origin of algebra is to take note of this transformation and therefore make it visible, as a propaedeutic to the philosophical exploration of its broader significance for exact science and philosophy. From the standpoint of the traditional practices of the history of exact science and philosophy, both when the study was first published and still today, the novelty of Klein's thesis, which is inseparable from its radicality, presents formidable challenges to those who would comprehend it. The proposition that an abstract idea like number is subject to historical change strikes at the heart of the most basic presupposition of the dominant conception of exact science, namely, that its basic concepts are essentially a priori and therefore timeless or, at the very least, invariant through time. Moreover, the methodological enlistment of history—which, as an empirical discipline, is devoted to "facts" whose very meaning, as facts, is that they could be otherwise and thus seem to be contingent or accidental—is hardly suited to an investigation of the "ideal" truths of mathematics. Rather than address these and other potential methodological objections, Klein's study, guided by the thesis that the conceptuality responsible for symbolic numbers is first found in François Vieta's invention of modern algebra as an "analytical art" in the sixteenth century, discloses the character of the conceptual transformation responsible for this innovation by presenting in its proper context the Greek mathematics that formed its conceptual horizon.

Klein accomplishes this by approaching, for the first time in the history of mathematics, the concepts of both Greek mathematics and the Greek philosophy of mathematics in their own terms. This means that rather than assume the universal mathematical applicability of the conceptual level represented by the modern symbolic mathematics made possible by Vieta's invention of algebra, Klein returned to the original Greek sources and rediscovered the basic arithmetical evidence that guided their concept formation. The recovery of that evidence permitted Klein to re-construct the Greek concept of number (ἀριθμός), independently of the conceptuality of the modern symbolic

concept of number, as a "determinate number" in the exact sense of a definite amount of definite items. And it also permitted him to re-construct the conceptual transformation that occurred with respect to precisely this concept of number when Vieta employed a *logistice speciosa*, a method that calculates with the "species-symbols" of "magnitude in general" rather than with determinate numbers, as part of his analytical art.

Klein's study broadens its philosophical perspective by showing the connection of Vieta's analytical art with Descartes's and John Wallis's development of the *mathesis universalis*. Again employing the original sources, Klein documents the eclipse of the fundamental ontological concerns of Greek science, especially the highest sciences of Platonic dialectic and Aristotelian first philosophy, in Descartes's and Wallis's writings. Klein shows that both Descartes's identification of the substance of the world, extension, with the symbolic subject matter of his "analytic geometry" and Wallis's absorption of the ratios and proportions of Euclidian geometry into an arithmetical "analytical art" conceived of entirely in symbolic terms, obviate, in "one stroke," the Greek methodological preoccupation with the mode of being proper to the objects investigated by means of the cognitively general methods of mathematics and philosophy. Especially the specific problem encountered by these general methods, namely, the nature of the "unity" belonging to mathematical and other kinds of multitudes, is eliminated with the symbolic rendering of unity enabled by the analytical art, and definitely so when this art is identified with symbolic mathematics in the "pure" *mathesis universalis*. The "pure unity" of the latter is now entirely symbolic, which means that its mode of being is identical with the unambiguous meaning of the symbols employed by its symbolic calculus.

In 1935 and 1936, Edmund Husserl wrote two papers that drew a connection between the formalization that characterizes the mathematics of modern physics and the crisis of European humanity, "The Crisis of European Sciences and Transcendental Phenomenology" and "The Origin of Geometry as an Intentional-Historical Problem." He proposed as a response to this crisis a radical "historical reflection" on the Galilean origin of modern physics and the Greek origin of the geometry Galileo employed. The goal of Husserl's reflection was to "reactivate" the original evidence that led to the establishment of these sciences and thereby to clarify their genuine meaning, which Husserl was convinced had become lost in the impulse toward formalization initiated by Galileo and in that toward idealization initiated in the Euclidean geometry relied upon by the Galilean impulse. Husserl remarked that never before had epistemology been viewed as a peculiarly historical task, and in fragmentary historical reflections he attempted to reawaken the evidence that led

to the original accomplishments that "anticipated" the meanings of these sciences, evidence that he maintained was somehow still present—though obscured due to what he termed 'sedimentation'—in the contemporary meanings that compose these sciences.

Husserl worked out the nature of the method required to undertake these "epistemological-historical" investigations, which he characterized as a "zigzag" reflection that moves from the present meaning of a science to historically prior meanings, then back again to the present meaning, all with the goal of "reactivating" in the prior meanings evidence that anticipated the contemporary meaning. He also worked out the philosophical basis of these investigations, distinguishing the "historiography" of contingent facts, which is rooted in an empirical science, from the "historicity" of a cultural tradition, including the cognitive achievements of a science, which (in the case of science) has as its medium the textual embodiment of meaning made possible by written language. Because the meanings embedded in this medium can be accessed only on the basis of a cognitively intentional relation that brings them to evidence, Husserl characterized the radical epistemological-historical reflection required to de-sediment and reactivate the original meanings of a science as an "intentional investigation" of their history. Eugen Fink, Husserl's assistant and close collaborator and the original editor of the two papers just mentioned, made explicit the connection between intentionality and history in them with the phrase 'intentional-historical', which he included in the title he gave to Husserl's second essay.

In 1940 Klein published "Phenomenology and the History of Science," which was the first discussion in the literature on Husserl's two essays. There he showed that, far from signaling a relapse into "historicism," Husserl's last essays represent an internally consistent deepening of his phenomenology's guiding concern, from beginning to end, with the problem of the non-empirical "origins" of cognitive meaning. Klein linked Husserl's programmatic announcement and execution of the epistemological-historical "reactivation" of sedimented meanings in these essays to Husserl's recognition of the need to deepen the genetic analyses of the meaning of logical objectivity presented in *Formal and Transcendental Logic* in order to get at its phenomenological origin. By establishing this linkage, Klein demonstrated that he saw more clearly than Husserl's other commentators that Husserl's late turn to history was not related to historicism's basic thesis of the historical contingency and therefore relativity of human knowledge. But Klein also "corrected" Husserl's account of the content of the historical reflection needed to reactivate the "sedimented history" of the origin of mathematical physics by adding a third task to the two enunciated by Husserl. In addition to the tasks of de-sedimenting

and reactivating the original evidence sedimented in 1) the geometry relied upon by the Galilean impulse to mathematization that led to the anticipation of physics as an "exact" science and 2) the evidence connected with this anticipation itself, Klein also identifies 3) the task of de-sedimenting and reactivating the original evidence that led to Vieta's establishment of modern algebra.

Klein's sketch of the third task seamlessly weaves a concise synopsis of the results of the mathematical-philosophical investigations in his *Greek Mathematical Thought and the Origin of Algebra* into the fabric of Husserl's articulation of the epistemological-historical methodology belonging to the *Crisis*'s project of de-sedimenting the origins of modern mathematical physics. Oddly enough, however, Klein makes no mention of his study in his 1940 essay and therefore the fact that this task had already been accomplished.

The scholarly curiosity of Klein's elision of what can only be characterized as his own contribution to the phenomenologically historical investigation of the origin of mathematical physics is compounded by his study's lack of reference to Husserl's phenomenological investigations of the origin of symbolic mathematics. The superficial similarity of their investigations, especially Husserl's *Philosophy of Arithmetic* but also his *Logical Investigations*, has led commentators to assert both Husserl's priority over and influence on Klein's investigations. Indeed, detailed study of Husserl's *Philosophy of Arithmetic* reveals not only that Husserl, like Klein, draws a basic distinction between determinate and symbolic numbers but also that they each arrive at the same conclusion regarding the independent origins of such numbers.

Careful study of these investigations discloses, however, that the undeniable similarity of Husserl's and Klein's accounts of the nature of and relationship between non-symbolic and symbolic numbers is underpinned by a fundamental difference that gets to the heart of their radically different accounts of the origination of the "logic" of symbolic mathematics. This difference is located in their respective accounts of the mathematical concept of unity. Husserl's account involves what, from the standpoint of Klein's study, can only be characterized as a major equivocation.

On the one hand, Husserl understands 'unity' in terms of its mathematical and logical function in the "pure" *mathesis universalis*, and thus as a formalized concept with absolutely no individual or materially generic content. On the other hand, he understands 'unity' to be that which is responsible for the perceptual apprehension of an object as an individual and thus as a concept that has individual and material ontological content. From the perspective of Klein's study, Husserl's equivocation becomes problematic when he attempts to account for the origin of the formalized meaning of unity on

the basis of a modification of the unity characteristic of its perceptual meaning. The source of this equivocation proves to be that Husserl's attempt relies upon Aristotelian abstraction and presupposes that this manner of abstraction has the ability to generate a concept of unity that is formalized.

The guiding thesis of the present study is twofold: that Klein's historical-mathematical investigation of the origin of algebra seeks to demonstrate the impossibility of an Aristotelian origin of the unity that makes possible the logic of symbolic mathematics, and that it is Husserl's articulation of the methodology belonging to the de-sedimentation of the sedimented meaning responsible for the origin of an exact science and the reactivation of the evidence in which this meaning is originally given that makes Klein's demonstration philosophically compelling.

The study's structure is dictated by its content and thus is divided into four unequal parts.

Part One treats Klein's interpretation of the historical character of Husserl's investigations in the *Crisis* by providing a detailed explication and analysis of their separation by Klein from historicism and his articulation of the phenomenological continuity of Husserl's genetic and "intentional-historical" investigations of the constitution of the unity proper to meaning in the exact sciences.

Part Two provides an overview of Klein's historical-mathematical study and substantiates the claim made in Part One that the third task he adds to Husserl's project of de-sedimentation had in fact already been completed in Klein's own study on Greek mathematics and, moreover, executed there using a method that is in implicit accord with the methodology laid out in Husserl's *Crisis*.

Part Three begins with a detailed exposition and analysis of Husserl's investigation of the nature of and relationship between non-symbolic and symbolic numbers in his *Philosophy of Arithmetic*. This discussion is followed by an equally detailed exposition and analysis of Klein's presentation of this relationship in his *Greek Mathematical Thought and the Origin of Algebra*. The detailed level of exposition and analysis of both of these studies is required, in part, by the relative lack of discussion of these texts in the literature, especially in relation to the main topic of concern, the origination of the logic of symbolic mathematics.

Part Four begins with a detailed comparative analysis of the structure and origin of non-symbolic numbers in Husserl's *Philosophy of Arithmetic* and Klein's *Greek Mathematical Thought and the Origin of Algebra*. At the conclusion of this analysis, a major digression is presented, the topic of which is Husserl's account of the origination of the logic of symbolic mathematics as

he elaborated it after *Philosophy of Arithmetic*. The purpose of this digression is to "correct" two prevalent standard views of the development of Husserl's thought subsequent to *Philosophy of Arithmetic*: 1) that his doctrine of "categorial intuition" overcomes the latter's psychologism in its account of the origin of the "collective unity" that composes non-symbolic numbers, and 2) that Husserl's analyses in *Formal and Transcendental Logic* present a mature phenomenological theory of judgment that provides the foundation for both the distinction and the unity of the formal logic and formal mathematics that make up the "pure" *mathesis universalis*. The results of this digression, together with those of the initial sections of Part Four, form the basis of the final analysis of Husserl's and Klein's respective accounts of the origination of the logic of symbolic mathematics. That analysis concludes with a synoptic account of the historicity of formalization on the basis of its origination in the pre-scientific life-world.

Following Part Four is a brief coda, in which we situate Husserl's logical "Platonism" in the context of Plato's own mathematical and eidetic "Platonism." On the one hand, Husserl's Platonism, which is connected with his "break" with psychologism, is distinguished from Platonism's typical formulation as the straightforward thesis of the existence of mind-independent ideal mathematical objects. On the other hand, it is distinguished from the supposition (in Plato's own Platonism) of the non-mathematical, eidetic being of the "one" that is responsible for the defining characteristic of the "one over many" unity proper to mathematical objects. Finally, Husserl's attempt to separate his thought from psychologism by appealing to the "numerical identity" of the content of logical cognition is shown to presuppose the eidetic being of the "one" supposed by Plato's own Platonism.

Part One

Klein on Husserl's Phenomenology and the History of Science

Chapter One

Klein's and Husserl's Investigations of the Origination of Mathematical Physics

§ 1. The Problem of History in Husserl's Last Writings

Some seventy years have passed since the first publication of two fragmentary texts on history and phenomenology that Husserl wrote in his last years,[1] texts that unmistakably link the meaning of both the crowning achievement of the Enlightenment (the new science of mathematical physics) and that of his own life's work (the rigorous science of transcendental phenomenology)

1. Edmund Husserl, "Die Krisis der europäischen Wissenschaften und die transzendentale Phänomenologie. Eine Einleitung in die phänomenologische Philosophie," *Philosophia* I (1936), 77–176 (reprinted as §§ 1–27 of *Die Krisis der europäischen Wissenschaften und die transzendentale Phänomenologie. Eine Einleitung in die phänomenologische Philosophie*, ed. Walter Biemel, Husserliana VI [The Hague: Nijhoff, 1954; 2d ed., 1976]; henceforth cited as '*Hua* VI' where reference is specifically to the German edition) and "Die Frage nach dem Ursprung der Geometrie als intentional-historisches Problem" [The Question Concerning the Origin of Geometry as an Intentional-Historical Problem], ed. Eugen Fink, *Revue internationale de Philosophie* I (1939), 203–25; subsequently republished in newly edited form as Beilage III in *Hua* VI, 365–86, here 379; English translation of the latter: "The Origin of Geometry," in *The Crisis of European Sciences and Transcendental Phenomenology*, trans. David Carr (Evanston, Ill.: Northwestern University Press, 1970), 353–78, here 370. Because the only version of the second text available to Klein when he wrote "Phenomenology and the History of Science" (see n. 3 below) was Fink's edition, page numbers will refer wherever possible to this text and then to Carr's translation. Where the deviation between Fink's edition and Biemel's precludes reference to Carr's translation of the latter, reference will be exclusively to Fink's text, in which case all translations will be mine. Exclusive reference to Biemel's edition and Carr's translation of it will signal the absence of the relevant passage in Fink's edition. To make these differences more readily perspicuous to the reader, references to the pagination of Fink's edition will be immediately preceded by 'F', whereas those to the pagination of Biemel's edition will be preceded by 'K' (i.e., *Krisis*); the English translation will be preceded by 'C' whenever it is Carr's. Note that where at issue is the main text of the *Crisis*, it is cited as *Crisis* with German and English page references, respectively.

See the prefatory note to the Bibliography for a clarification of the conventions used here in citing texts.

to the problem of their historical origination. It is striking that in the years following the original publication of these works and their republication in 1954 in Walter Biemel's Husserliana edition of the *Crisis*, commentary on them has, with one significant exception, passed over what Husserl articulated as the specifically *phenomenological* nature of the problem of history. It has been ignored in favor of mostly critical discussions of Husserl's putative attempt to accommodate his earlier "idealistic" formulations of transcendental phenomenology to the so-called "problem of history."

As it is typically understood, this problem begins with the notion of a contingent sequence of events that shape cultural formations and human experience in a manner that defies rational calculation. History in this sense becomes a problem when its contingency is understood to condition the intellectual content of cultural formations, such as philosophy and science. The problem here concerns the influence of the historically conditioned heritage of taken-for-granted ideas, meanings, and attitudes on the knowledge claims made by philosophy and science. When the intellectual content of the latter is understood to have as its insuperable limit the particular historical situation of the philosopher and the scientist, as well as of philosophy and science, the knowledge claims of both are correspondingly understood to be incapable of ever achieving "universality." Formulated in this manner, the "problem of history" assumes, as is well known, the guise of what since the beginning of the twentieth century has been called 'historicism'.

The reception of Husserl's last works has been preoccupied with the story of their departure from his own early rejection of historicism and his late attempts to establish what many have deemed oxymoronic and therefore impossible: a phenomenology of the apriori proper to the historical origination of meaning. Motivated by the goal of establishing phenomenology as a presuppositionless universal science of a priori meanings, Husserl's early thought had identified the "facticity" of history as among those presuppositions standing in the way of a "pure" phenomenology. Husserl's late turn to the problem of history has therefore led many to suspect that pure phenomenology and the historical preoccupation of his last texts are intrinsically incompatible.

§ 2. The Priority of Klein's Research on the Historical Origination of the Meaning of Mathematical Physics over Husserl's

Part One of this study is concerned with the major exception to the trend in the literature to overlook the significance assigned to history in Husserl's *Cri-*

sis alluded to above, namely, the work of Jacob Klein.[2] Its twofold aim is to elaborate Klein's understanding of the phenomenological problem of history sketched by Husserl in his last works[3] and to introduce Klein's own contribution to the understanding of the problem of the historical origination of the meaning of mathematical physics. The latter's contribution occurs in his little known but remarkable works on Greek mathematics and the origin of algebra.[4] On the assumption that Klein's contribution to that understanding came after his appropriation of Husserl's formulation of the phenomenological problem of history, the execution of this twofold aim would seem to be a fairly

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2. The only other thinker to pay sustained attention to the theme of history in Husserl's *Crisis* was Jacques Derrida, in his introduction to Edmund Husserl, *L'Origine de la géométrie*, trans. Jacques Derrida (Paris: Presses Universitaires de France, 1962; 2d ed., 1972); English translation: Jacques Derrida, *Edmund Husserl's Origin of Geometry: An Introduction*, trans. John. P. Leavey, ed. David B. Allison (Lincoln, Nebr.: University of Nebraska Press, 1989). Despite the merits of his discussion, Derrida fails to identify what Klein alone among Husserl's commentators correctly recognizes as the crux of the *phenomenological* problem of history in Husserl's late texts, namely, the formalization of the mathematical "language" of natural science accomplished by modern mathematics. We shall show that it is precisely Husserl's encounter with the crisis posed by the "unintelligibility" of the formalized language of modern mathematics and mathematical physics—where "intelligibility" is measured by our pre-formalized encounter with the world—that engenders his method of historical reflection on the origin of modern science. For a critical discussion of Derrida on this point, see Burt C. Hopkins, "Klein and Derrida on the Historicity of Meaning and the Meaning of Historicity in Husserl's *Crisis*-Texts," *Journal of the British Society for Phenomenology* 36, no. 2 (2005), 179–87. For a discussion of the merits of Derrida's discussion, see Joshua Kates, "Modernity and Intentional History: Edmund Husserl, Jacob Klein, and Jacques Derrida," *Philosophy Today* 49, no. 5 (Supplement 2005), 193–203.

For an instructive survey of the *Crisis* and the texts it comprises, see Ernst Wolfgang Orth, *Edmund Husserls 'Krisis der europäischen Wissenschaften und die transzendentale Phänomenologie'. Vernunft und Kultur* (Darmstadt: Wissenschaftliche Buchgesellschaft, 2001).

3. Jacob Klein, "Phenomenology and the History of Science," in Marvin Farber, ed., *Philosophical Essays in Memory of Edmund Husserl* (Cambridge, Mass.: Harvard University Press, 1940), 143–63; reprinted in Jacob Klein, *Lectures and Essays*, ed. Robert B. Williamson and Elliott Zuckerman (Annapolis, Md.: St. John's Press, 1985), 65–84. The later version will be cited in what follows as *PHS* with page reference.

4. See Jacob Klein, "Die griechische Logistik und die Entstehung der Algebra," *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, Abteilung B: *Studien*, vol. 3, no. 1 (1934), 18–105 (Part I), and no. 2 (1936), 122–235 (Part II); English translation: *Greek Mathematical Thought and the Origin of Algebra*, trans. Eva Brann (Cambridge, Mass.: MIT Press, 1969; reprint: New York: Dover, 1992); henceforth cited as *GMTOA*, with German and English page references, respectively (exceptions will be noted), and occasionally referred to simply as *Origin of Algebra*. See also Jacob Klein, "The World of Physics and the 'Natural' World," ed. and trans. David R. Lachterman, in *Lectures and Essays*, 1–34; henceforth cited as *WP*. (This German original of this text was delivered as a talk at the Physikalisches Institut of the University of Marburg on February 3, 1932. The manuscript, which was not published during Klein's lifetime, appears to have been lost.) See the following studies from Klein's *Lectures and Essays*: "On a Sixteenth Century Algebraist [Simon Stevin]" (35–42), "The Concept of Number in Greek Mathematics and Philosophy" (43–52), and "Modern Rationalism" (53–64).

straightforward matter. One would need only to show how the method and content of Husserl's path-breaking investigations influenced or otherwise provided the context for Klein's own research. However, Klein's work on the historical origination of the meaning of mathematical physics actually preceded Husserl's work on this same issue by a number of years.[5] Thus, Hiram Caton's felicitous characterization—in another context, and one that will be taken up shortly—of Klein's relationship to Husserl as "a scholarly curiosity"[6] proves apt here as well, since Klein's work on the history of mathematics represents an uncanny anticipation of Husserl's own work.

In 1959 Leo Strauss characterized Klein's magnum opus, "Die griechische Logistik und die Entstehung der Algebra," then still untranslated, as a work that is "much more than a historical study."[7] Strauss continued: "But even if we take it as purely a historical work, there is not, in my opinion, a contemporary work in the history of philosophy or science or in 'the history of ideas' generally speaking which in intrinsic worth comes within hailing distance of it. Not indeed a proof but a sign of this is the fact that less than half a dozen people seem to have read it, if the inference from the number of references to it is valid." Strauss's characterization of this work as "much more than a historical study," along with his comparison of it—without limiting it—to both the "history of philosophy" and the "history of ideas," is instructive here. For while it claims that Klein's treatment of his topic is of unparalleled historical import, the cryptic suggestion that its true significance transcends contemporary studies in the history of philosophy or science, as well as studies in the history of ideas generally, gives occasion to formulate a major thesis of the present study: that both the methodology and the content of Klein's mathematical studies fall outside the traditionally distinct methodological approaches to the likewise traditionally distinct domains staked out, respectively, by the history and the philosophy of science. Before developing

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5. As indicated in the previous note, Klein's "The World of Physics and the 'Natural' World" was given as a talk in 1932, and his "Die griechische Logistik und die Entstehung der Algebra" was published in two parts, in 1934 and 1936, respectively. Husserl began working on the *Crisis* in 1934, whereas his work on the origin of geometry dates from 1936; see the editor's introduction to Edmund Husserl, *Die Krisis der europaischen Wissenschaften und die transzendentale Phänomenologie. Ergänzungsband. Texte aus dem Nachlass 1934–1937*, ed. Reinhold N. Smid, Husserliana XXIX (Dordrecht: Kluwer, 1992), xi and lvi. The former was first published, in part, in 1936 and the latter, posthumously in 1939 in Fink's edition. See n. 1 above.

6. Hiram Caton, "Review of Jacob Klein's *Greek Mathematical Thought and the Origin of Algebra*," *Studi Internationali di Filosofia* 3 (1971), 222–26, here 225; henceforth cited as 'Caton' with page reference.

7. Leo Strauss, "An Unspoken Prologue to a Public Lecture at St. John's [In Honor of Jacob Klein, 1899–1978]," *Interpretation* 7 (1978), 1–3, here 3. Strauss penned these remarks in 1959, on the occasion of Klein's sixtieth birthday.

this thesis within what here will be argued to be the proper context for considering both the method and the content of Klein's mathematical studies, it is necessary to digress briefly so as to situate this context in relation to how the methods and the contents of the history of science and the philosophy of science are typically understood to differ. The goal thereby is to provide a context in contrast to which the radicality of Klein's approach to both historical and systematic issues in his mathematical studies can be demonstrated.

With respect to method the difference in question here concerns the traditional contrast between the "empirical" approach to science characteristic of the history of science and the "epistemological" approach characteristic of the philosophy of science. Accordingly, the history of science is usually defined by its investigation of the contingent series of mathematical, scientific, and philosophical theories involved in the formation and development of a given science. By contrast, the philosophy of science is usually defined by its investigation of the cognitive status of the philosophical problems posed by the employment of logic, mathematics, and metaphysics in the knowledge claims advanced by the systematic sciences. Corresponding to these methodological differences are the differences in content of the domains typically treated by the historical and the philosophical investigations of science. Thus, the content of the history of science reflects the changes over time that mark the development of a science, whereas the content of the philosophy of science reflects the temporal stability that defines scientific knowledge.

§ 3. The Importance of Husserl's Last Writings for Understanding Klein's Nontraditional Investigations of the History and Philosophy of Science

Rather than work within the context of this traditional understanding of the difference and indeed opposition between these methods and their domains, Klein's mathematical studies are characterized by a method—albeit one that largely remains implicit—that overcomes the opposition between historical explanation and epistemological investigation in the study of science. His studies are thus historical without being limited to empirical contingencies and epistemological without being cut off from the historical development of scientific knowledge. In other words, Klein's work overcomes the problem of history that leads to historicism by showing, in effect, that the disclosure of the "historicity" of scientific knowledge does not lead to an opposition between the contingency of history and the universality of knowledge. His work shows this by uncovering the heritage of ideas, meanings, and attitudes that underlie the basic concepts of the modern mathematics that makes mathematical

physics possible; that is, he uncovers aspects of what Husserl will refer to as the "historical apriori" (*Origin*, K380/C375) of modern physics. Yet it is Husserl who in his last works was the first to articulate explicitly the methodological issues involved in overcoming the opposition in question here. The assessment of both i) the scope and limits of Klein's implicit method and ii) the cogency of its results must take Husserl's reflections on this methodology as its point of departure. Husserl's later articulation of the "theory of knowledge . . . as a peculiarly historical task" (F220/C370), a task he assigns to his final formulation of transcendental phenomenology and its now defining goal of overcoming "[t]he ruling dogma of the principial separation between epistemological elucidation and historical explanation" (ibid.), provides the proper perspective from which to assess Klein's work. It is Husserl's formulation of the "universal apriori of history" (K380/C371) as "nothing other than the vital movement of the coexistence and the interweaving of original formations and sedimentations of meaning" (F221/C371) that serves as the "guiding clue" for overcoming the "ruling dogma" in question. The methodology that discloses this "vital movement" thus is indispensable for taking the measure of Klein's investigations, and it is to be found in Husserl's sketch of phenomenologically historical reflection. Husserl characterizes such reflection in terms of a "'*zigzag*' back and forth" from the "'breakdown' situation of our time, with its 'breakdown of science' itself," to the historical "*beginnings*" of both the original meaning of science itself (i.e., philosophy) and the development of its meaning leading up to the "breakdown" of modern mathematical physics (see *Crisis*, 59/58).

§ 4. Klein's Commentary on Husserl's Investigation of the History of the Origin of Modern Science

Klein himself provides the warrant for this account of the significance of Husserl's methodology for understanding his own mathematical studies in his article "Phenomenology and the History of Science" from 1940. After first explicating Husserl's articulation of the phenomenological problem of history in the original published versions of the *Crisis* and "The Origin of Geometry," Klein goes on to outline "[t]he problem of the origin of modern science" (*PHS*, 82) in a manner that corresponds to Husserl's formulation of the problem, save for one significant deviation. There Klein adds a third task to the two tasks that Husserl articulates in connection with this problem. Whereas for Husserl the problem of the origin of modern science involves the "reactivation of the origin of geometry" (83) and "the rediscovery of the prescientific world and its true origins," (84) according to Klein there is yet another aspect

to this problem. He articulates this aspect in terms of "a reactivation of the process of symbolic abstraction" (83) whose "'sedimented' understanding of numbers is superposed upon the first stratum of 'sedimented' geometrical 'evidences'" (83–84). Klein therefore positions this additional task between the twin tasks that Husserl articulates in the *Crisis.*[8]

Klein's introduction of this third task is significant for a number of reasons, all of which will be taken up here in due course. At this point, however, only one requires comment, namely that the task of the "reactivation of the process of symbolic abstraction" had in fact already been undertaken and indeed completed by Klein himself in "Die griechische Logistik und die Entstehung der Algebra." There can be no mistake about this. In the final section of his "Phenomenology and the History of Science" (see 79–83), Klein presents a synopsis of the development of the symbolic transformation of the traditional Greek theory of ratios and proportions, as well as of the ancient Greek "concept" and science of number, into François Vieta's "'algebraic' *art* of equations" (80). In addition, he discusses the "formalization" of Greek mathematics that was prepared for with the "anticipation" of an exact geometrical nature by Galileo and his predecessors and realized with the symbolic transformation of Euclidean geometry into Descartes's analytic geometry—the latter being made possible by Vieta's "invention" of modern mathematics. The formalization of Greek mathematics, upon which are "laid the foundations of mathematical physics" (82), is said by Klein to "have already lost the original intuition" (81) of the Greek mathematics underlying it. He traces this loss to modern mathematics' technique of operating with symbols. As a result of this, the "reactivation of the process of symbolic abstraction" (84) that makes possible the formalization of the mathematics that prepares the way for mathematical physics is held by Klein to involve, "by implication, the rediscovery of the original arithmetical evidences." For him these original evidences concern "the original 'ideal' concept of number, developed by the Greeks out of the immediate experience of 'things' and their prescientific articulation" (81).

What Klein lays out in this synopsis amounts to a précis of the "argument" of his work on Greek mathematics and the origin of algebra from

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8. Husserl is, of course, well aware of the importance of "thinking with . . . 'symbolic' concepts" (*Crisis*, 48/48) for the origination of mathematical physics, and Klein is aware that he is aware of this (see *PHS*, 81–82 nn. 43–44, 46–48, 50, 52, where Klein cites Husserl's discussion in the *Crisis* of both the importance of the "formalization" of arithmetic and geometry for the new physics and the resultant emptying of the original intuitive meaning of these disciplines). However, Husserl nowhere explicitly articulates the task of reactivating the original evidence that is sedimented in symbolic concepts and the calculational techniques that operate with them, as he does in the case of the sedimented meanings characteristic of the "idealization" of geometry.

1934–36. This fact calls attention to a second "scholarly curiosity" characteristic of Klein's relationship to Husserl, namely, his failure to provide any reference to that work in an article that articulates—in effect—both the historical design and the philosophical significance of its results in terms of Husserl's transcendental phenomenological formulation of the problem of history. In other words, in that article Klein situates his mathematical studies within the context of Husserl's understanding of the theory of knowledge as a historical task, the peculiar character of which is bound up with the phenomenological characterization of the "interlacement of original production and 'sedimentation' of significance[9] [that] constitutes the true character of history" (78).[10] This curiosity is compounded by the reference to this article in the 1968 English translation of his magnum opus, *Greek Mathematical Thought and the Origin of Algebra*.[11]

§ 5. The "Curious" Relation between Klein's Historical Investigation of Greek and Modern Mathematics and Husserl's Phenomenology

This second "scholarly curiosity" provides occasion to discuss a third and final curiosity, the context for which is provided by Hiram Caton's characterization of the relation of Klein's thought to Husserl's. In the aforementioned review of Eva Brann's English translation of Klein's book, Caton remarked upon Klein's "failure to cite Husserl as the source of his Husserlian terminology" (Caton, 225), that is, the terminology of the "theory of symbolic thinking" and the "concept of intentionality." It is Caton's contention that precedence for both of these should go to Husserl. In the case of the former, he appeals to Husserl's "remarkably similar theory in the *Logische Untersuchungen* (Vol. II/1, par. 20)." In the case of the latter, he points to how, "by citing the scholastic Eustachias as illustrating the sources of the thinking of Vieta and Descartes," Klein "ingeniously capitalizes on . . . [the] genealogy" of intentionality, which Husserl took "from Brentano, who in turn took it from medieval logic."

Before commenting on Caton's claims in light of our own estimation of Klein's relation to Husserl, it should be mentioned that the context of his remarks concerns the "question of the purpose and achievement of Klein's

9. Klein's rendering of *Sinn* as 'significance' or 'significant' will be followed in Part One in order to avoid the awkwardness that would be introduced to the text by changing the large number of citations of this word to its preferred translation as 'meaning'. In the remainder of this study, however, *Sinn* will be consistently translated as 'meaning'.

10. This is Klein's paraphrase of Husserl's statement found in *Origin*, F220.

11. Klein, *GMTOA*, 118 (the reference occurs only in the English translation).

historical method." Caton situates this question in terms of his judgment regarding the "overpowering scholarship of its [i.e., Klein's book's] argument" (222), a judgment he buttresses with the remark that "Klein's erudition is so great and thorough that one imagines that there are few men living who can move familiarly on his terrain: certainly I am not one of them" (226).[12]

Regarding the purpose and achievement of Klein's historical method, Caton maintains that while Klein's bypassing of Husserl's concept of intentionality in favor of the medieval Eustachias "lends plausibility to his claim to interpret these Renaissance figures [i.e., Vieta and Descartes] in accordance with the conceptuality of their *milieu*" (225), what Klein actually does is project "Husserl back upon Vieta and Descartes via Eustachias." Thus, Caton concludes that "although Klein's analysis of symbolic abstraction is acceptable, his claim to find it in the *self-conscious* thought of Vieta, Descartes and Stevin produces no conviction."

However, Caton is wrong in both the essentials and the details of his assessment of the significance of Klein's relation to Husserl, despite the acuity of his discernment of what indeed is "a scholarly curiosity" with respect to Klein's "failure" to acknowledge both Husserl's theory of intentionality and his theory of symbolic thinking. To begin with, the original "precedence"[13] for the "theory of symbolic thinking" is to be found in Husserl's sustained investigation of the relationship between the "authentic" and "symbolic presen-

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12. In *The Origins of Subjectivity* (New Haven, Conn.: Yale University Press, 1973), 224, Hiram Caton also avers that "[f]or understanding Descartes' mathematical background, and how it leads to his *Geometry* and to his stress on the mind's 'turning to itself' method, utility, and art, Klein's book is the best available." Caton's own considerable achievement in the study of Descartes's philosophy lends authority to this claim.

13. The significance of Klein's referring (in "Die griechische Logistik und die Entstehung der Algebra") neither to Husserl's phenomenological "theory" of intentionality nor to his phenomenological investigations of symbolic thinking has *nothing* to do with Husserl's thinking on these matters somehow being the unacknowledged source of or inspiration for Klein's thought in that work. In the early 1930s (when Klein wrote this work), Husserl had not yet linked either intentionality or symbolic thinking to the issue of their historical origination, whereas it is the remarkable achievement of Klein's thinking during this period to have established precisely this connection. Therefore, notwithstanding the "scholarly curiosity" of Klein's "failure" to acknowledge the "precedence" of Husserl's thought to his own with respect to the matters in general pertaining to intentionality and symbolic thinking, there can be *no question* of Husserl's thought having "priority" over Klein's on the specific issue of the connection of these matters to the history of mathematics. While the last part of this study will investigate the complicated matter of the significance of this precedence in detail, it needs to be stressed here that the conclusion Caton draws from it, that Klein—in effect—simply projected Husserl's theories of intentionality and symbolic thinking back upon the history of mathematics, cannot be maintained without distorting the nature of both the relation of Klein's thought to Husserl's and the originality of Klein's mathematical investigations.

tation of multiplicities," along with the relation of calculational technique and arithmetic, in *Philosophy of Arithmetic*,[14] and not in the few brief remarks to which Caton refers under the heading of "Thought without Intuition and the 'Surrogative Function' of Signs"[15] in the *Logical Investigations*.[16] Furthermore, Caton's attribution to Klein of the view that the Renaissance mathematicians were "*self-conscious*" of the effects wrought by their innovation of "symbolic abstraction" cannot withstand careful scrutiny of Klein's text. Such a view suggests that they were aware of the implications of their innovation for the shift in the "conceptuality" of the concept of number that Klein's research demonstrates. However, while Klein does indeed argue that a major change in the concept of number was precipitated by their innovation, he also argues that a fundamental lack of awareness of it characterizes both their self-understanding and the science born of this change.[17] Ultimately, "the purpose and achievement of Klein's historical method," as worked out in his text, possesses a subtlety that precludes assessing it by separating (as Caton does) its themes of a historical development "*immanent* to mathematics" (225) and a "philosophical purpose" supposedly beyond this development (225–26). This is not to suggest, however, that the connection between these two themes is readily apparent. Without the guiding clue for rendering Klein's methodology perspicuous that is provided by Husserl's articulation of the phenomenological problem and method of history, the internal connection between the historical development of a science's basic concepts and their philosophical meaning is not at all evident. Husserl's thesis that the clarification of the philosophical meaning of the mathematics that makes modern physics possible is inseparable from the investigation of the history of the development of its most basic concepts is indispensable for grasping this point. By not recognizing the operative role of this insight in Klein's work, Caton could not help but see a "tension" (225) between Klein's putative history of a technical innovation (i.e.,

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14. Edmund Husserl, *Philosophie der Arithmetik*, ed. Lothar Eley, Husserliana XII (The Hague: Nijhoff, 1970), 245; English translation: *Philosophy of Arithmetic*, trans. Dallas Willard (Dordrecht: Kluwer, 2003). Henceforth cited as *PA* with German page references, which are included in the margins of the English translation.

15. Edmund Husserl, *Logische Untersuchungen. Zweiter Band, Erster Teil: Untersuchungen zur Phänomenologie und Theorie der Erkenntnis*, ed. Ursula Panzer, Husserliana XIX/1 (The Hague: Nijhoff, 1984), 73; English translation: *Logical Investigations*, trans. J. N. Findlay, 2 vols. (New York: Humanities Press, 1970), I: 304 (Investigation I, § 20). Henceforth cited as *LI* with Investigation number in roman and German and English page references, respectively.

16. See § 154 below, where we discuss these remarks in detail.

17. See Caton's related statement (225) that, according to Klein, "the originators" of modern mathematics "were aware of the ontological dimension of the problem" presented by the symbolic concept of number.

"symbolic abstraction") immanent to mathematics understood (by Caton, not Klein) to be a science whose significance is ultimately "non-historical" (226), and his (again, putative) philosophical projection of a non-mathematical (in Caton's sense) "ontological dimension of the problem" (225) into this history.[18]

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18. See Part III, n. 146 below, where Caton's review of the English translation of *Greek Mathematical Thought and the Origin of Algebra* is discussed in detail.

Chapter Two

Klein's Account of the Essential Connection between Intentional and Actual History

§ 6. The Problem of Origin and History in Husserl's Phenomenology

Klein's interpretation of Husserl's articulation of the phenomenological problem of history in his last writings capitalizes on Husserl's lifelong concern with "the problems of origin" (*PHS*, 65) in order to argue that there is an "essential connection, as Husserl understood it," between "the approach to the 'true beginnings'" formulated in his earlier writings and his adumbration of "the aims which should control research in the history of science" in his last works. Klein locates this essential connection precisely in Husserl's concern, from beginning to end, with the constitutive problems of origin (true beginnings). According to Klein, Husserl's phenomenological preoccupation with the "ῥιζώματα πάντων, 'roots' of all things" (69), traced a continuous path from his early rejection of historicism as a means of accounting for the origin of logical, mathematical, and scientific propositions, to his late formulation of "the historicity (as the 'historical apriori') which makes intelligible not only the eternity or supertemporality of the ideal significant formations but the possibility of actual[19] history within natural time as well, at least of the historical development and tradition of a science" (74–75). Thus, in marked contrast to later commentators who see in Husserl's *Crisis* and "The Origin of Geometry" "the conflict between transcendental philosophy and histori-

19. The German term that Klein translates here (and elsewhere) as 'actual' is no doubt *faktisch*. Throughout this book, 'actual' is used to render *faktisch*, following Klein. Wherever it translates *wirklich*, the latter will be included in brackets following the English term. Wherever 'actual' occurs in a text cited that has been translated from the German and that is not Klein's (or does not occur in a work of Klein's or in the context of a discussion of his thought), 'actual' renders *wirklich*; in such cases, the German term is not included in brackets. See the Glossary below for translations commonly used in the present study.

cism,"[20] Klein aims to show that in these works "Husserl actually confronted the two greatest powers of modern life, mathematical physics and history, and pushed through to their common 'root'" (74).

On Klein's view the common "root" that Husserl uncovered is the "'sedimentation of significance'" (78). 'Sedimentation' is an important concept that Husserl introduced in his last writings to indicate the status of significant formations that are no longer present to consciousness but that nevertheless can still be made accessible to it. This status pertains both to the temporal modification of the experience of significant formations and the role that passive understanding plays in the apprehension of the signification of concepts and words. In either case, it is sometimes possible to render the sedimented formations present to consciousness again in a process called 'awakening'. In the case of the passive understanding of significant formations, because it does not reproduce the cognitive activity that originally produced their signification, Husserl contends that the original meaning becomes diminished and in some sense forgotten. Insofar as the original meaning has not completely disappeared, however, it can still be "awakened" by phenomenological reflection. The key to understanding Husserl's articulation of the phenomenological problem of history is to be found, on Klein's reading, in the former's account of the involvement of both kinds of sedimentation in the problem of "'constitutive origins'" (72) and the two distinct yet interrelated aspects of the content of what is sedimented in each case. Klein characterizes these aspects in terms of i) the "'intentional history'" (70)[21] of "the essential and objective pos-

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20. David Carr, *Phenomenology and the Problem of History* (Evanston, Ill.: Northwestern University Press, 1974), 238.

21. Klein's article makes repeated reference to "Husserl's notion of 'intentional history'" (*PHS*, 70; see 72–74, 76, 78, 82). However, Klein's consistent use of quotation marks when referring to "intentional history" is misleading, since the expression comes from him and not Husserl; hence, they are to be taken as scare quotes.

One possible source for Klein's expression may be a passage in *Formal and Transcendental Logic* where Husserl introduces the term 'sedimented history' (to which Klein refers on *PHS*, 72 n. 20). There intentionality is said to involve "a complex of accomplishments that are included as *sedimented history* in the currently constituted intentional unity and its current manners of givenness—a history that *one can always uncover by following a strict method*"; see Edmund Husserl, *Formale und transzendentale Logik*, ed. Paul Janssen, Husserliana XVII (The Hague: Nijhoff, 1974), 217; English translation: *Formal and Transcendental Logic*, trans. Dorion Cairns (The Hague: Nijhoff, 1969); henceforth cited as *FTL* with original page reference, which is included in the margins of both the German and the English editions cited. This method is articulated as "*uncoverings of intentional implications*" (ibid.). Thus, while there is no mention by Husserl of the term 'intentional history' in this passage (or in any other passage I know of), it is plain from what is stated here that he understands the method of uncovering the sedimented history of the accomplishments (*Leistungen*) responsible for the constitution of both an intentional unity and its manners of givenness as the "uncoverings of the intentional implications" that manifest this *history*.

sibility of each single significant phenomenon" (67) and ii) the "actual history" (69) connected to the "original 'presentation'" (73) of the significant phenomenon within "natural time." As will become clear, the "true character of history" (78) shows up for Klein when neither of these two "histories" is taken in isolation. Rather, the "essential necessity" of intentional history's "being subjected to a history in the usual sense of the term" is disclosed by Husserl in the *Crisis* and "The Origin of Geometry" when he "faced in those papers . . . precisely the relation between intentional history and actual history" (74).

§ 7. The Internal Motivation for Husserl's Seemingly Late Turn to History

In order to explicate how Klein arrives at his interpretation of Husserl's articulation of the phenomenological problem of history, we shall show that he situates the phenomenological motive for Husserl's putative "late" turn to history in the radicalization of his "early" concern with the problem of "constitutive origins." Then we shall make clear how Klein, alone among Husserl's commentators, correctly pinpoints the locus of the phenomenological problem of history articulated in Husserl's last works.

The locus in question is the inability of the constitutive analysis of the "internal temporality" (72) of each intentional "significant formation" (67) to reactivate "the 'original foundations,'" and therefore "the 'roots,' of any science and, consequently, of all prescientific conceptions of mankind as well" (78). For Klein, Husserl's phenomenology is *internally* motivated to widen

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Another possible source of Klein's expression is Eugen Fink's edition of "The Origin of Geometry"—again, the only version available when Klein wrote his article. There the expression *intentional-historisches Problem* occurs in the full title, "Die Frage nach dem Ursprung der Geometrie als intentional-historisches Problem," and once in the text: "Freilich hat diese ihrerseits selbst wieder Wissenstraditionen und ihnen entsprechende Seinsgeltungen, die wiederum in einem weiteren und noch radikaleren Rückfragen zu einem intentional-historischen Problem werden müssen" (*Origin*, F219). Klein's article does make one reference to "Husserl's 'intentional-historical' analysis of the origin of mathematical physics" (*PHS*, 79), though without citing the source of his quotation. The real author of the expression 'intentional-historical problem', however, was Fink, as the absence of the expression from the publication of Husserl's original version of the article in Biemel's edition of the *Krisis* makes clear. Klein, of course, had no way of knowing this when he wrote his article.

Notwithstanding the philological issue of the origin of the expression 'intentional history', its aptness in characterizing what both Fink's and Biemel's versions of "The Origin of Geometry" refer to as "das *eigentliche* Problem, das *inner-historische*" (*Origin*, F225/K386; only Fink's version is italicized) cannot be denied. That the 'inner' at issue in *inner-historische* has its basis in intentionality is clear from Husserl's reference in the *Crisis* to "the hidden unity of intentional inwardness which alone constitutes the unity of history" (*Crisis*, 74/73).

the scope of its inquiry into the origins of intentional objects beyond the evidence manifest in the analysis of their temporal genesis. This motivation is therefore provided *not* by any newfound interest in history on Husserl's part but by his recognition that the phenomenology of internal temporality is not up to the task of disclosing these original foundations. Thus, in marked contrast to those who argue that Husserl's turn to history in his last works has its source in his response to the situation whereby "the most radical and fundamental (i.e., going to the deepest roots and seeking the most extensive implications) rationalization of the *factual* is 'historically' not forthcoming,"[22] Klein identifies this source in Husserl's continued interest in the problem of accounting for evidence that discloses the origin of ideal significant formations that are *non-factual* and therefore, in precisely this sense, "rational." For Klein, then, the locus of the problem proper to history in Husserl's mature phenomenology is inseparable from the problem of the origin of non-factual meaning, that is, from the "ideal" meanings of Galilean Geometry and the "symbolic formulae" (*PHS*, 82) that make mathematical physics both possible and—so long as the origin of these meanings is investigated in the "genetic intentional analysis" of their "*temporal genesis*" (*FTL*, 278)—unintelligible.

Hence for Klein the phenomenological problem of history as sketched by Husserl does not involve what some have formulated as the question, "How is the facticity of history compatible with the claim of phenomenology that it leads to insights into essences which have unconditioned universality?"[23] It does not involve this question because it is precisely the "unconditioned universality" or the "*a priori*" status of the essences of any significant or meaning-formations—beginning with the "exemplary"[24] considerations of the ideal meanings that are constitutive of mathematical physics—which, when traced to their constitutive origins, are revealed to contain within themselves the "sedimented history" of their origination in "actual history." It will become clear, then, that for Klein, Husserl's articulation of the phenomenological problem of history does not lead to the problem of the opposition between the "facticity" of history and the "aprioricity" of essences, but rather uncovers their essential connection. Indeed, according to Klein, Husserl not only shows this essential connection, one that renders "untrue" the "generally

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22. Gerhard Funke, "Phenomenology and History," trans. Roy O. Elveton, in Maurice Natanson, ed., *Phenomenology and the Social Sciences*, 2 vols. (Evanston, Ill.: Northwestern University Press, 1973), II: 3–101, here 34 (my emphasis).

23. Ludwig Landgrebe, "A Meditation on Husserl's Statement: 'History is the Grand Fact of Absolute Being,'" *Southern Journal of Philosophy* 5 (1974), 111–25, here 118.

24. Husserl, *Origin*, K365/C353. Husserl understands his considerations to have "exemplary significance" for the "problems of science and the history of science in general, and indeed in the end for the problems of a universal history in general" (ibid.).

accepted opposition between epistemology and history, between epistemological origin and historical origin" (*PHS*, 78), but he also discloses its "universal and transcendental meaning" (74). This meaning uncovers "the real problem of historicity" in terms of the "nexus of significance between the '[transcendental] subjectivity at work'[25] and its intentional products (*Leistungsgebilde*)" (74), a nexus that yields the "interlacement of original production and 'sedimentation' of significance" (78).

25. 'Work' is Klein's translation of *Leistung* (see *PHS*, 67). This translation results in Klein's articulation of the "manner of being" of transcendental subjectivity in terms of "an 'intentionality' at work" (ibid.). It is interesting to note that Klein translates Aristotle's notion of ἐνέργεια as 'at work' and thus maintains that for Aristotle the "manner of being of an εἶδος is that it is altogether 'at work.'" See Jacob Klein, "Aristotle, an Introduction," in his *Lectures and Essays*, 171–95, here 181; this text is an enlarged version of a lecture first given in 1962 and published in Joseph Cropsy, ed., *Ancients and Moderns: Essays on the Tradition of Political Philosophy in Honor of Leo Strauss* (New York: Basic Books, 1964), 50–69. See also Jacob Klein, "Aristotle (I)," ed. Burt C. Hopkins, *New Yearbook for Phenomenology and Phenomenological Philosophy* III (2003), 295–313, esp. 310–11.

Chapter Three

The Liberation of the Problem of Origin from Its Naturalistic Distortion: The Phenomenological Problem of Constitution

§ 8. Psychologism and the Problem of History

Turning now to Klein's account of the problem of history in Husserl's early work, we find him maintaining that in "attacking 'psychologism,' Husserl was in fact facing the problem of 'history'" (65). Indeed, Klein maintains "that Husserl in criticizing the attitude of historicism [in 'Philosophy as Rigorous Science'] puts it on the same level with psychologism. In fact, the former is but an extension and amplification of the latter" (68). Thus:

> Any "naturalistic" psychological explanation of human knowledge will inevitably be the history of human development with all its contingencies. For in such an account any "idea" is deduced from earlier experiences out of which that idea "originated." In this view, the explanation of an idea becomes a kind of historical legend, a piece of anthropology. The *Logical Investigations* showed irrefutably that logical, mathematical, and scientific propositions could never be fundamentally and necessarily determined by this sort of explanation. (65–66)

For Klein, "Husserl's radical criticism of psychologism implies anything but a simple opposition between never-changing 'abstract' principles and ever-changing 'empirical' things" (68). Thus, he holds that Husserl's exposure of the inability of the psychologistic and historicistic appeals to the "natural time" of empirically conceived experience to account for the origin of the eternity or supertemporality of ideal meanings and significations did not rule out all connection of time to the problem of their origin. Rather, Husserl's recognition of the "naturalistic distortion" (66) of the account of the origin of ideal meanings in psychologism and historicism was followed by his at-

tempt to "liberate" the problem of their origin from this distortion by disclosing it in terms of "phenomenological" time.

On Klein's view the fact that the phenomenological descriptions grounding Husserl's account of the "internal temporality" of phenomenological time "were immediately interpreted as psychological descriptions . . . shows not merely that a great many readers of Husserl were not able to understand his thought, but that there is a definite affinity between psychological and phenomenological research" (69). Klein expresses this affinity in terms of the commonality of the "mental phenomena" with which each is concerned. In support of this he refers (69 n. 11) to Husserl's *Formal and Transcendental Logic*, where Husserl writes that the mistaking of "psychologically inner experience" for "transcendentally" inner experience "is a falsification that could not become noticeable before the advent of transcendental phenomenology" (*FTL*, 224). And thus: "every mode of intentionality, including every mode of evidence and every mode of the fulfilling of meanings through evidence, can also be encountered and treated psychologically"—and not just transcendentally—"*in the psychological attitude* toward experience."

Despite this affinity, however, Klein discerns the following:

> The real difference [between psychology and phenomenology, or "more exactly . . . between psychological phenomenology and transcendental phenomenology"] can only be found in the fundamentally different attitude of the thinker toward his objects: on the one hand, the psychologist considers them in "mundane apperception," taking them as existing elements or parts or qualities of the existing world; on the other hand, the phenomenologist deprives these same objects of their "index of existence." (*PHS*, 69)

The phenomenological reduction, which deprives the mind (*das Psychische*) of its "index of existence," of its being understood as a "natural object" existing in "natural time," considers "the mind as the transcendental subjectivity." And thereby "transcendental phenomenology, as the universal theory of 'constitution,' is primarily concerned with the problems of origin, the problem of true beginnings." Husserl's characterization of the latter in "Philosophy as Rigorous Science" as the *ῥιζώματα πάντων*[26] is noteworthy on Klein's view because in contrast to the traditional *ἀρχή*—which "in the 'classical' sense of the term" denotes the "perfect shape" of things—"the (Empedoclean) term . . . 'root' is something out of which things grow until they reach their perfect

26. Edmund Husserl, "Philosophie als strenge Wissenschaft," *Logos* I (1910–11), 289–341, here 340; English translation: "Philosophy as Rigorous Science," trans. Marcus Brainard, *New Yearbook for Phenomenology and Phenomenological Philosophy* II (2002), 249–95; henceforth cited as *PRS* with the original German page reference, which is reproduced in the margins of the English translation.

shape." Klein interprets this to mean that the "'radical'[27] aspect of phenomenology is more important to Husserl than its perfection," and he finds in this "the attitude of a true historian."

§ 9. Internal Temporality and the Problem of the Sedimented History of Significance

For Klein, then, Husserl's liberation of the problem of origin from the naturalistic distortion of psychologism and historicism was achieved through the phenomenological reduction, which brings about the phenomenologist's "fundamentally different attitude" over against the "empirical" psychologist's toward the mind or psyche. The result of this liberation was anything but Husserl's abandonment of the problem of history. On the contrary, his analysis of the "roots" of the origin of the "*eidos*," of the "*a priori* form" of each "significant formation" presupposed by both human knowledge and the misguided empirical attempts of psychologism and historicism to account for such meaning, resulted in the phenomenological problem of uncovering its "history of significance" (67).[28]

According to Klein, Husserl's awareness of this problem followed "the first step—first in the actual development of Husserl's thought, and first in any phenomenological analysis"—of "finding the 'invariables' within the absolute flow (the 'internal temporality') of the mind," of "determining the 'invariants' which remain unchanged by reason of an essential necessity." Thus Husserl's "reflection upon this kind of analysis, its implications and significance," led to the realization that, "[f]ar from being complete in itself, the finding and facing [of] an 'essence' requires a further investigation into its intrinsic 'possibility.'" This is the case because the "possibility" of an *eidos* is not exhausted by the process that initially yields it, that is, the "continuous and arbitrary 'variation' of a given 'example,' a variation that takes place in the 'freedom of pure fantasy.'" Rather, whatever is discovered "as having a definite significance—an essence, its 'inflections,' its essential characteristics, the compresent 'halo,' and so forth—has also a 'backward reference' to a more original 'significant formation.'" It is precisely this "backward reference" that allows for the uncovering of the history of significance of each significant formation, which "describes the 'genesis' of that mental product."[29] The "his-

27. Klein probably has in mind here the Latin etymology of 'radical' (*radic-*, *radix*, root); see Husserl's characterization of phenomenology in "Philosophy as Rigorous Science" as "the method of a radical science" (*PRS*, 340; see Klein, *PHS*, 65).

28. The reference, unattributed by Klein, is to Husserl, *FTL*, 184.

29. The source of this quote, as well as of Klein's paraphrase here, is Husserl's *FTL*, 184.

toricity"[30] at issue in this genesis is characterized by Klein as "a curious kind of 'history,' . . . a peculiarity of the mind, whose manner of being is nothing but 'work' (*Leistung*), a constructive work, tending toward the formation of 'units of significance'—an 'intentionality at work.'"

30. Klein does not use this term—'historicity' (*Historizität*)—here, though Husserl does. See *FTL*, 184.

Chapter Four

The Essential Connection between Intentional and Actual History

§ 10. The Two Limits of the Investigation of the Temporal Genesis Proper to the Intrinsic Possibility of the Intentional Object

Klein unpacks Husserl's account of how it is that the "intrinsic possibility" of an object's intentional unity "contains the 'sedimented history' of its 'constitution'" (72) in view of two "limits." These emerge in the analysis of the "universal eidetic 'form' of the intentional genesis" of each such object's unity, that is, in the analysis of "internal temporality" (72–73).[31] The first limit concerns the "general substratum of consciousness" that the "continuous modification of the retentional consciousness approaches and beyond which the 'prominence' of the object flows away" (73). The second limit concerns the "'past history' of the original 'presentation' of the object." Both limits point to sedimented meanings that can be "awakened" (*FTL*, 280) such that the "intentional genesis" of the meaning in question is "reproduced" as the "history" of

31. Husserl's phenomenological investigations of internal time-consciousness are well known and therefore need only be briefly rehearsed here. For Husserl, the consciousness of temporal phenomena is manifest in accordance with a structure he terms the 'living present'. The living present is a complex whole comprised of a focal Now phase that is horizonally girt by Past and Future phases. The living present, therefore, is *not* structurally equivalent with the Now phase of time since its structure is made up of all three phases of time. In addition, these three phases of time are not structurally independent. This means that "internal time" is not manifest in terms of the successive appearance and therefore successive ordering of its phases; rather, the three phases of time are interrelated and interdependent. Husserl expresses this state of affairs with the metaphor of "flow" or "flux." This metaphor suggests that the focal Now of time's living presence is manifest in terms of its inextricable reference to the "slippage" of both just-past Nows, which comprise the horizon of the past, and of the Nows to come, which comprise the horizon of the future. Husserl terms consciousness of the Now 'impression', whereas he terms consciousness of the "past" 'retention', and consciousness of the "future" 'protention'.

its constitution, a history that, "of course, did not take place within 'natural time'" (*PHS*, 72). For Klein, then, two "histories" are *initially* at issue in Husserl's phenomenological account of the intrinsic possibility of an object's intentional unity. The first history concerns the possibility of such an object's retaining its unity as an enduring "presence" once it has been presented to consciousness. This history concerns the object's intentional genesis as an objective "prominence," its persistence throughout the temporality that is the essential characteristic of the experience of that object. The second history concerns the possibility of the object's "original presentation" to consciousness. This possibility is more fundamental than that of its persistence as a "prominence," for what is at issue here is its presentation to consciousness *prior* to any modification in accordance with consciousness's structure of internal temporality. It is important to note, however, that neither of the possibilities or "histories" at issue here concern the "natural" existence of objects and their histories. This has been precluded by the phenomenological reduction's removal of the "index of existence" from the experience of both the object and its history. As a result, what is at issue in the priority of the original presentation of the intentional object to consciousness is decidedly *not* its being experienced "first" in a supposed natural succession of awareness. Rather, the priority involved here is *methodological*, in the sense—to be discussed in detail below—that the evidence uncovered by these analyses discloses an "indication" that points to a more original "possibility" belonging to the object than its enduring presence in experience.

For example, the significant formation 'S is P' is immediately presented to consciousness when one studies logic. So long as one is attentive to its significant formation, it is experienced as an objective prominence. When another object becomes the focus of attention, say, a knock at the door, the prominence of 'S is P' recedes into the general substratum of consciousness. When, after this interruption, one's logical studies are resumed, 'S is P' can be awakened and thereby made prominent once again. After intensive study, the awareness that this particular significant formation has a "past history," one that exceeds its sedimented history as a prominence in one's experience, becomes unavoidable as soon as one realizes that 'S is P' as a significant formation does not originate with its presentation to one's consciousness as an "immediate prominence." Therefore, subsequent to the initial phenomenological access to and analyses of the intrinsic possibility of an object's intentional unity as the eidetic form of its intentional genesis (which is manifest as its internal temporality), Husserl's analysis indicates the possibility of its "original presentation" to consciousness. As the discussion below will show, for Klein it is precisely this inquiry into the original presentation of an object's

intentional unity that "may reveal the essential necessity of a historical development within natural time" (73).

The phenomenological problem of evidentially uncovering the possibility of the enduring "prominence" of an intentional object, an object "given originally in the mode of immediate 'presence,'" is therefore what is at issue in the "awakening" of the sedimented meanings to which the first "limit" points. Inseparable from "this immediate 'presentation'" of the object is its being "followed, of necessity, by a 'retention' of the object, in which the object appears in the mode of 'just-having-been-experienced.'" The continuous retentional modification of the originally given "prominence" of an object "accompanies every living present," such that "the initial part of the constitution of an identical object" comes about as "one that, in the broadest sense, persists" (*FTL*, 280). The limit of this continuous modification, the general substratum of consciousness wherein the prominence of the object recedes, is what points to objects that are at once no longer prominent while still being capable of being made prominent again. Thus, when the "sedimented prominences" connected with the "possibility" of the object's givenness in the mode of immediate presence are "awakened," what is uncovered by the intentional analyses of the evidence experienced in these sedimentations is the "sedimented history" of its intentional genesis as an object given to inner experience precisely in the mode of immediate presence. Such evidence is therefore mediated by the retentional modifications that are inseparable from the object's initial givenness to inner experience in the mode of an immediate presence.

§ 11. The Transcendental Constitution of an Identical Object Exceeds the Sedimented Genesis of Its Temporal Form

According to Husserl, "every [retentional] modification obviously refers back, either immediately or mediately, to its absolute original mode—to a consciousness that, to be sure, is modified at once yet is no longer a [retentional] modification" (*FTL*, 280). And it is precisely here, at the "limit" manifest in this reference back to the "original 'presentation' of the object" (*PHS*, 73), Klein maintains, "that the 'evidence' experienced in the immediate presentation assumes the character of a transcendental problem of constitution." It is at this point that "the intrinsic 'possibility' of the object is revealed out of its categorial formations,[32] that the 'intentional genesis' leads back to the 'consti-

32. The phenomenological problem of revealing the intrinsic possibility of an object in terms of its "categorial formations" is connected to Husserl's theory of parts and wholes. Husserl initially formulated this theory in *Philosophy of Arithmetic*, where it is articulated with respect to the arithmetic activities of collecting and counting. This formulation was followed by his

tutive origins,' that the 'sedimented history' is reactivated into the 'intentional history.'"[33] Further, it is here that Klein states that "such a transcendental inquiry into an object may reveal the essential necessity of its being subject to a history in the usual sense of the term. In other words, it may reveal the essential necessity of a historical development within natural time."

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articulation of the theory in terms of "pure logic" in the *Logical Investigations*. In subsequent stages of the development of his phenomenology, i.e., in *Ideas I* and *Experience and Judgment*, Husserl returned to the theory in order to refine it in accordance with the new resources that on his view were made available by this development, most notably his theory of "formal ontology." However, the text Klein is working with, *Formal and Transcendental Logic*, does not contain an explicit discussion of the theory. Nevertheless, the theory is relevant to the "judicative formations" and "categorial formations" he treats in the context of the phenomenological development of transcendental logic. A brief consideration of Husserl's theory is therefore necessary here so as to provide a context for Husserl's (and Klein's) consideration of the constitution of the intentional object out of its categorial formations.

The theory of parts and wholes is formulated as the phenomenological project of providing an account of the logical unity of an object on the basis of the internal relations between the logical "parts" composing it and its initially experienced "wholeness." In the context of Klein's discussion, the logical unity in question refers to the object understood as an intentional unity, whereas its parts refer to its "categorial formations." Husserl calls the phenomenological status of these parts the 'moments' of the whole in order to signal that they cannot be given to experience independently of the whole to which they refer, the universal unity of the intentional object as it is initially given to experience. Moments, as non-independent parts of a whole, are thus characterized as being "foundationally related" to the whole insofar as their givenness is inseparable from and therefore "founded" upon the givenness of the whole. Yet because these moments make up the essential components of the intentional object's unity, Husserl likewise characterizes the whole in terms of its foundational relationship to the parts that, as its moments, make it up and therefore "constitute" it. Husserl characterizes the two-way relationship, manifest in the non-independence of the whole from the part and the part from the whole, in terms of a foundational relationship that is "reciprocally founding/founded." Husserl's concern in *Formal and Transcendental Logic* with the investigation of the intentional genesis of the categorial formations of the intentional object therefore represents in effect a deepening of the phenomenological problem of the constitution of the wholeness of an object out of its parts. By focusing his discussion upon this aspect of the problem of constitution, Klein thus sees what even today many readers of Husserl do not see: that Husserl's later "genetic" investigations of the intentional genesis of the "categorial formations" of the intentional object do not represent a self-critical departure from his earlier "static" phenomenological investigation of the logic of whole/parts relations. Rather, in these later investigations Husserl attempts to bring to completion his life-long preoccupation with the goal of accounting for the "constitutive origins" of ideal objects. Klein's recognition that the genetic investigations adumbrated in *Formal and Transcendental Logic* (like the static investigations before them) were seen by Husserl in his last works to fall short of the goal of accounting for these origins (though this did not cause him to question or abandon the phenomenological goal of accounting for the constitutive origins of ideal objects) likewise sets Klein's understanding of Husserl apart from his contemporary readers. Indeed, it is precisely Klein's account of this realization by Husserl, and its role in motivation of the need to turn to "actual history" in order to fulfill the enduring goal in question, that is the subject of his discussion here.

33. Regarding Klein's use of the expression 'intentional history', see n. 21 above.

Before considering more closely Klein's understanding of how Husserl worked out the "essential necessity" of intentional history's subordination to an actual history in his last works, it will be helpful to elaborate on the distinction, as well as the relation, that Klein articulates between: i) the sedimented history of the intentional genesis of an object's immediate presentation and ii) the reactivation of this sedimented history into intentional history in the investigation of the object's transcendental constitution. This discussion will highlight the basis of Klein's argument that Husserl recognized an essential connection between intentional history and the historical development that takes place within natural time.

Husserl's analyses of the constitutive origins of "the intrinsic 'possibility' of the identity of an object" suggest for Klein that this possibility *cannot* be revealed by the "'evidence' experienced in the immediate presentation" of the object (*PHS*, 73). The transcendental problem of intentional history, which Klein understands to emerge out of the phenomenological investigation into the origins that are constitutive of an object's intrinsic possibility as an identity, involves a mode of access to the object that exceeds its mode of being given as an intentional object that persists as a "prominence" present to inner experience. This is clear from the analyses in *Formal and Transcendental Logic* on which Klein bases his articulation of these issues (67 n. 5 and 73 n. 22). These analyses take the universality of an identical object to reveal—when "studied more closely"—"its peculiarity as the constitution of persisting categorial formations" (*FTL*, 280). According to Husserl's analyses, prior to this closer study, which investigates the history belonging to the significance of the formations, the "ideal formation" (183) of each identical object appears as "the finished products of a 'constitution' or 'genesis'" (184). Husserl's analysis of the initial appearance of these ideal constituents—as the universality of an identical object—refers to this appearance as the object's "immediate presentation" to inner experience as a persisting prominence. Klein situates the transcendental problem of the "intentional genesis" of the intrinsic possibility of this identity, as it "is revealed out of its categorial formations," in the investigation of the constitutive origins of these formations. According to Klein, then, it is precisely the problem of these constitutive origins that leads Husserl's inquiry back to an "intentional history" that exceeds the "universal eidetic 'form' of the intentional genesis" (*PHS*, 72–73) belonging to this identity's immediate presentation. Husserl's analysis of the need to investigate the genesis of these constituents makes it evident that the constitutive origins of these categorial formations, which comprise the intentional significance (*Sinn*) of an object's unity, exceed the sedimented history of this significance as it is "reproduced" in terms of its internal temporality. Husserl articulates this need as follows:

> The essential peculiarity of such products [i.e., the "ideal or categorial formations" of an identical object] is precisely that they are significances [*Sinne*] that bear within themselves, as a significance-implicate of their genesis, a kind of historicity; that in them, level by level, significance points back to original significance and to the corresponding noetic intentionality; that therefore each significant formation can be asked about its *essential significance-history*. (*FTL*, 184)

§ 12. The Distinction between the Sedimented History of the Immediate Presence of an Intentional Object and the Sedimented History of Its Original Presentation

The distinction Klein makes between the history of the significance belonging to the categorial formations of an object's identity, when this identity is immediately given to consciousness as the finished product of a constitution, and the "sedimented history" proper to this significance's original "constitution" by intentional "accomplishments," becomes clear when the two limits he identified in Husserl's analysis of any object's intentional genesis are considered. These two limits concern two distinct but nevertheless interrelated dimensions of an object's possibility as a "'significant' or 'intentional' unit" (*PHS*, 72). Our discussion has shown the first limit to involve the reactivation of the sedimented history that is responsible for an intentional object's persisting identity, a reactivation that reveals the intentional history of its immediate presentation to inner experience as a finished product of constitution. The "history" at stake in the persisting identity of an intentional object is therefore quite different from the history to which psychologism implicitly appeals and that which historicism explicitly invokes. The latter concept of history is empirical, which means that its meaning is inseparable from the contingency and accidental nature of natural "reality." Because the identity of a categorial formation is neither contingent nor accidental, empirical history can contribute nothing in the way of an account of the origin of this identity as something that persists as identical throughout "inner" experience, as an "ideal" object. The failure of empirical history to address on its own terms the origin of an ideal object does not, however, rule out this origin's having a history. Klein points out that Husserl initially sought to account for this history's source in the flowing away (via the continuous modification of retentional consciousness) into the general substratum of the consciousness of the intentional object's prominence "as one and the same (identical, 'invariant') object" (73). Our discussion has also shown that the second limit that Klein identified concerns Husserl's recognition that the reactivation of the sedimented history of just these continuous retentional modifications is *unable* to disclose the

"original presentation"—unmodified by retentions—of a categorial object. Klein thus links i) the problem of the "original presentation" of the object with ii) the investigation of the categorial formations comprising its significance as an initially given, finished product of a constitution, whose original presentation is investigated through the reactivation of the intentional history of these formations.

Once these two issues have been linked, the evidence experienced in the universal eidetic form of internal temporality—an evidence that reproduces the intentional genesis of an intentional object's enduring presence—can be seen to account for its constitution only in terms of the "possibility" of this enduring prominence for an "inner experience." The constitution of the intrinsic possibility of an object's original presentation to consciousness, prior to the sedimented "history" of its temporal modification, is therefore kept distinct from the constitution of its intrinsic possibility subsequent to its having been given to inner experience and modified in accordance with the internal temporality that essentially characterizes such experience. For Klein, then, Husserl's analyses show that the intrinsic possibility of an object's "original presentation" to consciousness and therefore "original" constitution in consciousness is "not itself a modification" of its "immediate presentation" to inner experience as an "enduring prominence" constituted by internal time. In other words, the constitution of an object in accord with the form of internal time is derivative, in the sense that the awakening of the intentional history of the sedimented history belonging to *this* constitution points to a more basic constitution, wherein the object's unity is originally fashioned. On the basis of this, Klein is able to reveal that for Husserl the derivative constitution of the object's identity is inseparable from its significance (*Sinn*) as an identity that already involves the retentional modifications of consciousness manifested by the universal structure of internal temporality. In contrast to this, the problem of the transcendental constitution of the "intrinsic possibility" of the object's significance points to the temporally unmodified but nevertheless "historical" intentionality that discloses the radical beginnings of the categorial formations that compose its significance. Such formations, however, do not initially appear, according to Husserl's analyses, as temporally unmodified. Rather, they are first encountered in accord with consciousness's retentional modification of their significance as unities whose formation has already been accomplished. It is therefore only when the problem of the constitution of the intrinsic possibility of the significance of these categorial formations is investigated with a view to its transcendental origin that "every [retentional] modification" of such formations is encountered as referring back to a consciousness that "is not itself a [retentional] modifica-

tion" (*FTL*, 280). In other words, the transcendental inquiry into the origin of its unity is led from the retentional modification of the experience of the enduring identity of a categorial formation to the phenomenological problem of its original constitution.

Chapter Five

The Historicity of the Intelligibility of Ideal Significations and the Possibility of Actual History

§ 13. The Problem of ʽΙστορία Underlying Husserl's Concept of Intentional History

In the foregoing discussion we have indicated that for Klein the transcendental inquiry into the problem of the "intentional history" of the categorial formations of the significance making up an object's identity "may reveal the essential necessity of its being subject to a history in the usual sense of the term." That is to say, the transcendental inquiry into the intentional history of an object's categorial unity may disclose an essential connection between the origin of this unity and its historical development within natural time. For Klein, "[h]istory, in the usual sense of the term, is not a matter-of-course attitude. The origin of history is itself a non-historical problem" (*PHS*, 72). This is the case because history in its usual sense is "the 'story' of a given 'fact.'" Thus, the telling of any such story about the "fact" of the historical attitude will of necessity presuppose, rather than account for, the "historical attitude" that gives rise to the "telling of the telling" of the story of the historical origin of this attitude: "Whatever historical research might be required to solve it [i.e., the origin of history], it leads ultimately to a kind of inquiry which is beyond the scope of a historian." Such research "may, indeed, lead back to the problem of inquiry, the problem of ἱστορία as such, that is, to the very problem underlying Husserl's concept of an 'intentional history.'"[34]

For Klein the connection between the problem of inquiry that underlies historical research and Husserl's concept of 'intentional history'[35] is estab-

34. Klein's linking of the meaning of "historical research" here to "problem of ἱστορία" includes the reference of the latter to its source in Plato's *Phaedo* (96a ff.).

35. See n. 21 above.

lished when Husserl—in the works that present the final phase of his thought—once again takes "up a task that psychologism could not solve with its own premises but had attacked in its own way" (74): the investigation of the origin of the unity of the "significant formation" of any intentional object. In these works, Husserl showed that the inquiry into the constitution of any "significant formation" as an invariant that transcends the natural time presupposed by psychologism is itself "but a mode of 'eternal' time: its identity is an intentional product of the transcendental subjectivity which is 'at work' through all the categorial determinations that constitute a significant unit." Hence, the inquiry into the origin of an invariant, as identically the same, leads Husserl to the problem of the "nexus of significance between the 'subjectivity at work' and its intentional products (*Leistungsgebilde*)." According to Klein, this problem "is the real problem of historicity taken in its universal and transcendental meaning." For the inquiry into the intrinsic possibility of the invariant—if it is to pursue this possibility in terms of the transcendental origin of its status as an intentional product—must push beyond the articulation of this possibility in terms of its intentional genesis as an identity that is "immediately present" to inner experience. It must do so if the origins of the categorial formations are to be investigated with respect to their original (unmodified by retention) presentation to experience. And it is precisely here that the inquiry into origins reveals that the "intentionality at work" that constitutes this original presentation "implies historicity (as 'the historical *a priori*')" as that "which makes intelligible not only the eternity or supertemporality of the ideal significant formations but the possibility of actual history within natural time as well" (74–75).

§ 14. Two Senses of Historicity and the Meaning of the Historical Apriori

Klein therefore understands Husserl's inquiry into the intelligibility of the intrinsic possibility of an invariant to involve 'historicity' in two distinct senses. One concerns the intentional history of an invariant's intrinsic possibility. 'Historicity' in this sense is indicated at the limit of the inquiry into the invariant's sedimented history, an inquiry guided by the eidetic form of its internal temporality. This limit concerns the original presentation of an intentional unity as a significant unit that has not yet been retentionally modified and, therefore, not yet structured as an enduring unity within internal time. The other sense of historicity concerns the "actual (*faktische*) history" that is indicated by the limit of the inquiry into the intentional history of precisely this unmodified origination of the invariant's intrinsic possibility. It

therefore concerns the "actual history" indicated when the categorial formations that manifest an ideal significant formation are interrogated with respect to their transcendentally original constitution. Because both senses of 'historicity' are inseparable from the phenomenological inquiry into the "intrinsic possibility" of the ideal signification belonging to the intentional object's identity, the status of the meaning of the 'history' in each of these senses of historicity is different from that of the "empirical" history of accidents and contingencies that may be associated with the identity of an object. Indeed, it is precisely the inseparable link between each of these senses of historicity and the inquiry into the intrinsic possibility of the ideality of an intentional object's significance that allows Husserl—and thus Klein following him—to speak of this historicity in terms of the historical apriori. The aprioricity of the apriori in question here cannot be derived in the manner in which it is typically derived, that is, from a supposed opposition between the contingent and the non-contingent. This opposition establishes the apriori as something that is somehow *prior* to the contingent. Such an understanding of the apriori is "formal" in a sense that is irrelevant in the present context, which is concerned with what is itself prior to all formal significance and therefore also to the apriori understood formally. As the apriori of the formal apriori, however, the historical apriori, in its very aprioricity, nevertheless remains inseparable from the intrinsic possibility of both formal significance and the formal apriori. It is in this latter sense, and in it alone, therefore, that the apriori of the phenomenological notion of the historical apriori is to be understood: that is, as the non-contingent and non-formal condition that makes intelligible both the "supertemporality" of ideal significant formations and the possibility of actual history in natural time. It is Klein's achievement—and in this he stands alone among Husserl's commentators—to have recognized that in Husserl's last works the condition of possibility for this condition, the condition of possibility therefore for the historical apriori, is the transcendental accomplishment of intentionality "at work." That this accomplishment is only accessible via the sedimentations of significance that are manifest as a given (historical) tradition is also something that, as we shall see, Klein alone has recognized.

§ 15. Historicity as Distinct from Both Historicism and the History of the Ego

Thus for Klein what is indicated at this second limit—the fact that Husserl regards the historicity of actual history as implicit in the historicity of "intentional history"—does not mean, as some have argued, that Husserl belatedly recognized "the engagement of consciousness in a particular historical situa-

tion."[36] It neither signals a reassessment of his earlier attack on historicism nor accords "considerable legitimacy to [historicism's] notion of the socially and historically conditioned character of consciousness."[37] In addition, for Klein the issue here is not a historical meaning derived from the "*universal genesis of the ego*," in which "[t]he ego constitutes itself for itself in, so to speak, the unity of a 'history.'"[38]

Regarding the first point, Klein's discussion of the "real problem of historicity" in Husserl's last works shows that Husserl does *not* call into question—in light of historicism's insistence on the social and historical conditioning of all significance—the ideal status of the significant formations, such as mathematical and scientific objects, that are the accomplishments of transcendental subjectivity. Rather, what Husserl does in these works is to reveal how the inquiry into the origin of the ideal significance inseparable from these objects has an "essential connection" to their "'creation' in actual history" (*PHS*, 76), a connection that "does not refer to any known or even knowable historical event" (78). This connection, to be discussed in greater detail below, is established on the basis of the inability of the reactivation of the "intentional history" of the original presentation of the categorial formations proper to the ideal significance of mathematical and scientific objects to bring to evidence the origin of this significance in an "intelligible" manner. For Klein, Husserl's transcendental inquiry into the origin of these significant formations discloses that the significance of these formations paradoxically "appear[s] almost devoid of 'significance,'" unless the connection to the actual history of their origin is investigated. The phenomenological inquiry into the origin of the significance of these formations discloses not only that this origin is inseparable from their "intelligibility" but also that these formations' status *necessarily* remains "emptied of significance" (*Origin*, F218/C368) unless the inquiry into the intentional history of their origin is extended to include the inquiry into their "actual" history. Klein's understanding of how Husserl's inquiries show that this paradoxical state of affairs is possible, that is, the state in which mathematical and scientific objects are at once significant and yet almost de-

36. Carr, *Phenomenology and the Problem of History*, 239.

37. Carr, "Translator's Introduction," in *Crisis*, xxxvii.

38. Edmund Husserl, *Cartesianische Meditationen*, in his *Cartesianische Meditationen und Pariser Vorträge*, ed. S. Strasser, Husserliana I (The Hague: Nijhoff, 1950), 109; English translation: *Cartesian Meditations*, trans. Dorion Cairns (The Hague: Nijhoff, 1960), 109; henceforth cited as *CM*, with German pagination, which is included in the margins of the translation. The German version of this text was not available to Klein when he wrote "Phenomenology and the History of Science." However, he may well have had access to the French translation: *Méditations cartésiennes. Introduction à la phénoménologie*, trans. Gabrielle Peiffer and Emmanuel Lévinas (Paris: A. Colin, 1931).

void of real significance, as well as his understanding of Husserl's "admirable attempt to restore the integrity of knowledge, of ἐπιστήμη" (*PHS*, 78), which is threatened by this state of affairs, will be considered shortly. At this point, it is important to note that rather than call into question the possibility of the ideality of mathematical and scientific objects, the problem of actual history in Husserl's last works emerges within the context of his response to "the demand, which has spread throughout the modern period and has finally been generally accepted, for a so-called 'epistemological grounding' of the sciences" (*Origin*, F218/C368).

This account of the emergence of the problem of actual history in Husserl's phenomenology also tells against the attempt to trace the meaning of history at issue here to the historicity of the transcendental ego's universal genesis (the second point above). This is the case because the status of the transcendental ego as "the universal *unity-form of the flux*" (*CM*, 109)—whereby "[i]n each present moment of the existence of an ego its past cooperates in the manner of *sedimentation*"[39]—would clearly have as its analog the "history" of the sedimented prominences that make possible an intentional object's givenness in the "mode of immediate 'presence.'" What is at issue, then, in the sedimentation of the ego's past would be the retentions of its universal unity-form, retentions that are inseparable from its constitution as an enduring unity present to inner experience. Thus, the reactivation of the intentional historicity of these sedimented retentions would be disanalogous to the reactivation of the sedimented history of the "absolute mode" of the unmodified (by retentions) "original presentation" to consciousness of the categorial constituents of an intentional unity. For Klein, as shown above, Husserl's analyses establish the phenomenological problem of actual history in terms of its essential connection with the inquiry into the intentional history of just such unmodified original presentations. Hence, the historicity of the transcendental ego's enduring presence as a "universal unity-form" would be incapable of providing a basis, analogical or otherwise, for the connection between actual history and intentional history that Klein maintains Husserl established in his last works.[40]

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39. Landgrebe, "A Meditation on Husserl's Statement," 116.

40. This state of affairs raises the interesting question of whether the possibility of the "immediate presence" of the transcendental ego's unity likewise refers back to its "absolute original mode" of unmodified presentation to consciousness, that is, to a sedimented history that, in a manner analogous to the sedimented history of the intentional object's original presentation, likewise "may reveal the essential necessity of a historical development within natural time." The implications of this question, however, will not be pursued in this study.

Chapter Six

Sedimentation and the Link between Intentional History and the Constitution of a Historical Tradition

§ 16. Maintaining the Integrity of Knowledge Requires Inquiry into Its Original Historical Discovery

According to Klein, then, it is Husserl's phenomenological inquiry into the transcendental constitution of the origins of the ideal formations proper to mathematical and scientific objects that reveals that the "'evidence' of all the 'significant formations' belonging to a science such as geometry" presupposes "the link between 'intentional history' and actual history" (*PHS*, 76). For Klein, this link is established by Husserl on the basis of the following considerations: i) the "ideal 'intentional units'" at issue in these significant formations are the product (*das Erwirkte*) of an "accomplishment" (*gelingende Ausführung*; *Verwirklichung*) that arises in their "anticipation" (*Vorhabe*)[41]—not in their "retention"; ii) "'accomplishment or [*sic*][42] what is anticipated means evidence to the active subject: herein the product shows itself originally as itself'"; iii)

> since the product, in the case of geometry, is an ideal product, "anticipation" and the corresponding "accomplishment," as acts of the subject . . . , are founded upon the "work" of transcendental subjectivity: the ideal formations of geometry are products of the "intentionality at work." "Anticipation" and "accomplishment" *translate* into terms of "reality" what actually takes place within the realm of "transcendental subjectivity";

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41. Carr translates *Vorhabe* as 'project'. See Husserl, *Origin*, K356/C208 *et passim*.

42. The German text translated by Klein here reads "Gelingende Verwirklichung *einer* Vorhabe," which makes it probable that the 'or' here is a typographical error, since the correct translation would be 'accomplishment *of* what is anticipated'. The original version of Klein's text published in *Philosophical Essays in Memory of Edmund Husserl* also prints an 'or' here.

and iv)

> the constitution of those ideal "intentional units" presupposes, of necessity, the whole complex of experiences leading to the situation in which geometry as a science is capable of being "anticipated" *and* "intended."[43] In other words, "*science, especially geometry, as a subjective intentional product, had to have some definite historical beginning*,"[44] i.e., a beginning within the course of actual history. At this definite moment the "original foundation" (*Urstiftung*) of geometry occurred.

Klein fills out the "necessity" belonging to the presupposition of a definite historical beginning that is inseparable from the constitution of these ideal "intentional units" with two additional presuppositions. They are disclosed by Husserl's transcendental inquiry into the origin of the ideal significant formations belonging to a science such as geometry. Both presuppositions are inseparable from the "anticipation" and "accomplishment" that yield these formations on the basis of the "work" of transcendental subjectivity. They are inseparable in the sense that the transcendental inquiry into the "nexus of significance" between this subjectivity and its "intentional products" points to the "discovery" of geometry in an "anticipation" of a "first geometer" whose accomplishment brought about "geometry as a supertemporal product of the mind."

Before addressing Klein's account of these additional presuppositions, however, it is necessary to consider more closely the inquiry into the origin of the ideal significant formations "pointing to" the "discovery" in question. What is at issue here is not a known or knowable historical event. Rather, it is Husserl's realization that an inquiry into the constitutive origins of the significance of the ideal formations of mathematical and scientific objects guided by exclusively "epistemological" concerns *cannot* fulfill "the presupposition that the foundations of [their] deductive structure have truly been produced and objectified in original evidence, thus have become universally accessible acquisitions" (*Origin*, F216/C366). Husserl's phenomenological investigations have, of course, taken it for granted from the beginning that the significance

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43. Klein refers here to *Formal and Transcendental Logic*, where Husserl writes: "genetic intentional analysis . . . is directed to the whole concrete nexus in which each consciousness and intentional object as such in each case stands. Immediately the problem becomes extended to include the other intentional references, those belonging to the *situation* . . . and to include, therefore, the immanent *unity of the temporality* of the life that has its '*history*' therein, in such a fashion that every single conscious lived-experience, as occurring temporally, has its own 'history'—that is: its *temporal genesis*" (*FTL*, 278).

44. In Biemel's version of "The Origin of Geometry," the phrase *als subjektive Leistungsgebilde* found in Fink's version (F208) is replaced by *mit diesem Seinssinn* (K367). Also, only *historischen Anfang* (F208) appears in italics in the passage in Fink's version, which Klein quotes and translates here.

(*Sinn*) of the deductive structure of these ideal formations *necessarily* had to have been produced and objectified in such original evidence. Indeed, Husserl never wavers in holding this "assumption" to be inseparable from the integrity of knowledge itself. However, only in his last investigations does one find the realization that "what is lacking [in the 'epistemologically' guided inquiry into this original evidence] is precisely what had given and had to give significance to all propositions and theories, a significance arising from the primal sources that can be made evident again and again" (K376–77/C367). Moreover, this realization is manifest in the paradoxical situation that these investigations describe—the situation Klein characterizes in terms of what "has been becoming increasingly the state of affairs in recent centuries and is the case now" (*PHS*, 78)—that the "'sedimentation of significance' can reach such a degree that a particular science, and science in general, appear almost devoid of 'significance.'" In this case, the primal sources whose evidence is productive of the foundations of scientific propositions and theories is "lacking." And it is just this realization that leads Husserl's transcendental inquiry into the intelligibility of the significant formations of the science of geometry to uncover the reference to the *essential necessity* of their having to have been the product of a "discovery." The intelligibility of such formations is therefore inseparable from the original production of the significance that is now at stake in the transcendental inquiry into their intelligibility.

§ 17. Two Presuppositions Are Necessary to Account for the Historicity of the Discovery of the Ideal Objects of a Science Such as Geometry

The need to inquire into the "discovery" of the original evidence that is the source for the sought-after intelligibility of a science such as geometry therefore has its basis in the fact that such original evidence is *not* forthcoming so long as the "ruling *dogma* of the separation between epistemological elucidation and historical . . . explanation" (*Origin*, F220/C370) is maintained. According to Klein, it is precisely Husserl's inquiry into the "transcendental problem" of the "constitutive origins" of the original evidence in question here that overcomes this ruling dogma by disclosing that such evidence implies "historicity." On Klein's view, this historicity is disclosed by the two additional presuppositions alluded to above, presuppositions that are essentially connected with the constitution of the ideal "intentional units" of a science such as geometry having "of necessity" some definite historical beginning. The first presupposition concerns the discovery that geometry is inseparable from "a characteristically articulated world, . . . the acquaintance with a definitely

shaped and featured 'material,' . . . in short, the experience of 'things'" (75). The second presupposition concerns the first geometer's "'anticipation' (*Vorhabe*) of what comes into being through his 'accomplishment' (*gelingende Ausführung*)," namely, geometry as "a supertemporal product of the mind" (when the mind, deprived of its index of existence, is understood as transcendental subjectivity). By not recognizing and inquiring into these two presuppositions, the phenomenological inquiry into the transcendental constitution of the origins of the evidence that yields the ideal significant formations belonging to a science such as geometry becomes incapable of disclosing the "true significance" (*Crisis*, 53/53) of these formations, "the significance that is authentic, true to the origin."

§ 18. Sedimentation and the Constitution of a Geometrical Tradition

For Klein, Husserl's analysis of the "essential connection" between these two presuppositions and the discovery of geometry is but the first step in his account of the constitution of the historicity of geometry as a science, a constitution whereby geometry's "ideal objectivity" becomes "the property of many individuals" (*PHS*, 76). Only when this occurs does geometry and any science dealing with ideal objects become capable of assuming the meaning of a "tradition" with a "historical development." It is the first step because "the original evidence, experienced during the first actual production," does not "transcend the personal sphere of the subject" (77). Thus, what is at issue in the "discovery" of geometry is not the finding of some historical "fact"—understood as an abiding possession of many subjects—but the transcendental conditions of possibility that give rise to such "facts," conditions of possibility that are necessarily inseparable from establishing the intelligibility of geometry as it is *presently* experienced. For the significant formations whose ideality manifests the intentional objects of the science of geometry as it is "handed down" in the guise of a tradition are inseparable from the original production that *had to* initiate the historicity of its tradition. And it is precisely the latter that is referred to when, in Husserl's transcendental inquiry into the origin of the significant formations of the science of geometry, it is shown that the full intelligibility of the ideal significance of these formations is not forthcoming so long as the inquiry is guided exclusively by "epistemological" interests. This state of affairs, which comes to light in Husserl's analysis of the role and status of geometry in the foundations of Galileo's physics, is captured in his term 'sedimentation'. When used in this context, 'sedimentation' describes the superficial and passive under-

standing of the significance of a science such as geometry that accrues to its significant formations as a result of a "'forgetting'" (*Origin*, F212)[45] of the original evidence that produced these formations.

According to Klein, "[a]t least three steps are required" (*PHS*, 76) for geometry to reach the stage of an ideal objectivity "capable of being handed on." The first step involves the state of affairs whereby "the original evidence, experienced during the first actual production [of geometry as a supertemporal product of transcendental subjectivity], passes over into a 'retentional' consciousness and finally fades away into forgetfulness." This evidence, which "presupposes, of necessity, the whole complex of experiences leading to the situation in which geometry as a science is capable of being 'anticipated' *and* 'intended'" (76), that is, presupposes the "experience of things" and the "handling" (75) of their "shape and measurability" according "to a more or less satisfactory technique," "does not disappear completely: it can be reawakened, and the 'active' remembrance of the original production of any ideal significant formation carries with it the evident experience of the sameness of that formation, carries furthermore the insight into its unlimited reproducibility" (76–77). A "second necessary—and decisive—step" is required, however, if such experience is "to transcend the personal sphere of the subject," namely, "the embodiment of that experience in words, which makes it communicable to other subjects" (77). Such embodiment enables these others "to reproduce the same evident experience out of their own mental activity." Thus, for Klein the "'ideal significant unit' acquires its peculiar manner of existence only through speech and in speech." A last step is required "in order to secure the *lasting* existence of the 'ideal objects,' to establish their perfect 'objectivity.'" This step involves "the translation of the spoken word into the written word." According to Klein, "[a]t this stage the real history of a science may begin."

Klein summarizes Husserl's account of the real history of a science in terms of its being, "of necessity, not only the history of 'progress,' of the accumulation of knowledge, but also a history of failure." For the "means which se-

45. The explicit connection between 'sedimentation' and 'forgetting' is found only in Fink's edition of "The Origin of Geometry" (F212: "Alle Sedimentierung ist in einer gewissen Weise ein 'Vergessen'"; translation: "in a certain way, all sedimentation is a 'forgetting'"), and not in Biemel's publication of presumably Husserl's original version of the essay. Klein, of course, had no way of knowing this when he wrote "Phenomenology and the History of Science." However, in a lecture delivered in 1973, he remarked upon the absence of the sentence in question in the version of "The Origin of Geometry" published by Biemel, and expressed the following view of the matter: "I assume, however, that this sentence [found in Fink's] is based on Husserl's own words, uttered in conversation with Fink." See Jacob Klein, "Speech, Its Strength and Its Weaknesses," in his *Lectures and Essays*, 361–74, here 372.

cure the objectivity of a science at the same time endanger its original integrity." Specifically:

> No science, in its actual progress, can escape the "seduction" emanating from the spoken and written word. For the signifying function of a word has, by its very nature, the tendency to lose its revealing character. The more we become accustomed to words, the less we perceive their original and precise "significance": a kind of superficial and "passive" understanding is the necessary result of the increasing familiarity with spoken and written words. The original mental activity, the production of significance, embodied in sounds and signs, is not reproduced in the course of actual communication. Yet it is there, in every word, somehow "forgotten" but still at the bottom of our speaking and our understanding, however vague the meaning conveyed by our speech might be. The original "evidence" has faded away but has not disappeared completely. It need not be "awakened" even; it actually underlies our mutual understanding in a "sedimented" form.

The "reactivation" of the original evidence, "in order to restore the full significance of all the previous steps leading to a given stage within the development of a science" (77–78), is what for Klein "constitutes the true character of history" (78). Indeed, for Klein it is the "interlacement of original production and 'sedimentation' of significance"[46] that manifests the only "legitimate form of history: the history of human thought" (78). As a result, "the main problem of any historical research is precisely the disentanglement of all these strata of 'sedimentation,' with the ultimate goal of reactivating the 'original foundations.' . . . Moreover, a history of this kind is the only legitimate form of epistemology." Thus, "the problem of history cannot be restricted to the finding out of 'facts' and their connection. They embrace all stages of the 'intentional history.' History, in this understanding, cannot be separated from philosophy."

§ 19. The Historical Apriori of Ideal Objects and Historical Facts

For Klein the historical apriori at issue in Husserl's last works concerns both the historicity of transcendental subjectivity's original production of the ideal significant formations that form the basis of a science such as geometry and the transcendental conditions of possibility that constitute such a science

46. This is Klein's paraphrase of the following passage in Fink's edition (*Origin*, F220): "Geschichte, wie wir sie verstehen, ist nichts anderes als die lebendige Bewegung des Miteinander und Ineinander von ursprünglicher Sinnbildung und Sinnsedimentierung"; translation: "history, as we understand it, is nothing other than the vital movement of the original formation and sedimentation of meaning with and into each other." In Biemel's version (K380), *von vornherein* (from the beginning) is substituted for *wie wir sie verstehen*.

with the status of a "historical fact" (*Origin*, F220/C371)—that is, of something that is "tradition and at the same time a handing-down" (F220/C370). On Klein's view, the historical apriori and the historical fact are each grasped differently.

On the one hand, Klein shows that the key to grasping the historical apriori of the origin of a science such as geometry lies in the "historicity" of the twin presuppositions necessarily entailed in this origin: i) the origin's necessary "anticipation" by a "first" geometer and ii) a "characteristically articulated world," that is, the "experience of things" and the "handling" of their "shape and measurability" according to "a more or less satisfactory technique." The historicity here emerges from the problem of "'epistemological' grounding or clarification" (*Origin*, K381/C373) insofar as the production of the original evidence that "had given and had to give" significance to geometry's propositions and theories is not made manifest so long as these twin presuppositions are not invoked. For Klein, it is precisely the historicity implicit in these presuppositions that provides the basis for rendering intelligible what according to Husserl would otherwise remain unintelligible—namely, the epistemological presupposition that the "deductive structure" of the ideal significant formations of geometry's propositions and theories "have truly been produced and objectified in original evidence." Clearly, what is at issue in the inquiry into the historicity thematized here is neither a known nor a knowable "historical event," but rather the historical apriori that the transcendental inquiry into the "intelligibility" of the ideal significant formations discloses as inseparable from their significance as such.

On the other hand, Klein shows that the key to grasping the historical apriori that manifests the transcendental conditions of possibility that enable a science such as geometry to assume the status of a "historical fact," of a tradition that is still being handed down, lies precisely in Husserl's account of the written embodiment of the "ideal significant units" comprising its propositions and theories. It is therefore the translation of these "ideal significant formations," from their peculiar manner of existence in speech into the written word, that both secures their lasting existence as "ideal objects" and also establishes the possibility of the "sedimentation" of their original evidence. That is, the possibility of their assuming the guise of a tradition capable of being "handed down" is coincident with the forgetfulness of the evidence that is inseparable from the "original establishment" (*Urstiftung*) of these very "ideal significant formations." Thus what Husserl calls the "universal apriori of history" (*Origin*, K380/C371) can be seen to consist in "nothing other" (F220/C371) than this "interlacement of original production and 'sedimentation' of significance" (*PHS*, 78).

§ 20. The Historical Apriori Is Not a Concession to Historicism

Hence Klein shows that rather than representing a radical departure from Husserl's earlier critique of historicism, this critique remains in force in Husserl's formulation of these issues in his last works. It does so because the connection they establish between history and both the origination of a science such as geometry and the existence of its "ideal objects" does *not* have its basis in "the relativity of everything historical, of all historically developed apperceptions" (*Origin*, F221/C373), as historicism maintains, but rather in the following state of affairs:

> All factual history remains unintelligible, always merely drawing its conclusions naively and straightforwardly from facts; it never makes thematic the general ground upon which all such conclusions rest, has never investigated the immense structural apriori that is proper to it. Only the disclosure of the essential general structure lying in our present and then in every past or future historical present as such, and, in its totality, only the disclosure of the concrete, historical time in which we live, in which our total humanity lives with respect to its total, essentially general structure—only this disclosure can make possible historical inquiry which is truly understanding, insightful, and in the genuine sense scientific. This is the concrete, historical apriori that encompasses everything that exists as its historical becoming and having-become or exists in its essential being as tradition and handing-down. (*Origin*, F221/C371–72)[47]

Thus, the apriori in Husserl's concept of a historical apriori is *not* rooted in the attempt to overcome the supposed opposition between the a priori status of the "ideal significant formations" that compose the propositions and theories of a science such as geometry and the contingency of historical facts. Rather, it is rooted in the necessary connection between the very "aprioricity" of the "ideal significant formations" in question and the "actual" history of both their origination and their historical development. It is precisely the latter state of affairs, or the being in question of the "intelligibility" of the "ideal objectivity" of these "significant formations," that motivates the need to extend the transcendental phenomenological inquiry into the origin of such intelligibility, moving beyond the scope of the question of its "epistemological"

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47. In Fink's edition, the second sentence reads: "Nur die Enthüllung der konkreten historischen Zeit hinsichtlich ihrer totalen *wesensallgemeinen Struktur*, die alles Seiende im historischen Gewordensein und Werden oder in seinem wesensmässigen Sein als Tradition und Tradierendes umgreift, kann eine *wirklich* 'verstehende Historie' ermöglichen, die die verborgene 'Vernunft in der Geschichte' an den Tag bringt." Translation: "A *truly* 'understanding historical inquiry,' which brings to the light of day the 'reason concealed in history,' is possible only in the disclosure of the concrete historical time with regard to its total, *essentially universal structure*, which encompasses the historical having-become and becoming of everything that exists, or its essential being as tradition and handing-down."

grounding. The being-in-question of intelligibility, which Husserl regarded as characterizing the "crisis" situation of the European sciences that prevailed in his day, leads to the question and questioning of origin that results in turn in the "exposition of the horizon" of the present (F223/C374). According to Husserl, what is disclosed in this exposition "is not something learned, not knowledge that had ever been current and then sank back [into oblivion]," but something whose "horizon-certainty had to be already there in order to be capable of being explicated thematically" as "a past present" (F220/C374). Indeed, "[t]he thematically emphatic explication of this implicit meaning, the reactivation of the inner tradition of meaning, the awakening of a sediment that is perhaps centuries old, is historical disclosure" (F220).[48] And it is precisely here—and nowhere else—that the "necessary connection" becomes manifest between the immediate presentation of the significant formations of a science such as geometry—formations that manifest an "ideal" aprioricity—and the historical apriori. Inseparable from both the origination of their significance and its "present" status as a "historical fact" in a tradition that is still being handed down, the historical apriori makes intelligible both the non-contingent meaning of science and its existence as a historical fact.

48. The passage in Biemel's edition reads as follows in translation (*Origin*, C370): "Every explication and every transition from making explicit to making evident . . . is nothing other than historical disclosure."

Chapter Seven

Klein's Departure from the Content but Not the Method of Husserl's Intentional-Historical Analysis of Modern Science

§ 21. The Contrast between Klein's Account of the Actual Development of Modern Science and Husserl's Intentional Account

Klein's discussion in "Phenomenology and the History of Science" does not "follow Husserl's pattern" (*PHS*, 79) in his last works of providing an "'intentional-historical' analysis of the origin of mathematical physics," an analysis that for Klein, "although not based upon actual historical research, is on the whole an amazing piece of historical 'empathy.'" Thus, rather than follow Husserl and analyze the foundations of Galileo's physics by treating "Galileo's name [as] somewhat of a collective noun, covering a vast and complex historical situation," Klein tries "to give a general outline of that actual historical development" of mathematical physics. In so doing, he situates his account of this development within the context of his articulation of the significance of Husserl's late confrontation with the problem of "the relation between intentional history and actual history" (74), and thus within the context of Husserl's analysis of the "increase of 'sedimentation' [that] follows closely the establishment of the new science of nature, as conceived by Galileo and Descartes" (79). By proceeding in this manner, Klein operates on the assumption that Husserl's phenomenological analysis of the problem of "true beginnings" has "adumbrated the aims which should control research in the history of science" (65), an assumption that discloses the understanding of history here as being inseparable from philosophy itself. In addition, Klein's outline of the actual historical development of mathematical physics also operates on the assumption of the aptness of Husserl's characterization of the method of historical reflection in the *Crisis.* This is because, for Husserl, the

problem of "sedimentation" emerges from out of the "unique situation" (78) that held sway for him at the time he wrote the *Crisis*, the situation in which both a particular science and science in general "appear almost devoid of 'significance.'" This method characterizes historical reflection as involving "the '*zigzag*' back and forth" from the "'breakdown' situation of our time, with its 'breakdown of science' itself," to the historical "*beginnings*" of both the original meaning of science itself (i.e., philosophy) and the development of its meaning which leads to the "breakdown" of modern mathematical physics.

By focusing on the implications of Husserl's investigations of the historical apriori for research in the history of science, Klein's discussion of Husserl's phenomenology leaves aside both Husserl's mostly undeveloped suggestions regarding the implications of these investigations for "the problems of a universal history in general"[49] and his more developed suggestions that (Husserlian) transcendental phenomenology represents the "final establishment" of the Greek "primal establishment" of philosophy (*Crisis*, 73/72).[50] By contrast, Klein's outline of the actual historical development of mathematical physics does provide an account—albeit in the form of an extremely condensed discussion—of an aspect of the problem of the origin of modern science that Husserl's "careful, if incomplete, analysis" (*PHS*, 74) in "The Origin of Geometry" and the *Crisis* does not "seem to appreciate" fully (70). Klein acknowledges that Husserl "realizes that the discovery of a formal symbolism by Vieta in his establishment of algebra (*ars analytice*, *logistice speciosa*) is at the basis of modern mathematics as well as modern science." However, Klein maintains that Husserl fails to appreciate, "in this connection, the importance of Stevin's algebraic work and, strangely enough, the Cartesian idea of a *mathesis universalis*, based at least partly on Stevin and leading directly to the corresponding, if modified, Leibnizian concept." Klein's outline thus supplements Husserl's account of the sedimentation involved in Galileo's "'Euclidean' approach to the world" (83) with an account of the "'sedimented' understand-

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49. See n. 24 above.

50. Regarding this last point, however, it is interesting to note that the last of the three tasks articulated in Klein's outline of the actual historical development of modern mathematical physics—all of which will be discussed in the final part of this study—concerns the problem of "sedimentation" posed by the origin of the hypothesis of an "exact" nature. Carrying out this task involves—for Klein no less than for Husserl—the "rediscovery of the prescientific world and its true origins." Klein's apparent approbation of this task thus gives rise to the question of whether the disclosure of these "true origins" compels him, as it does Husserl, to embrace transcendental phenomenology as the "final establishment" of Greek philosophy. This question will be addressed in the final part of our study. In addition to posing this question, that last part will also explore the possibility that Klein's research into the history of mathematics may provide guidance for the discovery of sedimented meanings at work in Husserl's understanding of transcendental phenomenology as the "final establishment" of Greek philosophy.

ing of numbers" (84) that made the discovery of the formal symbolism possible. Although Husserl's analyses did not pursue the sedimentation inseparable from this discovery, he nevertheless recognized the discovery's importance, having characterized it as leading to a "completely universal 'formalization'" (*Crisis*, 44/45) of nature by mathematical physics.

§22. Sedimentation and the Method of Symbolic Abstraction

Klein's account of the sedimentation involved in the discovery of formal symbolism focuses on what he calls the "method of *symbolic abstraction*" (*PHS*, 83). He writes:

> It is the method used consciously by Vieta in his establishment of a "general" algebra and by Descartes in his early attempt to set up the *mathesis universalis*. It amounts to a symbolic understanding of magnitudes *and* numbers, the result of which is an algebraic interpretation of geometry. The roots of this development can be found in the adoption of the Arabic system of numeration which leads to a kind of indirect understanding of numbers and ultimately to the substitution of the ideal numerical entities, as intended in all Greek arithmetic, by their symbolic expressions.

The "indirect" understanding of numbers in the method of symbolic abstraction amounts to a "complicated network of sedimented significances [that] underlies the 'arithmetical' understanding of geometry" (84), an understanding that "is superposed upon the first stratum of 'sedimented' geometrical 'evidences,'" which Husserl analyzes in his last works. As we have already pointed out, Klein positions the task of "the reactivation of the process of symbolic abstraction and, by implication, the rediscovery of the original arithmetical evidences" between the two other tasks he identifies within the tripartite problem of the origin of modern science. His articulation of these other tasks adheres to Husserl's presentation of the sedimentation that "follows closely the establishment of the new science of nature" (79), tasks formulated in terms of both "the intentional-historical reactivation of the origin of geometry" (83) and "the rediscovery of the prescientific world and its true origins" (84). But whereas for Husserl the latter is articulated in terms of an "intentional-historical" analysis that links "all our considerations to his [i.e., Galileo's] name, in a certain sense simplifying and idealizing the matter" (*Crisis*, 58/57), Klein's discussion of the "rediscovery" in question outlines the "more exact historical analysis [that] would have to take into account how much of his thought he owed to his 'predecessors.'" Moreover, Klein maintains that the history "anticipated" by the "'anticipation' of an exact nature" (*PHS*, 84) is precisely "the development of the method of symbolic abstraction." This development goes beyond Galileo, "who has not yet at his disposal the powerful instrument of

symbolic formulae" (82), and comes to characterize the modern scientific attitude itself.

§ 23. The Establishment of Modern Physics on the Foundation of a Radical Reinterpretation of Ancient Mathematics

Klein's outline of the "more exact historical analysis" maintains that "[t]he establishment of modern physics is founded upon a radical reinterpretation of ancient mathematics, handed on through the centuries and acquiring a new dignity in the middle of the sixteenth century" (79). Crucial to this "reinterpretation" is the careful study of Euclid's *Elements*, along with the rediscovery of Archimedes, Apollonius, and Diophantus. In addition, the "publication and translation of Proclus' commentary on the first book of Euclid allows a fusion of the traditional theory of ratios and proportions with the 'algebraic' *art* of equations" (80). He continues:

> The algebra (leading back, at least partly, to a Greek tradition represented by Diophantus and Anatolius) and especially the *Arithmetic* of Diophantus are understood as an immediate application of the theory of ratios and proportions. Moreover, the (Eudoxean) "general" theory of proportions, as laid down in the fifth book of Euclid, seems to indicate that the "vulgar" algebra as well as the *Arithmetic* of Diophantus is but a remnant of a more general theory of equations, of a *true* and more general algebra.

On Klein's view, it is François Vieta "who works out the logical and mathematical consequences of this insight and becomes thus the 'inventor' of modern mathematics." Vieta's invention is based in part on his application of the Greek procedure of geometrical analysis to the method Diophantus used in his *Arithmetic* to solve indeterminate numerical equations. As a result, "Vieta postulates a reckoning (*logistice*, λογιστική) using not number but merely 'species' (taking over the Diophantean term 'species,' εἶδος, applied by Diophantus to the various powers of the unknown)" (81). He thus "opposes a 'restored' and 'pure' algebra, the *logistice speciosa*, to the commonly used Diophantean *logistice numerosa*. At the same time, this pure algebra represents, in his mind, the general theory of proportions." The latter, "[d]escribed by Proclus as the 'highest' mathematical discipline, . . . in the form of Vieta's pure algebra becomes from now on the fundamental discipline not only of mathematics but of the system of human knowledge in general." In order "to designate this highest mathematical discipline," Barocius, "the Latin translator of Proclus, . . . uses the term *mathesis universalis*, referring to it on the margin as *scientia divina*." Thus, Klein says, "[i]t is from this source that Descartes, and the entire seventeenth century, have derived the term and the conception of a 'universal science' which includes all possible sciences of man."

Diophantus's method consisted in "setting up an indeterminate equation which is immediately converted into a determinate one by the arbitrary assumption of a numerical value" (80). According to Klein, "[t]his equation has a purely numerical character: apart from the unknown quantity, the 'given' quantities as well as the coefficients of the unknown are definite numbers." Diophantus is said to have referred frequently to "the easily performed checking of the result [of equations that he had solved] in these terms: καὶ ἡ ἀπόδειξις φανερά (and the demonstration [the 'proof'] is obvious)." Because in Greek mathematics a "demonstration" means the "synthesis" that both follows from and reverses the preceding "analysis," "Vieta calls the Diophantean solution an 'analytical' process, referring himself to the traditional definition of analysis as 'the way from the unknown taken as known, through the consequences, down to something which is known.'"[51] However, Vieta's "analytical" interpretation of Diophantus's method represents an innovation, for Greek analysis was limited to treating the unknown in terms of magnitudes. Thus, Vieta's innovation of a "restored" and "pure" algebra is based on the assumption "that the 'general' method behind the 'Diophantean analysis' must be applicable to the numerical *as well as to the geometrical procedure*" (80–81). In creating an *ars analytice* that reckons not with numbers but with their species, Vieta, on Klein's view, "introduced for the first time, fully conscious of what he was doing, the notion of a mathematical *symbol* and the rules governing symbolic operations: he was the creator of the mathematical *formula*" (81).

§ 24. Vieta's and Descartes's Inauguration of the Development of the Symbolic Science of Nature: Mathematical Physics

As a result of Vieta's invention of a "general" algebra, the *mathesis universalis* "bears from the outset a *symbolic* character." And although the method of "symbolic abstraction" used by Vieta to establish symbols and symbolic operations "preserved . . . the original 'ideal' concept of number, developed by the Greeks out of the immediate experience of 'things' and their prescientific articulation," his innovation nevertheless prepared the way for the loss of this "original intuition" of numbers. According to Klein, this is precisely what happens to his "immediate successors, Ghetaldi, Harriot, Oughtred, and Wallis (partly under the influence of Stevin and, as far as Wallis is concerned, of Descartes' *Geometry*)," with the result that "[t]he technique of operating with symbols replaces the science of numbers." Moreover, Klein holds that Descartes,

51. The source of the Greek definition of analysis in Klein's quote is Pappus, ed. Hultsch, II 634. He also refers to the scholium to Euclid xiii, prop. 1–5.

"aiming at the all-comprehensive *mathesis universalis*, and following the algebraic doctrine of Stevin, transforms the traditional understanding of Euclidean geometry into a symbolic one, which transformation is at the basis of his analytic geometry" (81–82).

Klein thus considers it the symbolic understanding of numbers and magnitudes first made possible by Vieta and Descartes, respectively, that "inaugurates the development of a symbolic science of nature, commonly known as mathematical physics" (82). For Klein, it is the method of symbolic abstraction underlying this understanding that solves "the problem of finding the adequate means" (83) for "the anticipated conversion of *all* 'natural' appearances into geometrical entities," an anticipation at work in Galileo's "concept of an 'exact' nature as a great book written in mathematical characters" (82). Klein's general outline of the "actual historical development" of modern science deviates from Husserl's "intentional-historical" analysis thereof by explicitly calling attention to the sedimented understanding of the numerical entities intended by Greek arithmetic. What is at issue in this deviation, however, is not Klein's recognition of something that Husserl failed to recognize, namely, the importance of the role of symbolic formulae in the universal formalization of nature accomplished by mathematical physics.[52] On the contrary, what Klein claims to see that Husserl does not is the explicit linkage of mathematical physics' use of symbolic concepts to an arithmetical sedimentation, which—in addition to the geometrical sedimentation uncovered by Husserl's intentional-historical analyses—is likewise involved in the establishment of the new science of nature. Thus, Klein's outline of the actual history of this establishment represents a more complex view of the matter than Husserl's "simplifying and idealizing" view insofar as the former's extremely condensed account of the method of symbolic abstraction suggests that a "network" of sedimented significances underlies the origin of modern science.

Klein's outline also deviates from Husserl's intentional-historical account by noting the importance of the nominalistic school of the fourteenth century to what Husserl's analysis refers to as the Galilean anticipation of the "possibility of reducing *all* appearances to geometrical entities" (82). Here Klein mentions "Nicolaus Oresmus (Nicole Oresme), whose work *De uniformitate et difformitate intensionum* has profoundly influenced all following thinkers up to Galileo, Beeckman, and Descartes" (83). Klein also questions Husserl's analysis of the Galilean "Euclidean" approach to the world when he calls attention to the fact that the latter understood the geometrical conversion of appearances to involve, in addition to the geometrical magnitudes on which Husserl's analy-

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52. See n. 8 above.

sis focuses, their treatment as "definite ratios and proportions." This latter deviation is of major significance because, on Klein's view, it is precisely Vieta's understanding of his "general" algebra as an immediate application of the Greek theory of ratios and proportions that provides the means for realizing what Husserl's analysis refers to as the Galilean geometrization of nature.

§ 25. Open Questions in Klein's Account of the Actual Development of Modern Science

Klein's general outline of the actual historical development of modern science raises a number of questions. To begin with, there is the question of precisely what the method of symbolic abstraction entails. Klein mentions that this method "is quite different from the ancient ἀφαίρεσις." Yet beyond hinting that this method leads to "a kind of indirect understanding of numbers and ultimately to the substitution of the ideal numerical entities, as intended by Greek arithmetic, by their symbolic expressions," Klein sheds no light on it in his discussion. How the modern method of abstraction differs from the ancient method, to say nothing of what is involved in the ideal numerical entities intended by Greek arithmetic, is not considered. Indeed, insofar as Klein explicitly ties the sedimented understanding of numbers underlying the modern "arithmetical" understanding of geometry to the "task" of reactivating both the process of symbolic abstraction and the "original arithmetical evidences" at issue in Greek mathematics, it would appear that he regards the "rediscovery" of these evidences as something that had yet to be made at the time he wrote "Phenomenology and the History of Science." In addition, it remains unclear why it is that Vieta understands his *logistice speciosa* to be a "restored" algebra in opposition to both the "vulgar" algebra and the Diophantean *logistice numerosa*. And, finally, the question remains open whether—and if so, how—what Klein characterizes as the "*third* task arising from the attempt to reactivate the 'sedimented history' of the 'exact' nature" (84), that is, the task of "the rediscovery of the prescientific world and its true origins," differs from Husserl's formulation of this problem. In other words, the question of the relation of Klein's thought to Husserl's is raised by Klein's very deviation from the "pattern" of Husserl's intentional-historical analysis of the sedimented history of modern science—when Klein turns to the actual history of this sedimentation.

Regarding the first set of questions, it is important to recall that prior to writing "Phenomenology and the History of Science"[53] Klein had in fact

53. In a letter to Klein, dated November 10, 1939, Marvin Farber, editor of *Philosophical Essays in Memory of Edmund Husserl*, invited him to submit a paper to the volume.

already done extensive historical research on both the process of symbolic abstraction and the ideal numerical entities intended by Greek arithmetic. Indeed, his work on these issues was already completed[54] before both

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He wrote that Husserl's son, Gerhart, "has written to me about your ability to have a paper ready for the E. H. memorial volume within a week, or very soon thereafter," and that "unusual circumstances . . . make it possible at this late date to consider another paper." In reply to Farber, Klein wrote on November 12, 1939: "Although the time is very short I can get the article written before the deadline. I shall be grateful to you, if you can extend the time limit to the end of November." Farber eventually extended the deadline to December 5, in response to Klein's November 27, 1939 telegram to Farber requesting an extension to that date.

In his letter to Farber of November 12, Klein described his proposed paper as follows: "The subject of my paper would be something like Phenomenology and History with special reference to the History of science. I have in mind the *Philosophica* essay which you mention in your letter and, in addition, Husserl's article "Die Frage nach dem Ursprung der Geometrie als intentional-historisches Problem" published in the *Revue internationale de philosophie* (Janvier 1939). (It goes without saying that I should have to refer to other publications of Husserl as well.)" "I should like to add that my intention is not to give simply a commentary on those texts but also to examine the notion of History of science as such."

All of the correspondence referred to and cited above may be found among Klein's papers, which are housed in the St. John's College Library in Annapolis, Maryland. I wish to express my thanks to Mr. Elliot Zuckerman, the literary executor of Klein's estate, for permission to cite from Klein's correspondence.

54. The first part of Klein's "Die griechische Logistik und die Entstehung der Algebra" was published in 1934, while the second part was published in 1936. According to Klein's wife, Else "Dodo" Klein, his research for these two extended articles was done during the late 1920s. (Else Klein's recollection is contained in the transcript of a tape recording [the original tape is apparently lost] among Klein's papers, which are housed in St. John's College Library, Annapolis, Maryland.) In a letter to Leo Strauss, dated April 17, 1934, Klein mentions that "the entire part relating to antiquity" was "really done a year ago," but that, "since then the 16th century has intervened." He characterizes the work dealing with antiquity as "really amounting to a discrete whole," and relates that the just mentioned "intervention" permitted him to grasp, "for the first time, a sense of the shape of the whole thing." See "Korrespondenz Leo Strauss – Jacob Klein," in Leo Strauss, *Gesammelte Schriften*, ed. Heinrich Meier (Stuttgart: Metzler, 2001), III: 455–605, here 497–500 (letter 25), esp. 499.

Klein's own assessment (in that same letter) of the entire work is interesting, esp. in light of the obscurity into which it fell soon after its publication: "This presently bears the pompous title 'Greek Logic and the Origins of the Language of Algebraic Formulae,' comprising a little more than seven proof sheets and, at that, compressed to an extreme state of terseness. Much about this work is *not good*. Much is certainly good. How this work will be received by the available public is, unfortunately, only too easy to predict. Since it will be obvious that really substantial work went into this project, 'respect' will not be withheld. The mathematicians will certainly not be persuaded with respect to some of the points. Perhaps, they will accept some of the arguments, and, even, those that are not unimportant. The philosophers will find scarcely anything with which to make a beginning. Most importantly, only a very few—on account of the materials—will understand this book. (So far as I am competent to judge, I can only say: this work is the first attempt to develop a *fundamentally* different approach to the history of the exact sciences and philosophy as it is ordinarily practiced. Probably no one will notice this point, just as it's likely no one will point out the actual, factual weaknesses in this work.)" See also n. 5 above.

Husserl's *Crisis* and Fink's version of "The Origin of Geometry" were originally published.[55] This chronology raises a number of interesting issues and questions, especially when one compares Klein's "general outline" of the actual development of modern science in the "Phenomenology and History of Science" with his analyses of Greek mathematics and the origin of algebra in his *Greek Mathematical Thought and the Origin of Algebra*. As has already been indicated above, what Klein delineates in the "Phenomenology and History of Science" as the "task" of "the reactivation of symbolic abstraction and, by implication, the rediscovery of the original [Greek] arithmetical evidences," he himself had already completed in effect in the early 1930s and published in the German original of *Greek Mathematical Thought and the Origin of Algebra*. This chronology therefore raises the additional question of the relation of Klein's investigations in the latter work to Husserl's phenomenological investigations prior to the publication of the latter's final investigations in the *Crisis* and "The Origin of Geometry." Moreover, the similarity of Klein's treatment of symbolic thinking to Husserl's treatment—first noted by Caton[56] and in effect corroborated by Klein himself when he observes that "Husserl's logical researches amount in fact to a reproduction and precise understanding of the 'formalization' which took place in mathematics (and philosophy) since Vieta and Descartes paved the way for modern science" (*PHS*, 70)—adds considerable force to this question. The remainder of this study will be devoted to investigating the answers to each and every one of these questions.

55. See n. 1 above.
56. See § 5 above.

Part Two

Husserl and Klein on the Method and Task of Desedimenting the Mathematization of Nature

Chapter Eight

Klein's Historical-Mathematical Investigations in the Context of Husserl's Phenomenology of Science

§ 26. Summary of Part One

Part One of our study explored Klein's interpretation of Husserl's turn to the problem of history in his last works. We argued that Klein, alone among Husserl's commentators, recognized that this turn is in harmony with Husserl's lifelong investigation of the phenomenological origins of the ideal meaning formations that make both philosophy and science possible. We also argued that prior to Husserl's account (in the final phase of his work) of the essential connection between historical inquiry and the quest for the epistemological foundations of scientific knowledge, Klein's own investigations of the history of mathematics recognized the same essential connection. We showed, however, that the priority and thus independence of Klein's investigations in relation to Husserl's is a complicated affair.

To begin with, Klein does not hesitate—though necessarily after the fact—to situate his own mathematical investigations in terms of Husserl's articulation of the phenomenological problem of the sedimentation of significance. As we have seen, this problem concerns forgetting the original evidence belonging to the origination of the meaning formations that make a given science (e.g., geometry) possible. Klein accepts Husserl's argument that sedimentation is inseparable from both the primal establishment of science and the historicity of its phenomenal status as a tradition. We have also seen that Husserl characterizes the method of historical reflection that reactivates the forgotten original evidence as involving a back-and-forth or zigzag movement. Beginning with what for Husserl was the present crisis situation of the sciences, reflection strives to uncover the original accomplishments that gave and had to give their formations meaning. As for the crisis itself, we singled out Husserl's account of

the role that the unintelligibility of the epistemological foundations belonging to the meaning formations that make science possible played in the breakdown situation of his time. Finally, we advanced the thesis—but not yet supported it—that the *operative* method of Klein's mathematical investigations is captured by Husserl's articulation of the peculiar zigzag movement characteristic of the method of historical reflection. And we suggested that it is precisely this method that permits—explicitly in Husserl's case and implicitly in Klein's—their historical investigations to overcome the traditional opposition between the epistemological investigation and the historical explanation of science, which is to say, to overcome the problem of historicism.

Despite the fact that Klein situates his own investigations in terms of Husserl's account of the historicity belonging to the a priori foundations of the meaning formations of science, he nevertheless argues that Husserl's account is incomplete. Klein locates the source of this incompleteness in Husserl's "intentional-historical" analysis of the origins of mathematical physics. Thus, while Klein credits Husserl with showing that there is an essential connection between 1) the reactivation of the intentional history proper to the intrinsic possibility of the unity of the meaning formations of scientific objects presented to consciousness and 2) the actual history of their development within natural time, he finds Husserl's intentional-historical investigation of this connection wanting. Despite his appreciation for what he refers to as the historical "empathy" that characterizes Husserl's intentional-historical investigation of the sedimentation inseparable from the development of modern physics, Klein provides a general outline of this development that he purports is based on its actual history. His outline deviates from Husserl's discussion of this development in the *Crisis* on two related counts. First, he identifies the "superposition" of a sedimented understanding of numbers upon the stratum of sedimented geometrical evidence analyzed in Husserl's account of the Galilean spirit of modern physics. Secondly, he explicitly singles out the task of reactivating the process of "symbolic abstraction" responsible for this sedimentation. We noted that Klein's outline is extremely condensed and that it amounts to a précis of the results of his own research—which he does not mention in his discussion of Husserl—on Greek mathematics and the origin of algebra published *before* Husserl's *Crisis*. Thus, we called attention to the curiosity of Klein's failure to mention his own mathematical investigations in connection with his discussions of this issue, a failure that is highlighted by his formulation of both the reactivation of the sedimented evidence in question and the consequent rediscovery of the original arithmetical evidence as a *task*. This formulation suggests, of course, that neither the reactivation nor the rediscovery had been achieved at the time of

his writing about them, a state of affairs that, as will be shown in detail, clearly was not the case.[1]

§ 27. Klein's Failure to Refer to Husserl in *Greek Mathematical Thought and the Origin of Algebra*

We also called attention to the curiosity of Klein's failure to refer to Husserl in his *Greek Mathematical Thought and the Origin of Algebra*. Because Husserl's published works on mathematics at the time Klein published this work had not yet established the connection between the history of mathematics and the investigation of the epistemological foundations proper to mathematical knowledge, the issue of Klein's failure was not seen to involve any question of his unacknowledged debt to Husserl on this score. Klein's precedence in establishing this connection is thus secure and therefore manifestly not what is in question with respect to the relation of his thought to Husserl's. Rather, what is in question is the relation between i) Husserl's pre-*Crisis* logical investigations of the foundations proper to the formalized meaning formations that make possible both modern symbolic mathematics and the *mathesis universalis* of modern philosophy and ii) Klein's own investigations of the historicity of these foundations.

Klein himself invites scrutiny of this relationship when he writes:

> Husserl's earliest philosophic problem was the "logic" of symbolic mathematics.[2] The paramount importance of this problem can be easily grasped, if we think of the role that symbolic mathematics has played in the development of modern science since the end of the sixteenth century. Husserl's logical researches amount in fact to a reproduction and precise understanding of the "formalization" which took place in mathematics (and philosophy) ever since Vieta and Descartes paved the way for modern science. (*PHS*, 70)

Klein goes on to say that "In all of this Husserl is the great interpreter of modern thought—he reveals its hidden implications and presuppositions, he follows and judges its essential tendencies" (71). Just how easy it is to grasp the "paramount importance" of the problem of the "'logic' of symbolic mathematics," however, is an open question. Certainly, it was easy enough for Klein,

1. Unlike the curiosity regarding the absence of any reference to Husserl in Klein's *Greek Mathematical Thought and the Origin of Algebra*, which, as we shall argue below, provides the occasion for raising fundamental questions about the relationship between historical explanation and epistemology in both Husserl's phenomenology and Klein's thought, the curiosity connected with Klein's lack of reference to his own work in his 1938 essay lacks philosophical significance. This is the case because the latter essay employs the basic argument of his own work, even though it remains unmentioned.

2. Klein refers in a footnote here to Husserl's *Philosophy of Arithmetic*. This is the only reference to this work that I know of in either his published or unpublished writings.

since his mathematical investigations in *Greek Mathematical Thought and the Origin of Algebra* dealt with precisely the role of symbolic mathematics in the development of modern science. Indeed, Klein's investigations purport to trace this development on the basis of the shift in the conceptuality of the concept of number that occurred with the modern innovation of formalization and the consequent invention of the mathematical symbol. Yet, as we have also noted, Klein makes no reference in that work to Husserl's thought in his articulation of this shift. Inasmuch as Klein does not hesitate to recognize a close connection between Husserl's logical research and the formalization that he maintains took place in modern mathematics and philosophy, one is left wondering why Husserl's thought, especially his investigations of the logic of symbolic mathematics, did not warrant so much as a footnote in Klein's study.

Klein's 1939 essay, "Phenomenology and the History of Science," makes clear his awareness not only of these dimensions of Husserl's thought but also of their significance for understanding the history of modern mathematics. While this essay was written *after* the publication of his own mathematical studies, it is unlikely that Husserl's thought was unknown to him in the early 1930s, when they were first published. Indeed, it is more than likely that Klein was, at the very least, familiar with Husserl's thought when he was engaged in the research for his mathematical studies.[3] So we are faced with the question: why did Klein not refer to Husserl in his *Greek Mathematical Thought and the Origin of Algebra*?

A closer look at the passage referred to above, combined with an anticipatory sketch of the main lines of the argument and conclusion of Klein's mathematical studies, provides a plausible answer to this question, an answer that the remainder of our study will endeavor to substantiate. What Klein says in this passage is that Husserl's logical research "*amount[s] to* in fact a reproduction and precise understanding of the 'formalization' which took place" (my emphasis) in modern mathematics and philosophy; he does *not* say that Husserl was aware of the fact that this is what his researches amounted to.

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3. Klein's wife, Else "Dodo" Tammann Klein (who, it is significant in this context to mention, had been married to Husserl's son Gerhart), reported that Klein worked on this research during the late 1920s. She also reported that Klein had visited Husserl in Freiburg in 1919 with the hope of studying with him. The postwar housing situation prevented Klein from being able to rent a room there (preference was given to returning veterans), so he went to Marburg to study with Nicolai Hartmann, under whose direction he completed his dissertation in 1922. Moreover, in 1933 Klein became a friend of the Husserl household at the invitation of Husserl's daughter, Elisabeth "Ellie" Rosenberg (née Husserl), who had taken one of the seminars on Plato that Klein had offered in order to support himself. Else Klein's reminiscences of her husband are contained in the untitled typed transcript of the tape recording (see Part I, n. 54) among Jacob Klein's papers.

And therein lies a crucial difference. Klein's main thesis in *Greek Mathematical Thought and the Origin of Algebra* is that the proper evaluation of the *epistemological* significance of this formalization cannot take place independent of a consideration of its *historical* context, which is inseparable from it. Klein argues that such a consideration *cannot* take place if the historical inquiry into this context "starts out from a conceptual level which is, from the very beginning, and precisely with respect to the fundamental concepts, determined by modern modes of thought" (*GMTOA*, 20/5). An inquiry that presupposes the "*fact* of symbolic mathematics" will therefore miss the conceptual transformation of Greek mathematics that makes this fact possible.

Klein's thesis and argument, of course, represents what we called attention to in Part One as his uncanny anticipation of Husserl's central thesis in the *Crisis* that the theory of knowledge is a peculiarly historical task. Prior to these texts, Husserl explicitly rejected the connection between epistemological elucidation and historical explanation. For instance, in *Philosophy of Arithmetic* he writes: "The periods within which the origination of number systems and number sign systems falls are unknown to any historical tradition. Therefore there can be no thought of a reproduction of the historical development" (*PA*, 245). So the perspective from which Husserl's logical research could be seen as "in fact" a "reproduction and precise understanding" of the formalization that took place in modern mathematics and philosophy is a perspective that Husserl himself *lacked* when he was engaged in that logical research. Presumably, Klein did not lack this perspective during his own research into the formalization that made modern symbolic mathematics possible. However, for him to connect this perspective with Husserl's research at that time, he would no doubt have been compelled—by "the things themselves"—to elucidate the *historical* dimension absent from Husserl's own research, a dimension that Husserl himself did not begin to recognize until his historical investigations in the *Crisis*. Such an elucidation would involve, in effect, the "desedimentation"[4]

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4. Neither Husserl nor Klein uses the term 'desedimentation' to refer to the methodical process of reflectively uncovering the sedimented history of meaning bound up with the origin of a science. Rather, as we have seen, Husserl and Klein (following Husserl) both use the term 'reactivation' to refer to this process. Jacques Derrida was the apparent innovator of this term, using it once in his *Edmund Husserl's Origin of Geometry: An Introduction*. There he refers to "an appropriate de-sedimentation" (35/50) as that which makes possible "a phenomenology of the experience" of the origin of "geometrical idealities." Caton also uses the term once in his review of Klein's *Greek Mathematical Thought and the Origin of Algebra*, where he parenthetically characterizes "the philosophic rationale for Klein's historical procedure" as "de-sedimentation," which he describes as involving "a historical investigation which exhibits" the "origin" of "our own 'intentionality'" (Caton, 225). In the present study, the term 'desedimentation' is used to refer to the methodical uncovering of the sedimented history of meaning inseparable from the origin of the scientific concepts that compose the origin of a science,

of the meaning in Husserl's work prior to the *Crisis* that Husserl himself identifies in the *Crisis* as the sedimented historical condition for the formalization accomplished by modern mathematics and philosophy. Given both what we shall see is the complexity of Klein's argument in *Greek Mathematical Thought and the Origin of Algebra* and its fundamental conclusion, we can speculate that avoiding the added burden of situating this work in terms of Husserl's early (pre-*Crisis*) research would have been a most reasonable, and indeed prudent, course for its young author to take.

§ 28. Critical Implications of Klein's Historical Research for Husserl's Phenomenology

A brief anticipation of the main conclusion of Klein's book and the arguments on which it is based will provide support for our speculation about the reason for the absence of any reference to Husserl in the work. This conclusion may be stated as follows: "the conceptual difficulties arising within mathematical *physics* today" (19/4) have their source in the fact that "the fundamental *ontological* science of the ancients is replaced by a *symbolic* discipline whose ontological presuppositions are left unclarified" (193–92/184). According to Klein, "The condition for this whole development is the transformation [effected by its 'formalization'] of the ancient concept of ἀριθμός and its transfer into a new conceptual dimension" (194/185). This transfer results in the ancient ontological understanding of "the things in this world . . . as countable beings, . . . [and] the world itself as a *taxis* determined by the order of numbers," being replaced by a discipline whose ontological investigations are guided by "the *structure* of the world which is grasped by means of a symbolic calculus and understood as a *'lawfully' ordered course of events*." Hence, for Klein, with this transfer the "whole complex of [ontological] problems which presented itself to the ancients because their 'scientific' interest was centered on questions concerning the mode of being of mathematical objects is obviated at one stroke" (126/122). Klein argues that the fundamental methodological preoccupation of ancient science with the meaning of 'abstraction' is replaced in "modern mathematics, and thereby also the modern interpretation of ancient mathematics," by "its attention first and last to *method as such*" (126/123). On Klein's view, the objects of ancient mathematics are yielded by their "abstraction" from the things proper to "'natural' prescientific experience" (125/120), with the result that "the conceptual character of any concept is determined" by its dependence on such experience. In the sharpest possible contrast to this,

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while 'reactivation' is used for the reawakening of the original evidence in which original meanings of these concepts are "anticipated."

modern mathematics "determines its objects *by reflecting on the way in which these objects become accessible through a general method*" (127/123). Thus, for Klein, whereas "ancient mathematics is characterized precisely by a tension between method and object" (126/122), which it attempted to resolve on the basis of the "'general' applicability" (127/122) of its method to the "'generality' of the mathematical objects themselves," that is, "on the basis of an ontology of mathematical objects" (127/122–23), the "new science" is no longer concerned with the

> thing presumed by the concept [as] an object of *immediate* insight. Nothing but the internal connection of all the concepts, their mutual relatedness, their subordination to the total edifice of science, determines for each of them a *univocal* sense and makes accessible to the understanding their only relevant, specifically scientific, content. (125/121)

In sum, Klein concludes that ancient mathematics is *not* symbolic. It always deals with "*determinate* numbers of units of measurement, and it does this *without any detour through a 'general notion' or a concept of a 'general magnitude*.'" Modern mathematics *is* symbolic in the sense that it identifies "the object represented with the means of its representation," and it replaces "the determinateness of an object with a *possibility* of making it determinate, such as would be expressed by a sign which, instead of *illustrating or making intuitive* a determinate object, would *signify* possible determinacy" (127/123).

As will be developed below, Husserl's pre-*Crisis* research on the logic of symbolic mathematics and formalization has two basic characteristics. First, this research is preoccupied with the problem of the foundation proper to the logic operative in the symbolic cognition characteristic of both modern mathematics and modern formal logic. Secondly, the key to resolving this problem for Husserl is a phenomenology of the origin of the basic concepts (and their fundamental relations) that are presupposed, but unaccounted for, by these modern formal disciplines. Husserl characterizes as *formal ontology* the discipline that is concerned with the formal objectivity investigated by both modern mathematics and formal logic. And Husserl's phenomenology is driven, from beginning to end, by the conviction that the latter's proper foundation can be provided only on the basis of

> the *meaning-relation of all categorial meanings* to something individual, which considered noetically is directed to individual evidences, to *experiences*—a relation springing from the genesis of the meaning in question and one that therefore characterizes every possible kind of example used by formal analytics. (*FTL*, 190)

He goes on to say that such evidence and experiences "surely *cannot be insignificant for the meaning and possible evidence of the laws of analytics*, in-

cluding the highest ones, the principles of logic. Otherwise, how could these laws claim *formal-ontological* validity?"

When these two characteristics of Husserl's thought are seen from the angle of the brief—and necessarily condensed—synopsis of the results of Klein's research on the historical context of modern formalization presented above, two things stand out. First, Klein's account of the historical context of the origin of symbolic mathematics pertains to the very same basic concepts and relations that Husserl investigates. Secondly, Klein's conclusion that the symbolic discipline of modern mathematics originates from unclarified ontological presuppositions calls into question the basic conviction of Husserl's phenomenology. That is, it calls into question Husserl's conviction that the proper foundation of formal ontology originates in the modification of evidence drawn from the experience of individual objects.

These preliminary considerations point to the following conclusion. The angle from which Husserl's pre-*Crisis* research on logic is seen to have a fundamental significance for understanding the formalization that is the defining trait of both modern symbolic mathematics and the *mathesis universalis* to which this mathematics gives birth is an angle whose logical conclusion calls into question the basic conviction underlying Husserl's phenomenology of formal ontology. This is to say, the perspective provided by Klein's historical understanding of the formalization of meaning in modern mathematical thought calls for a fundamental critique of the basic intention of Husserl's phenomenology. Such a critique would have as its focal point the latter's fundamental aim to account for the origin of the basic concepts proper to formal ontology in the *immediate* experience and intuition of individual objects. What Klein's research shows is that, above all, the origin of these concepts occurs at a higher conceptual level than the ontology of individual objects on which both ancient Greek science and Husserl's phenomenological science are based.

Because Husserl evidently was unaware—both prior to his *Crisis* and, as we shall endeavor to show, in these texts themselves—of the implications for his phenomenology that lay sedimented in the meaning of the ontology of individual objects, to relate the argument and basic conclusion of *Greek Mathematical Thought and the Origin of Algebra* to this phenomenology would have, of necessity, led to its fundamental critique. For obvious reasons, such a critique would have taken its author very far afield from the explicit topic of his own research. Thus, it is reasonable to assume that Klein judged it best to leave it to others to undertake the task of such a critique. It is just this task, however, that we propose to take on and see through in the remainder of our study.

Chapter Nine

The Basic Problem and Method of Klein's Mathematical Investigations

§ 29. Klein's Account of the Limited Task of Recovering the Hidden Sources of Modern Symbolic Mathematics

In *Greek Mathematical Thought and the Origin of Algebra*, Klein situates his historical investigation of mathematics in terms of the "fact that it is impossible, and has always been impossible, to grasp the meaning of what we nowadays call physics independently of its mathematical form" (*GMTOA*, 19/4). He maintains that this is the case because, "[a]fter three centuries of intensive development, it has finally become impossible to separate the content of mathematical physics from its form" (18/3). This state of affairs is the result of the "intimate connection of the formal mathematical language with the content of mathematical physics," a connection that stems from "the special kind of conceptualization which is a concomitant of modern science and which was of fundamental importance in its formation" (19/4). Klein argues that because of this connection, any discussion of the problems faced by contemporary mathematical physics must have as its necessary propaedeutic "the limited task of recovering to some degree the sources, today almost completely hidden from view, of our modern *symbolic* mathematics." Thus, his study completely bypasses "the fundamental question concerning the inner relations between mathematics and physics, of 'theory' and 'experiment,' of 'systematic' and 'empirical' procedure within mathematical physics"—though, "[h]owever far afield it may run, its formulation will throughout be determined by this as its ultimate theme."

According to Klein the "creation of the formal language of mathematics is identical with the foundation of modern algebra." His study thus sets as its task an inquiry "into the origin and the conceptual structure of this formal language." Klein indicates that this origin may be traced to diverse sources, including the absorption by the West from the thirteenth to the middle of the

sixteenth century of the Arabic science of "'algebra' (*al-g' abr wa'l-muqābala*) in the form of the theory of equations," and "the special influence of the *Arithmetic* of Diophantus on the content, but even more so on the form, of this Arabic science" (19–20/4–5). However, Klein holds that "it was not until the last quarter of the sixteenth century" (20/5) that modern algebra was invented, when "Vieta undertook to broaden and to modify Diophantus' technique in a really crucial way," with the result that Vieta "thereby became the true founder of modern mathematics." Thus, even though prior to Vieta a form of algebra developed that "continually elaborated [it] in respect to techniques of calculation . . . , its self-understanding" failed "to keep pace with these technical advances" (148/151).[5] Klein writes that, consequently, "This algebraic school becomes conscious of its own 'scientific' character and of the novelty of its 'number' concept only at the moment of direct contact with the corresponding Greek science, i.e., with the *Arithmetic* of Diophantus"—that is, at the moment of Vieta's "introduction of a general mathematical symbolism" (149/152).

Klein notes that while conventional accounts of the history of this development recognize the significance of the "revival and assimilation" of Greek mathematics in the sixteenth century, "they always take for granted, and far too much as a matter of course, the *fact* of symbolic mathematics." As a result, they "are not sufficiently aware of the *character* of the conceptual transformation which occurred in the course of this assimilation and which constitutes the indispensable condition of modern mathematical symbolism." As a result, "most of the standard histories attempt to grasp Greek mathematics itself with the aid of modern symbolism, as if the latter were an altogether 'external' form which may be tailored to any desirable 'content.'" Their inquires, therefore, begin from a conceptual level that is foreign to the fundamental concepts of their subject, determined as it is by "modern modes of thought." Thus, for Klein, "[t]o disengage ourselves as far as possible from these modes of thought must be the first concern of our enterprise."

Klein articulates the goal of his investigation of modern algebra's origin as follows: "Hence our object is not to evaluate the revival of Greek mathematics in the sixteenth century in terms of its results retrospectively, but to rehearse or to make present the actual course of its genesis prospectively." Rather than take up the immense task of attempting to trace all the diverse sources responsible for this, which would include "the 'Arabic' positional system of ci-

5. Klein outlines this development as follows: "From Leonardo of Pisa (beginning of the thirteenth century), *via* the 'cossic' school and up to Michael Stifel (1544), Cardano (1545), Tartaglia (1556–1560), and Petrus Nonius (Pedro Nuñez, 1567)" (*GMTOA*, 147/151).

phers, which had been spreading in the West since the twelfth century" (184 n. 130/277 n. 259), Klein focuses his investigation on Vieta's appropriation and transformation of the Diophantine technique. According to Klein, the latter represents, "as it were, a piece of the seam whereby the 'new' science is attached to the old" (20/5–6). Indeed, he rules out the attempt to investigate more than this "seam" because "[t]he acceptance of this sign language [i.e., the Arabic ciphers] in the West *itself presupposes a gradual change in the understanding of number*, whose ultimate roots lie too deep for discussion in this study" (184 n. 130/277 n. 259).

§ 30. Klein's Motivation for the Radical Investigation of the Origins of Mathematical Physics

As we suggested above and shall demonstrate below, Klein's prospective rehearsal and making present of the genesis proper to the results of the revival of Greek mathematics in the sixteenth century traces precisely the back-and-forth zigzag movement that Husserl identifies as the basic characteristic of the method of historical reflection, a method, moreover, that Husserl argues is required in order to redress the then current crisis situation of European science. Before turning our attention to this, however, it is instructive first to consider the account that Klein's 1932 talk, "The World of Physics and the 'Natural' World," provides of what is rather tersely referred to in *Greek Mathematical Thought and the Origin of Algebra* as "the problem which mathematical physics faces today." Presented in 1932 (and thus more or less contemporaneous with his writing of his *Origin of Algebra*), the lecture contains a much more detailed elaboration than the latter of the motivation for turning to the history of science in general and of mathematics in particular in order to respond to the problem confronting contemporary physics.[6] This problem is described there as follows:

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6. Richard Kennington, in his review of the volume in which the text of Klein's talk appears ("Rev. of J. Klein, *Lectures and Essays*," *Review of Metaphysics* 41 [1987], 144–49), rightly observes that Klein's talk "is a useful introduction" (145) to *Greek Mathematical Thought and the Origin of Algebra*.

Given the final form of Klein's talk, it is interesting to note that he had originally intended to speak to an audience of philosophers, but that the organizer of the talk, Gerhard Krüger, apparently also invited physicists. Klein describes the change in the theme of his talk in a letter to Krüger dated December 17, 1931 ("Selected Letters from Jacob Klein to Gerhard Krüger, 1929–1933" [German/English], ed. and trans. Emmanuel Patard, *New Yearbook for Phenomenology and Phenomenological Philosophy* VI [2006], 308–29, here 318–19):

> But I have meanwhile continuously considered the *theme* of the talk, because your proposal, and especially the "official" participation of the physicists, does not really correspond to my original plan. I first wanted to speak only to the "philoso-

> [P]hysics now sees itself faced by questions in its own fundamental work which have always been taken to fall within the domain of philosophy. In its own right physics raises questions about the limits of possible knowledge and the epistemic sense of scientific statements and experimental results. Consequently, it now considers turning to "philosophy" as a reliable and valid court of appeal, if not for solutions to these questions, then at least for advice or new points of view. (*WP*, 2)

Notwithstanding this encroachment of physics upon the traditional domain of philosophy, "at the present time physics and philosophy . . . oppose one another more or less uncomprehendingly" (1). This is the case because "it is clear that no agreement [between physicists and philosophers] about the meaning of the most fundamental concepts which both physics and philosophy employ can be achieved" (2). According to Klein, the root of this disagreement can be traced to the situation that while both physics and philosophy deal with the problem of the world, each speaks about it in a language incomprehensible to the other. Thus,

> the physicists are inclined—not always, certainly, but for the most part—to regard the language of philosophy as unscientific, while the philosophers—not always, to be sure, but frequently enough—suspect themselves of something like bad conscience . . . simply because they are incapable of getting to the bottom of the physical concepts amidst the formalistic thicket of differential equations, tensor calculus, or group-theory.

For Klein, the result of this is that "[b]y the nineteenth century a real and hence mutual understanding between philosophers and physicists concerning the methods, presuppositions, and the meaning of physical research had already become basically impossible" (1).

Klein writes that the consequence of this situation is

> a long-standing controversy over how the experiential bases of physics fit together with its specific conceptuality. [. . .] The reciprocity of experiment

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phers" about "history." But now the basis of the talk has changed, and I considered how one could satisfy *everybody* without running the risk of satisfying *nobody*. *Provisionally*, I have come up with the following:

1) I shall by no means speak about special problems of the latest physics (causality, etc.), and this for very good reasons, which I shall spell out for you in person. Instead, I shall say something—to the physicists—about the remarkable relation of present-day physics to modern "philosophy." I shall trace the problem of philosophy of physics back to the problem history of physics.

2) This makes it necessary to speak—to both the physicists *and* the philosophers—about the particular situation of the 17th century.

3) Finally, I shall then speak—to the philosophers—about the foundation of mathematical natural science in its relation to the "natural" world and about the significance of the historical horizon.

> and theory, of observation and hypothesis, the relation of universal constants to the mathematical formalism—all of these issues point again and again to the two antithetical tendencies pervading modern physical science and giving it its characteristic stamp. [. . .] Nowadays, depending on the side one takes, one speaks of Empiricism or Apriorism; physicists themselves customarily side with the so-called empiricists and confuse apriorism with a kind of capriciously speculative philosophy. (6–7)

Rather than "take sides in this controversy" (7), Klein maintains that "[w]e must first of all try to find a common ground, a basis of shared questions, such that our questions are not in danger of missing their target from the start" (3). Since a common ground is not to be found in the present, he argues that "we have to consider whether we can find it in the past." Indeed, he finds it in the seventeenth century, "an age that did not know this hard and fast division between philosophy and physics," because for the thinkers of that age "the true philosophy coincides with the true science of the structure of this world." Noting that it was not until the middle of the eighteenth century that "the paths of the new science of nature and the new philosophy parted, even though their common origin could never be forgotten" (4), Klein maintains that "the contemporary tense division just noted between physics and philosophy has its roots in precisely this *history* of the two disciplines, a history which leads them from an original unit to an increasing mutual estrangement."

Klein therefore says that "we must try to gain purchase on that common ground by going back to the initial situation, the situation of science in the seventeenth century; from this we might *possibly* gain a measure of enlightenment concerning present-day difficulties, even if we simply come to understand the *nature* of these difficulties better." Klein defends the turn to the historical foundations of physics by calling attention to the fact that "physics itself, even in its most recent phase, has been forced again and again to look back to the past in order to recognize the limited character of many of its basic concepts." He cites as examples the "designation of 'classical physics' . . . [which] arises from the debate between quantum mechanics and relativity theory and the basic concepts of Galilean and Newtonian mechanics," along with "the debates between the mechanistic and the energetic conceptions within physics," which "led to the historical investigations of Mach and Duhem." However, Klein sees that what has to be done now is to "make this turn to historical origins even more radical. Not only is this demanded by the issue itself, it is most intimately connected with the basic presuppositions of our knowledge of the world" (4–5).

§ 31. The Conceptual Battleground on Which the Scholastic and the New Science Fought

Klein begins his radical turn to history "by picturing the general situation of science in the seventeenth century: A *new* science, desirous above all of being a science of Nature and moreover a 'natural' science *opposed an already extant science*" (5). The extant science was, of course, the "traditional and dominant science of the *Scholastics*." The founders of the new science, "[a]s has been emphasized time and again . . . were moved by an original impulse quite alien to the erudite science of the Scholastics." Nevertheless, for Klein the claim of men like Galileo, Stevin, Kepler, and Descartes to communicate "true science, true knowledge, necessarily took its bearings from the firmly-established edifice of traditional science." Consequently, "[t]he battle between the new and the old science was fought on the ground and in the name of the *one*, *uniquely true*, science. One or the other had to triumph; they could not subsist side by side." On Klein's view, "What especially characterizes this battle is not only the *common* goal marked by those most general presuppositions, viz., the one, unique science, but, over and above this, a definite uniformity of the weapons with which the battle was fought" (6). Thus, on the one hand, for him this battle was fought by its antagonists in the name of the one true science. On the other hand, the proponents on each side were largely in accord among themselves regarding the way in which "the contents designated by their concepts" were to be interpreted, "the way in which the concepts *intend* what is meant by them whenever they are employed." Klein refers to this accord as "the conceptual framework or intentionality (*Begrifflichkeit*)[7] in which their antithetical opinions are expressed." According to him, "[t]his

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7. The translator of Klein's Marburg talk, David Lachterman, follows Eva Brann, the translator of Klein's *Greek Mathematical Thought and the Origin of Algebra*, by rendering *Begrifflichkeit* as 'conceptual framework' and 'intentionality' here, though he primarily uses the latter word. The reasons for rendering this key term in Klein's thought more straightforwardly as 'conceptuality' are discussed in my essay "Eva Brann and the Philosophical Achievement of Jacob Klein," in Eric Salem and Peter Kalkavage, eds., *Essays in Honor of Eva Brann* (Philadelphia: Paul Dry Books, 2007), 106–19, esp. 110–14. These reasons may be summarized as follows. First, Klein himself maintains the distinction in *Greek Mathematical Thought and the Origin of Algebra* between *Begrifflichkeit* and the first and second intentions of the scholastic Eustachias that he enlists in his description of the transformation in modern symbolic mathematics of the *Begrifflichkeit* proper to the ancient concept of number. Secondly, Klein could easily have used the term 'intentionality' (*Intentionalität*) instead of *Begrifflichkeit*, but obviously had good reasons for not doing so. The main reason is no doubt connected with the fact that Husserl's concept of intentionality brings with it philosophical suppositions from which Klein wished to maintain a distance (see below, Part IV, §§ 207–8). In the interest of terminological consistency, we shall replace Lachterman's rendering here of *Begrifflichkeit* as 'intentionality' with 'conceptuality'.

accord has all too frequently been overlooked." Once this accord is taken into account, "[t]he only issue is: Which of them handled these weapons most suitably, which of them filled in the conceptuality common to both with contents in harmony with it?" The new science won the battle hands down, of course, because the Scholasticism of the sixteenth and seventeenth centuries, in Klein's words, was "unable to detect the tension between the *contents* of its concepts and the *use* it made of these." Specifically, its unquestioning employment of "the physics of 'substantial forms'" points for Klein to what "an unquestioning understanding of oneself always exhibits," namely, "a failure to comprehend one's own presuppositions and thus a failure to grasp what one pretends to know." Klein writes that "[t]his is the danger to which science is always exposed," and it is "the danger to which science in the sixteenth and seventeenth century succumbed as no other science had done before."

For Klein, then, the turn to the historical origins of the new science, that is, to the origins of mathematical physics, has "to keep this general situation of science in the seventeenth century constantly in mind. It determines in the most basic way the horizon of this new science, as well as its methods, its general structure. It determines, above all, the [conceptuality] of its concepts as such." Thus, Klein understands the return to the common origin of modern science and philosophy in the seventeenth century—in order to better understand the present-day difficulties and indeed the gulf between these two disciplines—to disclose that "What is primarily at stake is an understanding of the *particular* [conceptuality], the *particular* character of the concepts with whose aid the mathematical physics which arose in the seventeenth century erected the new and immense theoretical structure of human experience over the next two centuries" (7). Only in this way can the nature of the "fundamental conceptual shift which took place in the modern era, a shift we can nowadays scarcely grasp," be determined such that the "special character of these new concepts can be brought to light" (8). Klein identifies the *particular* conceptuality at issue here as that of Scholasticism.

According to Klein, the Scholasticism contemporary with the emergence of the new science in the seventeenth century understood itself to be "reproducing ancient doctrine, especially ancient cosmology, in exactly the same way it was understood and taught by the Greeks, that is, by Aristotle" (7). On Klein's view, the new science also interpreted the ancients in this way, that is, in terms of contemporary Scholastic science, although it was "certainly not content with this. Rather, it called upon the things themselves in order to rebuke the untenable doctrines of the Scholastic science." In addition, the new science "went back to the sources of Greek science, neglected by Scholastic science; these sources, too, were interpreted in terms of the [conceptuality] it

shared with Scholastic science. And this interpretation of the legacy of ancient teachings, involving a characteristic modification of every ancient concept, is the basis of the whole concept formation of the new science" (7–8). In other words, for Klein the conceptualities proper to both Scholastic science and the new science are related to "the specific [conceptuality] of Greek science" (17). Consequently, Klein maintains that the special character of the conceptuality proper to the new science "can be brought to light in two ways" (8). Either "we can contrast the Scholastic science of the sixteenth and seventeenth centuries with genuine Aristotelianism," or "we can confront Aristotle himself as well as the other sources of Greek science, most importantly Plato, Democritus, Euclid, Archimedes, Appolonius, Pappus, and Diophantus, with the interpretations" given to them by the moderns.

Both of these ways involve a turn to the history of science—or, more precisely, to the history of ideas—in order to attempt to overcome the present gulf between physics and philosophy by taking up the "scientific" problem of the natural world prior to their emergence as distinct disciplines. In both his Marburg lecture and in his *Greek Mathematical Thought and the Origin of Algebra,* Klein opts for the latter approach, one that involves the attempt to clarify the present problem confronting contemporary physics by articulating the transformation of ancient concepts, and therefore the transformation of the specific conceptuality of Greek science, which resulted in the specifically modern conceptuality that makes this physics possible.

Klein's method can thus be seen to represent, in effect, the anticipation of the "back-and-forth" zigzag movement of the historical reflection Husserl characterized as the most appropriate method for responding to the crisis situation of modern science. And while Klein's articulation of the motive for the turn to history makes no mention of a crisis in contemporary science, his description of the problem of modern physics comes remarkably close to stating what Husserl will identify as the basis of the crisis, namely: the unintelligibility to contemporary epistemology of the meaning belonging to the formalized concepts that make physics possible.

Chapter Ten

Husserl's Formulation of the Nature and Roots of the Crisis of European Sciences

§ 32. Klein's Uncanny Anticipation of Husserl's Treatment of the Historical Origins of Scientific Concepts in the *Crisis*

We shall now highlight Klein's uncanny anticipation in his *Greek Mathematical Thought and the Origin of Algebra* of Husserl's formulation of the nature and roots of the crisis of European sciences, together with the method of historical reflection, by elaborating Husserl's thought on these matters. We shall discuss the fragmentary nature of Husserl's desedimentation of the origins of modern mathematical physics with a view to showing that Klein's account of the genesis of modern algebra "desediments" precisely those aspects of Husserl's historical analyses of the origins of modern mathematics that remain fragmentary in his *Crisis*.

§ 33. Historical Reference Back to Origins and the Crisis of Modern Science

Husserl's investigation of the origin of the intrinsic possibility belonging to the objective unity of any meaning formation was shown in Part One to extend to the a priori structure of its genesis as an intentional unity. It was also shown that the latter holds the key to the insight that Husserl's turn to history in his last writings is the consistent outcome of the phenomenological project of investigating the radical beginnings proper to the things themselves. Far from representing a significant departure from his early rejection of the ability of psychologism and historicism to account for these beginnings, Husserl's late turn to history is motivated by his realization that the investigation of the origins of certain things themselves is not exhausted by uncovering the sedimented history of their genesis in the stream of consciousness. The backward reference (*Rückbeziehung*) to the "past history"

belonging to the original presentation of the ideal meaning formations proper to mathematical and scientific objects proves unable to account for their possibility so long as the genesis of this history is "reactivated" in accord with immanent time's a priori form. The original presentation of such meaning formations therefore transcends the limit of their temporal genesis. This limit is the general substratum of consciousness and it is uncovered in the a priori form of the continuous retentional modifications in which their objective prominence as meaning formations is maintained. Thus, the genesis of these meaning formations transcends the a priori temporal form of the individual stream of consciousness.

That for Husserl the possibility of the ideal meaning formations proper to mathematical and scientific objects transcends the individual stream of consciousness is clear from the distinction he draws between "the intrapsychically constituted structure" (*Origin*, K370/C359) of such "'ideality'" and its "intersubjective being of its own as an ideal object." He characterizes the former in terms of the structure of immanent time insofar as he holds "the original being-itself-there" of ideality, "in the immediacy of its first production," to turn "into the passivity of the flowing fading consciousness of what-has-just-now-been." On Husserl's view, "this 'retention' disappears, but the 'disappeared' passing and being past has not become nothing for the subject in question: it can be reawakened." Husserl likewise understands the reawakening of ideality in accordance with the structure of immanent time, that is, as a "recollection" (K370/C360) that establishes "the capacity for repetition at will" of "the identity" proper to the structure of ideality. However, for Husserl "even with this, we still have not gone beyond the subject . . . we still have no 'objectivity' given." The latter for him requires, as we have seen in Part One (§ 18), first oral and then written communication, whereby "[i]n the unity of the community of communication among several persons the repeatedly produced structure becomes an object of consciousness, not as a likeness, but as the one structure common to all" (K371/C360).

According to Husserl it is precisely the transcendence—vis-à-vis the intrasubjective genesis of ideality—of the objectivity of the meaning formations of mathematical and scientific objects that is responsible for the primal establishment of a science and therewith a scientific tradition. Husserl holds that this primal establishment can be accessed methodically through the historical reflective method of phenomenology, which provides a "historical reference back" to "the concrete, historical apriori that encompasses everything that exists as historical becoming and having-become or exists in its essential being as tradition and handing-down" (K380/C372). It is Husserl's phenomenological quest for the true beginnings of the ideal meaning formations of mathe-

matics and scientific objects, of the ideal meaning formations proper to the mathematical objects that are inseparable from the possibility of mathematical physics, that renders *necessary* the connection between the sedimented history that belongs to the genesis of these formations' objective prominence in the stream and the whole of consciousness and the actual history of their beginnings, which is handed down by the tradition to which these meaning formations belong.

We need look no further than to the phenomenological problem of the genetic origin proper to ideal meaning formations in order to find the motivation for Husserl's turn to history. This is not to say, however, that Husserl's notion of "concrete history" can be reduced to the epistemological problem of "genetic" origins, but that, instead, Husserl came to recognize that this problem conceals "the deepest and most genuine problems of history" (K379/C370). As he puts it, so long as the "ruling dogma of the separation in principle between epistemological elucidation and historical . . . explanation, between epistemological and genetic origin" is maintained, these problems will remain concealed. This separation is therefore unwarranted on Husserl's view, and its overcoming is necessary in order to uncover the concrete historical apriori identified in his last works, an apriori inseparable both from the beginnings proper to the development of the ideal meaning formations that make mathematical physics possible and from the very intelligibility of the formations themselves.

Overcoming the separation between epistemology and history is therefore rooted in the fact that the ideal meaning formations that render the science of mathematical physics possible are, for Husserl, at present unintelligible. Their unintelligibility is first manifest in the "feeling of obscurity that asserts itself" (*Crisis*, 55/55) concerning the epistemological grounding of this science. Husserl's "historical deliberation or meditation" in the *Crisis* should therefore be understood as his *personal* attempt to unpack this feeling. He says as much when he discusses the historical manner of their investigation: "A historical meditation backwards of the sort under discussion is thus actually the deepest kind of self-meditation aimed at self-understanding in terms of what we are truly seeking as the historical beings we are" (73/72). This self-meditation manifests itself in "a constant critique, which always regards the total historical complex as a personal one, [in which] we are attempting ultimately to discern the historical task that we can acknowledge as the only one that is personally our own" (72/70).

Husserl's personal self-meditation unpacks this "feeling of obscurity" by tracing it to its roots in what he felt to be the present crisis situation of the sciences. In articulating this crisis, he focuses on two interrelated points. The

first concerns the "feeling of hostility among the younger generation" (4/6) in their evaluation of what the sciences or science in general can mean for human existence. "Merely fact-minded sciences," we are told, "make merely fact-minded people," the implication being that, having abstracted from everything subjective, neither can possibly have anything meaningful to say about "questions of the meaning or meaninglessness of the whole of this human existence." The second focal point concerns his experience of the unintelligibility of the "completely universal 'formalization'" (44/45) to which the instinctive and unreflective praxis of theorizing begun in Galileo's day leads. As a result, "until now" "there has been no unambiguous characterization of what in fact, and in a way practically understandable in mathematical work, a coherent mathematical field is." This lack of clarity for Husserl then spills over into the self-enclosed idea of a *mathesis universalis* first made possible by this formalization and its development into a formal logic of the 'something in general' (*Etwas-überhaupt*).

To be sure, the first focal point for Husserl is grounded in the second, which he identifies as the source of the "breakdown" of science and the consequent crisis situation of European humanity. Yet the breakdown is the consequence not of formalization per se but of how it is understood and employed by practitioners of the science of mathematical physics that formalization first made possible. When the latter is not carried out in a "fully conscious" (46/47) manner, "dangerous shifts in meaning" occur. Husserl characterizes these shifts as follows. First, what is in truth a method whose goal is to achieve knowledge *about* the natural world is mistaken for the reality of the world itself. Secondly, the tradition that provided the basis for the invention of this method is mistakenly treated as self-evident, which results in unquestioned assumptions that allow "elements of obscurity to flow into its [i.e., the method's] meaning." And, thirdly, owing to both of these mistakes, human subjectivity is misunderstood as an epiphenomenon of the formalized natural world and, consequently, it loses its autonomy as a being possessing a relation to the world independent of scientific intervention.

Husserl's diagnosis of these shifts in meaning is frequently understood to be made possible by his later emphasis on a pair of phenomenological concepts that he had not stressed in his earlier work: the environing world (*Umwelt*) and the life-world (*Lebenswelt*). Husserl's concern with the life-world, so the story goes, is what allows him to call attention to both the unwarranted abstractness of the modern scientific view of the world and its derivative status in relation to the ontologically more original life-world. On our view, however, this account is wrong in a number of crucial respects. First, it reverses the phenomenological order of things. Rather than bring to his

analysis of modern science a fully developed concept of the life-world—which, in its contrast to the overly formalized meaning of the scientific view of the world, would permit him to articulate science's dangerous shifts in meaning—Husserl's analysis in fact moves in the opposite direction. That is, the point of departure for Husserl's analysis of modern science is precisely his personal *experience* of its unintelligibility, an experience that initially has nothing to do with a developed concept of the life-world. Secondly, Husserl's thematic consideration of the life-world is mediated by his critique of the manner in which modern science is traditionally handed down. Thus, Husserl's critical turn to history is *not* motivated by his concept of the life-world. Instead, this concept—or, more precisely, this *phenomenon*—becomes thematic only on the basis of Husserl's historical questioning back to the beginnings of the modern scientific tradition. Thirdly and finally, Husserl's historical deliberation on the unnoticed occlusion of the life-world that he comes to maintain characterizes these beginnings—and the consequent need to render evident for the *first time* their "true meaning" in relation to this phenomenon—articulates a *task* rather than an *argument*. So, rather than argue that the true meaning of modern science is to be located in its derivative status vis-à-vis the life-world, Husserl's *Crisis* endeavors to lay the methodological groundwork for *showing* that and how this meaning is inseparable from its origin in the life-world. And this means that for Husserl the *phenomenological* access to both these beginnings and the life-world is only arrived at through historical critique. In other words, for Husserl the access to—or better: the 'constitution' of—the life-world is inseparable from his investigation of the concrete apriori of history.

For Husserl the unintelligibility of modern science initially shows up in terms of the *referential* obscurity of the formalized meaning-structures that make it possible. This obscurity is in turn bi-directional. On the one hand, *what* in the world these structures refer to is not at all clear to Husserl. On the other hand, insofar as these structures, as a function of their expression of a *mathesis universalis*, also refer to themselves as a self-enclosed "mathematical manifold" that seems to exclude—in principle—all relation to the natural world, *how* in the world it is even possible for this manifold to be applied to the natural world is likewise unclear to Husserl. That the formalized meaning-structures that characterize physics must refer to the natural world is never in doubt for Husserl. Nor does he ever doubt whether, in order for this reference to occur, the "universal manifolds" that characterize the *mathesis universalis* must lend themselves to being applied to the "definite manifolds" of pure space and time, manifolds that function as the system of axioms that make possible the predictions of modern physics. What is in doubt, however, is the foundation that makes possible both this reference and application.

And it is precisely Husserl's experience of the *feeling* of obscurity flowing from the dubiousness of this foundation that motivates what for him holds the promise of resolving it, that is, motivates the need for a historical reference back to the beginnings of the scientific tradition, a tradition that, in its present ungrounded state, he now *experiences* in its "breakdown." To stress Husserl's expression of the felt dimension of his encounter with the obscurity of modern physics is not to suggest that feeling per se somehow provides him with a direct link to the historical reference back that he maintains is all that "could be of help here" (16/17). Rather, it is to suggest that this feeling itself is conditioned by the lack of fulfillment of "the demand, which has spread throughout the modern period and has finally been generally accepted, for a so-called 'epistemological grounding' of the sciences" (*Origin*, K377/C368). Thus the claim here is that it is Husserl's endorsement of this demand, combined with the inadequacy of an exclusively epistemological attempt to meet it, that results in his feeling in question and his realization that "in the case of the sciences genuine historical explanation comes together with 'epistemological' grounding or clarification" (K381/C373). It is precisely in this sense, we maintain, that the felt obscurity in question motivates the need for a historical reference back to the beginnings of the scientific tradition in order to provide the epistemological grounding that Husserl seeks for modern physics.

That the formalized meaning-structures that make modern science possible must refer to the world is rooted in Husserl's conviction that there is a continuity between what philosophy in its primal establishment (*Urstiftung*) conceived of as its task and what now needs to be accomplished in order to bring about its final establishment (*Endstiftung*). That is, for Husserl a continuity must exist between the "exalted idea of universal knowledge concerning the totality of what is" (*Crisis*, 11/13) of ancient Greek philosophy and the modern attempt "to put metaphysics or universal philosophy on the strenuous road to realization" (13/15). Establishing this continuity involves bringing the "reason latent" in the *telos* that is inseparable from the primal establishment of philosophy "to the understanding of its own possibilities and thus . . . to the insight of the possibility of metaphysics as a true possibility." Hence, the overarching task of Husserl's historical deliberation is to restore faith in "reason itself and its (object) 'that which is'" (11/13), a restoration made necessary by the "collapse" of reason in "the actual situation of the present" (16/17) and the consequent skepticism about the possibility of reason, its ideas, and the worldly objects to which they relate.

Philosophy in Husserl's sense is therefore inseparable from history. And history is in turn inseparable from "science as it is given in its present-day form" (59/58). Indeed, it is precisely by following the reference back to science's

beginnings as a tradition that Husserl hopes to uncover the meaning of its development and thereby the true meaning of its accomplishment. Tracing this development, as we have already suggested in Part One and will now articulate in greater detail, requires a method that proceeds "back and forth in a zigzag pattern." It moves backward from science's present form to its beginnings and then forward to its present situation, such that relative clarification of the one yields some clarification of the other, which in turn provides further clarification of the former. Husserl's aim here is to reactivate the "original activity" (*Origin*, K372/C361) that produced the meaning formations presently experienced in terms of their felt obscurity. Only by proceeding in this way is the "hidden unity of intentional inwardness that alone constitutes the unity of history" (*Crisis*, 74/73) revealed. It is revealed because, when reactivated, the beginnings of present science *themselves* refer back to their beginnings, namely, to the primal beginnings of science as such in Greek philosophy and mathematics. Thus, for Husserl the task of revealing the intentional unity of history holds the promise of bringing about the final establishment of the *telos* that belongs to the primal establishment of philosophy, that is, the establishment of philosophy as the universal science of what is.

§ 34. Husserl's Reactivation of the Sedimented Origins of the Modern Spirit

For Husserl philosophy, as the universal science of what is, has but one goal: intuitive knowledge of what is. As we have seen, both what in the world the formalized meaning formations of mathematical physics refer to and therefore make intuitable, and how in the world this reference and corresponding intuition is possible, is obscure on Husserl's view. He traces this obscurity back to the fact that the formalized meaning at issue in modern mathematics is made possible by the progressive "*emptying of its meaning*" (44/44) in relation to the "real" (35/37), that is, to the intuitive givenness of the things manifest to everyday sense experience in the environing world. Husserl's historical reflection on the beginnings of the development of modern, Galilean science reveals that it is first made possible by this progressive emptying of meaning. The meaning formations of the mathematics that make physics possible are themselves made possible by their "becoming liberated from all intuited actuality" (43/44), including the "magnitudes" (44/44) that "numbers are supposed to signify" and of course from the intuitively given shapes of actual things. More precisely, the ideal shapes of Euclidean geometry are substituted for the intuited shapes of things, while

algebraic calculation with "'symbolic' concepts" (48/48) that express numbers in general—as opposed to determinate numbers—excludes the "*original* thinking that genuinely gives meaning to this technical process and truth to the correct results" (46/46).

Husserl's investigations in the *Crisis* of the emptying of meaning that makes modern physics possible are fragmentary, however. Their focus is on the origin of geometry and on what he refers to as the "sedimentation" (52/52) involved in the Galilean impulse to treat Euclidean geometry in a taken-for-granted, and therefore straightforward, manner. Husserl uses the term 'sedimentation' to designate the "constant presuppositions . . . [of the] constructions, concepts, presuppositions, theories" that characterize the significations of the meaning formations of a science—in the case at hand, of Galilean natural science—insofar as they are not "'cashed in'" (*Origin*, K376/C366), that is, reactivated in terms of the original activities that produced their meaning. Cashing in the meaning formations in question requires that we eventually reactivate the "historical beginning" (K367/C356) that this science "must have had," which in the case of Galilean natural science means that we eventually have to reactivate the origin of the Euclidean geometry that was taken for granted when its meaning formations were first established.

Husserl's fragmentary analyses of the "*origin of the modern spirit*" (*Crisis*, 58/57)—in which, as we have seen, he links to Galileo's name "all of our characterizations . . . in a certain sense simplifying and idealizing the matter"—therefore serve in effect to desediment the meaning formations accomplished by this spirit and thereby to reactivate their historical beginnings. Husserl's desedimentation of these meaning formations cashes in both the direct and indirect impulse of the Galilean spirit to mathematize the world by tracing this accomplishment back to its origin in "the sphere of immediately experiencing intuitions and the possible experience of the prescientific life-world" (42/43). Husserl's access to the latter is mediated by the historical reference back to beginnings that issues from the obscure or unintelligible meaning formations of present-day mathematical natural science, the reference back he traces in accord with the back-and-forth zigzag pattern that characterizes his epistemological-historical method. Thus, it is not as if Husserl, sitting in his study, was somehow able to conjure up the direct experience of the prescientific life-world, the primordial experience of which would then provide the basis for a comparison disclosive of the abstract view of the world presumably found in the meaning formations that make up mathematical physics. Rather, it is his experience of the empty meaning formations of physics that—when combined with his expectation that they must somehow be ultimately founded in a reference to (or, more precisely, an intention toward) the world that is ca-

pable of being intuitively fulfilled at some level—leads to his discovery (or, more properly, his re-discovery) of the prescientific life-world and its true origins.

However, rather than rehearse Husserl's well-known analyses that lead to the rediscovery of the life-world in his *Crisis*, what is necessary here is to thematize their salient results and highlight their fragmentary character. Regarding the former, Husserl shows that the Galilean impulse rests on both a *direct* mathematization of the appearances of bodies and an *indirect* mathematization of their sensuous modes of givenness as they show up in the intuitively given environing world. Husserl's attempt to cash in the ideal meaning formations of the pure shapes of Euclidean geometry, which Galileo took for granted as the "true" shapes of nature, reveals that our direct experience of nature *never* yields geometrical-ideal bodies but "precisely the bodies that we actually experience" (22/25). The abstractive directing of our regard to the mere shapes of these bodies cannot yield what the "pure geometry" (21/24) pregiven to Galileo understands as "geometrical ideal possibilities," nor can their arbitrary transformation in fantasy. This is because even though the latter yields "'ideal' possibilities" in a certain sense, these possibilities remain tied to sensible shapes, which can only manifest their transformation into other sensible shapes and thus not the "ideal possibilities" in question.

The method of operating with the pure or ideal shapes that characterizes Euclidean geometry therefore does not point directly back to the sensible shapes of the bodies we actually experience in the life-world, but rather to "the methodology of determination by surveying and measuring in general, practiced at first primitively and then as an art in the prescientific, intuitively given environing world" (24/27). It is therefore the practice of perfecting such measuring, "of freely pressing toward the horizon of *conceivable* perfecting 'again and again'" (23/26), that yields "*limit shapes* [Grenzgestalten] as invariant and never attainable poles" toward which the sequence of perfecting tends. Euclidean geometry is then born when "we are interested in these ideal shapes and are consistently engaged in determining them and in constructing new ones out of those already determined." This is the geometry that was pregiven to Galileo as an established tradition, one that as such was taken for granted. Consequently, the original activity in which its ideal meaning formations were accomplished, the original activity that on Husserl's telling his historical mediations reactivated in order to cash in and therefore make intuitively evident their reference to the world, remained concealed to Galileo. So when Galileo mathematized the intuitive shapes of bodies directly and their sensuous manners of appearing indirectly by substituting for

them the "anticipation" (*Origin*, K367/C356) of their true being in the ideal shapes of Euclidean geometry, the original intuition of the sensible shapes of bodies, along with their transformation into limit shapes by the practice of measuring, became sedimented. As a consequence of Galileo's methodical construction of the "true nature" through the substitution of the ideal shapes of Euclidean geometry for the experience of sensible shapes proper to bodies, the original intuition of the latter was lost. It is this state of affairs that Husserl has in view when he characterizes Galileo as "at once a *discovering* and a *concealing genius*" (*Crisis*, 53/52).

Husserl's analyses of the mathematization that makes possible the "'method of the true knowledge of nature'" (43/43) takes cognizance of the fact that "one thing more is important for our clarification." This "one thing more" is the "'arithmetization of geometry'" (44/44). Aided by "the algebraic terms and ways of thinking that have been widespread in the modern period since Vieta," this arithmetization transforms the ideal shapes of Galileo's Euclidean approach to the world into algebraic structures whose symbolic formula-meaning displaces—"unnoticed" (44/45)—the signification of magnitudes. Husserl considers this the "*decisive accomplishment*" (42/43) of the natural scientific method, which in accord with its "complete meaning" makes possible the anticipation of systematically ordered, determinate predictions about the practical life-world. Accomplished through the hypothetical substruction of undetermined generality by the mathematical idealities that comprise the formula-meaning, the determinate prediction arises through the projection of empirical regularities in advance of the immediately experienced intuitions that ground the possibility of knowledge of the prescientific life-world. Consequently, "[t]his arithmetization of geometry leads almost automatically, in a certain way, to the *emptying of its meaning*" (44/44). Husserl points out that this unnoticed emptying of meaning eventually "becomes a fully conscious methodical deplacement, a methodical transition from geometry, for instance, to pure *analysis*, treated as a science in its own right" (44/45). Thus, this process of methodical transformation leads beyond arithmetization to "a completely universal 'formalization,'" which, in the guise of a *mathesis universalis*, transcends both the pure theory of numbers in algebra and that of magnitudes in analytic geometry. Thought of in "empty, formal generality," manifolds are "conceived of as defined by determinate modalities of the something-in-general." And although Husserl does not explicitly mention it here, the theory of the "formal-logical idea of a 'world in general'" at issue in a "systematic development" (44/45) of the *mathesis universalis* is what he elsewhere refers to as the task of the discipline of "formal ontology" (*FTL*, 94). In contrast to his analyses of the Galilean geometrization of nature, however, Husserl's clarifi-

cation of the meaning formations that make up the *mathesis universalis* and formal ontology does not attempt to reactivate the historical beginnings of the original accomplishment that makes them possible. For instance, he makes no attempt to cash in the sedimentation of meaning that is inseparable from the displacement of the immediate intuitive experience of the life-world that takes place in the arithmetization of geometry, a displacement accomplished in Descartes's formulation of the *mathesis universalis*.

Despite the fragmentary character of Husserl's analyses of formalization, it is clear that he is of the view that the desedimentation of the "*actual coordination* among the mathematical idealities" (*Crisis*, 42/43) that makes formalization and hence the mathematization of the life-world possible would lead to immediate intuitions in the life-world. This is evident from the following passage in the *Crisis*: "If one still has a vivid awareness of this coordination in its original meaning, then a mere thematic shift of one's regard toward this meaning is sufficient in order to grasp the ascending orders of *intuitions* (now posited as approximations) indicated by the functionally coordinated quantities (or, more briefly, formulae); or rather, one can, following these indications, vividly render present the ascending order of intuitions." Likewise, as we have already mentioned, Husserl expresses the conviction in *Formal and Transcendental Logic* that

> the *meaning-relation of all categorial meanings* [Meinungen] to something individual, which considered noetically is directed to individual evidences, to *experiences*—a relation springing from the genesis of the meaning in question and one that therefore characterizes every possible kind of example used by formal analytics—surely *cannot be insignificant for the meaning and the possible evidence of the laws of analytics*, including the highest ones, the principles of logic. Otherwise, how could those laws claim *formal-ontological* validity . . . ? (*FTL*, 190)

However, neither the *Crisis*-texts nor, as we shall see, *Formal and Transcendental Logic* contain, respectively, concrete analyses that vividly render present the ascending order of intuitions in the original meaning of symbolic formulae-meaning or that trace the meaning-genesis of the categorial meanings employed by formal analytics back to individual objects and individual evidence. The question of whether the absence of such analyses is a reflection of their fragmentary nature, or whether it points—as we have suggested—to deeper and perhaps unresolved issues in Husserl's phenomenology, is something we shall not pursue until we have discussed in detail Klein's investigation of the origin of modern algebra.

§ 35. Husserl's Fragmentary Analyses of the Sedimentation Responsible for the Formalized Meaning Formations of Modern Mathematics and Klein's Inquiry into Their Origin and Conceptual Structure

We can now state our most basic thesis concerning Husserl's project of "cashing in" the formalized meaning formations of modern mathematics and Klein's inquiry into the origin and conceptual structure of this formal language. Our thesis is that Klein's execution of this task represents precisely the desedimentation of this "*decisive accomplishment.*" Husserl's analyses identify the "unnoticed emptying of meaning" that accompanies this accomplishment, but do not pursue its desedimentation. To support our thesis, we shall have to show that Klein's research pursued and achieved precisely this desedimentation, when he distinguished "Vieta's 'Ars analytice' from its Greek foundations, and, thus, revealed the conceptual transformation which is expressed in it" (*GMTOA*, 20/6). Yet because, as we suggested above, this transformation on Klein's view entails unclarified ontological presuppositions, we shall argue that the desedimentation of the formalized meaning formations of modern mathematics is not tantamount to cashing in their meaning in terms of their genesis in the individual objects of the pregiven life-world. Thus, we shall argue that, in the case of these meaning formations, Klein's research shows that their desedimentation does not realize Husserl's aim of tracing their meaning-genesis to evidence based in something individual. In other words, we shall show that Husserl's conviction that the latter plays a crucial role in the origin of the formalized meaning formations of modern mathematics is rooted in the unclarified ontological presuppositions that Klein shows to be sedimented in its origin, and that we demonstrate (based on Klein's research) are consequently sedimented in Husserl's phenomenology.

Chapter Eleven

The "Zigzag" Movement Implicit in Klein's Mathematical Investigations

§ 36. The Structure of Klein's Method of Historical Reflection in *Greek Mathematical Thought and the Origin of Algebra*

We shall now set about demonstrating the implicit "zigzag" movement belonging to the method of historical reflection operative in Klein's desedimentation of the formalized meaning formations that characterize modern mathematics. This movement can best be seen by way of an overview of the structure of his investigations in *Greek Mathematical Thought and the Origin of Algebra*. Rather than begin his investigation with a consideration of what he argues is the proximate origin of modern mathematics, namely, Vieta's assimilation and transformation of Diophantus's *Arithmetic*, Klein begins with a consideration of the Neoplatonic literature "which forms its [i.e., the *Arithmetic*'s] background." His rationale for this is that "we must first of all see the work of Diophantus *from the point of view of its own presuppositions*" (20/6). Klein discloses these in terms of the categories of Neoplatonic mathematics and its classification of mathematical sciences, and shows that these "go back to the corresponding formulations in Plato." Yet because the Neoplatonic categories and classifications "were such as to prevent the integration of the *Arithmetic* into this literature," and because the corresponding formulations in Plato are not "identical with them," Klein's task of seeing Diophantus's work "from the point of view of its own presuppositions" must first desediment the Neoplatonic and Platonic background of Diophantus's *Arithmetic*.

Uncovering these presuppositions results in desedimenting the basis in Plato's philosophy for the Neoplatonic division of the science of numbers into theoretical and practical parts, that is, theoretical arithmetic and practical logistic (the art of calculation). This leads, in turn, to Klein's reactivation of the source of Plato's account of a very different configuration of this science, wherein *both* arithmetic and logistic are assigned theoretical and practical

parts. Klein shows that for Plato "Both theoretical disciplines arise directly, on the one hand from actual *counting*, and on the other from *calculating*" (21/6). Klein also shows that the Neoplatonists ignored Plato's "postulation of a theoretical logistic as a noetic analogue for, and as the presuppositions of, any art of calculation" (21/7), because of an essential peculiarity of "the ἀριθμός-concept which forms the basis of all Greek arithmetic and logistic," including therefore of Plato's postulation of a theoretical logistic as well. It is this state of affairs that leads Klein to desediment the ἀριθμός (number) concept operative both in Plato's thought in particular and in Greek mathematics in general. Klein accomplishes this by reactivating the most basic meaning of this concept along with the original process of counting that, in making "use of a prior knowledge of 'counting-numbers' which are already in our possession" (22/7), gives rise to the basic problem of theoretical arithmetic: unfolding the true presuppositions of this practical activity. His reactivation of the most basic meaning of the ἀριθμός shows that it "*never* means anything other than 'a definite amount of definite objects.'"[8] This meaning forms the basis of the theoretical mathematical understanding of the ἀριθμός that makes counting possible, inasmuch as this activity is possible only on the basis of "definite amounts of 'undifferentiated' objects, namely assemblages of 'pure' units or monads, which must already be in our possession prior to the practical activity of counting."

Klein's reactivation of the original meaning of the ἀριθμός concept operative in Greek mathematics shows that Greek arithmetic is originally concerned with the theoretical "problem of the possibility of such assemblages, i.e., the question how it is possible that *many* 'ones' should ever form *one* definite amount of 'ones.'" This "leads to the search for the εἴδη with definite 'specific properties' such as will give unity to, and permit a classification of, all definite amounts." Greek arithmetic, then, is not a theory of numbers but a theory of the εἴδη of numbers.[9] Klein also shows that for Greek mathematics the art of

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8. Klein renders the Greek word ἀριθμός as *Anzahl* to distinguish it from *Zahl*, which he uses for the modern symbolic number concept. *Anzahl* and *Zahl* are translated as 'number' in the English translation of *Greek Mathematical Thought and the Origin of Algebra*, notwithstanding Brann's "Translator's note," wherein she states that "*it is a chief object of this study to show that Greek* 'arithmos' *and modern* '*number*' *do not mean the same thing*, that they differ in their *intentionality*, for the former intends *things*, i.e., a number of them, while the latter intends a *concept*, i.e., that of quantity" (vii). (Regarding the choice of 'intentionality' as the translation of *Begrifflichkeit*, see n. 7 above.) In order to reflect in English the distinction at issue in Klein's German choice of terms, *Anzahl* will be translated in this study as 'definite amount' or 'definite amount of definite objects', depending on the context, while *Zahl* will be translated simply as 'number'.

9. In his *The Evolution of the Euclidean Elements* (Dordrecht: Reidel, 1975), 134, Wilbur Richard Knorr substantiates this claim, and holds that "Jacob Klein reasonably inter-

calculating, and therefore theoretical logistic, considers these determinate amounts "only with reference to their 'material,' their ὕλη, that is, with reference to the units as such. The possibility of theoretical logistic is therefore totally dependent on the mode of being which the pure units are conceived to have" (22/8).

Klein's reactivation of the meaning of the fundamental problems of Greek mathematics shows that the reason why the Neoplatonists ignored Plato's postulation of a theoretical logistic is rooted in the latter's account of the indivisibility of the mode of being proper to these pure units or monads. Specifically, "the use of fractional parts of the unit of calculation, which is unavoidable in calculations, cannot be justified on the basis of such monads" (21/7). Klein's desedimentation of Plato's account of this mode of being discloses that it is rooted, in turn, in the ontological presuppositions of his philosophy. Klein reactivates these presuppositions by showing, first, the Pythagorean horizon of "cosmological 'mathematics' and its connection with the ἀριθμός concept" (22/8) that inform Plato's ontological presuppositions. Second, he reactivates "the significance which is attached to 'the ability to count and calculate' in Platonic philosophy." This leads to a consideration of the "privileged position he assigns to the theory of number" for the ontological problem of participation (μέθεξις), a problem "to which his 'dialectic' necessarily leads, without, however, being of itself able to provide a solution." The theory of number, however, is shown to lead to the discovery of a fundamental solution to this problem, a solution that is based on the distinction between eidetic and mathematical ἀριθμοί. Klein thus shows that Plato's philosophy repeats in the realm of the ideas themselves "the Pythagorean attempt at an 'arithmological' ordering of all being," a repetition, moreover, "which amounts to a decisive correction of the Eleatic thesis of the 'One.'"

Klein's desedimentation of the ontological presuppositions proper to Plato's conception of eidetic and mathematical ἀριθμοί provides the basis for his reactivation of Aristotle's critique of the mode of being attributed to them in Platonic philosophy. Rather than have a being that is "self-subsistent and originally 'separate' from sense perception," Aristotle, on Klein's view, "shows that the 'pure' units are merely the product of a 'reduction' [αφαίρεσις] performed in thought, which turns everything countable into 'neutral' material" (22–23/8). Having no being of their own, "[t]heir indivisibility is only an expression of the fact that counting and calculating always presuppose a last, irreducible 'unit,' which is to be understood as the

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prets . . . Plato's intent to define with full scientific rigor the fields of the theory of number (i.e., logistic and arithmetic) without reference to the concepts of number or numbered things."

given measure" (23/9). Aristotle's critique therefore allows the introduction of "a new and 'smaller' measure," which provides the ontological basis for realizing the demand for a "scientific" logistic that is made, but cannot be realized—because of the indivisibility of the unit—in Platonic philosophy.

Having desedimented the work of Diophantus "from the point of view of its own presuppositions," Klein then shows that his *Arithmetic* has the status of a "theoretical logistic." As such, it "always retains a dependence on the Greek ἀριθμός concept, although it apparently incorporates a more general, pre-Greek 'algebraic' tradition as well." The reactivation of the Diophantine technique provides the context for Klein's reactivation of Vieta's transformation of this technique, which in turn amounts to the desedimentation of the Greek context for his innovation of the algebraic symbolism that makes modern mathematics possible. This desedimentation has as its consequence both the reactivation of the process proper to the "symbolic abstraction" that makes possible the symbolic language of algebra and modern mathematics and the articulation of the unclarified ontological presuppositions that are inseparable from this process. Finally, Klein desediments these presuppositions as they function in Stevin's, Descartes's, and Wallis's thought, which allows him to "trace out the general transformation, closely connected with the symbolic understanding of number, of the 'scientific' consciousness of later centuries."

By way of a summary, we can articulate the main lines proper to the zigzag movement of Klein's historical reflections as follows. The questions facing the physics of Klein's day, in the guise of the philosophical unintelligibility of the significance of its cognitive achievement, motivate a historical reflection on the seventeenth-century origins of its basic concepts. This reflection represents the beginning of the "zig" movement, whose aim is to reactivate these origins in order to prepare the way for the reflective "zag" movement, which, guided by the insight into the basic concepts attained by the previous "zig" movement, will be better able to understand the problems facing physics in its historical present. However, for the initial reflective "zig" movement to accomplish its aim, it must extend its scope beyond the seventeenth century, back to the origins of the Greek concepts that played a crucial role in the origination of physics. This extended "zig" movement reactivates four—namely, Neoplatonic, Platonic, Pythagorean, and Aristotelian—levels or strata sedimented in these origins, each of which allows the corresponding reflective "zag" movement to clarify the key concepts of Greek mathematics that were transformed by the originators of modern mathematics in the seventeenth century.

Chapter Twelve

Husserl and Klein on the Logic of Symbolic Mathematics

§ 37. Husserl's Systematic Attempt to Ground the Symbolic Concept of Number in the Concept of *Anzahl*

As Klein notes, the logic of symbolic mathematics was Husserl's first philosophical problem. Husserl's investigations in *Philosophy of Arithmetic* seek to establish the foundation of symbolic mathematics, which he also calls 'universal arithmetic', on the authentic concept of cardinal number (*Anzahl*).[10] To anticipate the results of our own investigation of Husserl's treatment of this problem in the next chapter, it begins with the assumption of the logical equivalence, in the sense of the identity of their object, of the contents of the authentic and symbolic concepts of number: Husserl initially presents each as referring to the determinate unity of a determinate multitude of units, albeit directly in the case of the authentic concept of cardinal number and indirectly in the case of the symbolic concept of number. Husserl's investigation seeks to show that the foundation of symbolic mathematics, and thus its logic as well, lies in the authentic concept of *Anzahl*.

In the final chapters of *Philosophy of Arithmetic*, however, Husserl concludes that the calculative algorithms manifested by the sense-perceptible signs of the symbolic calculus have a "signitive" status that rules out their foundation in arithmetical concepts generally and in the concept of cardinal number specifically. In other words, Husserl rejects his initial thesis of the logical equivalence proper to the contents of the concept of *Anzahl* and the concept of symbolic number, and therefore no longer thinks it possible to ground the logic of the calculation with the latter in the concept of the former.

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10. See Part III, n. 2, for a discussion of this translation.

§ 38. Klein on the Transformation of the Ancient Concept of Ἀριθμός (*Anzahl*) into the Modern Concept of Symbolic Number

Our overview above of the content of Klein's method of historical reflection in his *Greek Mathematical Thought and the Origin of Algebra* allows us to see that its investigations purport to reach historically the same conclusion that Husserl's investigations arrive at systematically, namely, that the logic of symbolic mathematics cannot be grounded in the concept of *Anzahl*. Klein's research establishes that the Greek ἀριθμός, defined as the determinate unity of a determinate multiplicity of units, is the paradigm of the pre-modern, non-symbolic concept of 'number' as *Anzahl*. The identity of their conclusions in this regard has led one commentator, J. Philip Miller, to assume that Klein simply uses the "categories" generated by Husserl's a priori analyses in *Philosophy of Arithmetic* "in studying the actual history of mathematical thought."[11] Klein himself, however, points the way to a different interpretation of the reason that his research in *Greek Mathematical Thought and the Origin of Algebra* arrives at the same conclusion as Husserl's "a priori" analyses, namely, his view, discussed above, that Husserl's foundational approach to the problem of the logic of symbolic mathematics, in effect, amounts to "a reproduction and precise understanding of the 'formalization' which took place in modern mathematics and philosophy" (*PHS*, 70). And this occurrence, of course, is precisely what Klein's *Origin of Algebra* purports to present within a major strand belonging to its historical development.

Our analyses of Husserl's and Klein's accounts of non-symbolic numbers will demonstrate, moreover, that Miller's assumption cannot withstand critical scrutiny. Despite the fact that both Husserl and Klein define the concept of *Anzahl* as the determinate (delimited) unity of a determinate multitude of units, they nevertheless define these units in radically different ways. According to Husserl, the "unity" of the "units" in the multitude that composes an *Anzahl* is established "logically," by their "falling" (abstractively) under the materially empty (and therefore "formalized") concept of the *Etwas* ('anything'; later, *Etwas überhaupt*, or 'something in general'). According to Klein, the "unity" of the "units" composing an *Anzahl* as ἀριθμός is established ontologically, in the precise sense of the theoretical supposition of a multitude of beings that are both identical and indivisible. Miller's claim

11. J. Philip Miller, *Numbers in Presence and Absence: A Study of Husserl's Philosophy of Mathematics* (The Hague: Nijhoff, 1982), 132. The passage in full runs as follows: "Although Husserl's own analyses [i.e., in *Philosophy of Arithmetic*] move on the level of a priori possibility, Klein's work shows how fruitful these analyses can be when the categories they generate are used in studying the actual history of mathematical thought."

that Klein's study uses the categories of Husserl's analyses overlooks their different accounts of this very basic "category."

The guiding aim of the present study is to establish the immense importance of these different accounts of the "unity" of the units that compose an *Anzahl*, that is, a non-symbolic number, for Husserl's and Klein's respective accounts of the origination of the logic of symbolic mathematics. We shall demonstrate that Husserl's account of this "unity"—and, indeed, any "unity"—as a formalized concept, prevents his thought (both his pre- and properly phenomenological thought) from accounting for the origination of formalized unity. Because Husserl's account of what is properly the content of a non-formalized concept, the concept of *Anzahl*, introduces a formalized concept to account for the "unity" of its component "units," his thought, from beginning to end, remains incapable of providing an account of the origin or genesis of "formalization" that satisfies his own requirements regarding such an account. These requirements are stipulated in his demand for a "theory of judgment" that provides foundational evidence derived from the "pre-formalized" experience and givenness of individual objects. Husserl's presupposition of "formalized" unity and therefore formalization in his account of the units in a non-symbolic number, and in his account of unity per se, consequently prevents his analysis from satisfactorily establishing the phenomenological origin of the logic of symbolic mathematics—or so it will be our burden to establish in support of an underlying argument of this study.

Klein makes no such presupposition about the unity of the units composing non-symbolic numbers. Indeed, the fundamental finding of his *Greek Mathematical Thought and the Origin of Algebra* is that precisely the historical-philosophical discovery of this presupposition not only permits it to be withdrawn, but that its withdrawal holds the key to providing a philosophical account of the origination of the logic of symbolic mathematics that does not tacitly assume that the formalization coincident with this origination has already occurred.

§ 39. Transition to Part Three of this Study

Owing to the priority of Husserl's systematic investigations of the structure and origin of the concept of *Anzahl*, of the structure and origin in the case of the concept of symbolic number, and of his conclusion that the logic of symbolic mathematics cannot be grounded in the concept of *Anzahl* (and, indeed, in any concept), Part Three of our study begins with a detailed exposition and analysis of Husserl's *Philosophy of Arithmetic*. Our account of these issues will provide the context for the detailed exposition and analysis of Klein's histori-

cal-philosophical investigations of what we suggest here are the very same issues. And these expositions and analyses will provide the basis for Part Four's demonstration of our thesis that, while it is Klein, and not Husserl, who provides the most compelling account of the philosophical origination of the logic of symbolic mathematics, it is Husserl who (in his *Crisis*) provides the philosophical rationale that renders Klein's account the most compelling.

Part Three

Non-symbolic and Symbolic Numbers in Husserl and Klein

Chapter Thirteen

Authentic and Symbolic Numbers in Husserl's *Philosophy of Arithmetic*

§ 40. The Shortcomings of *Philosophy of Arithmetic* and Our Basic Concern

Husserl's *Philosophy of Arithmetic* is an attempt to establish the foundation of arithmetic by means of a psychological account of what he refers to as "cardinal number [*Anzahl*][1] in the true and authentic sense of the word"

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1. In *Philosophy of Arithmetic*, Husserl uses the German terms *Zahl* and *Anzahl* interchangeably, e.g.: "Die allbekannte Definition des Begriffes der Zahl – so dürfen wir konform mit der gemeinüblichen Sprechweise kurzweg für Anzahl sagen" (*PA*, 14). While both may be rendered as 'number', the latter has a more determinate meaning than the former and would best be translated as 'counting number' or 'amount or quantity of things'. Yet because such a translation would be awkward, we adopt the convention of translating *Anzahl* as 'cardinal number', defined as 'the number used in simple counting to indicate how many items there are in an assemblage'. As a consequence of our adoption of this convention, we deviate from Dallas Willard, the English translator of *Philosophy of Arithmetic*, who renders *Anzahl* variously as 'whole number', 'number', and 'cardinal number'. Husserl himself, however, seems to provide warrant for the convention we have chosen. On the very first page of his introduction to *Philosophy of Arithmetic*, he equates *Anzahlen* with cardinal numbers. Thus, in connection with the observation that the "Begriff der Zahl ist ein vielfacher" (10), he articulates as one such concept "die Anzahlen oder Grundzahlen (numeralia cardinalia)" and goes on to say with respect to *Anzahlen* that "the other characteristic names they also bear—basic [*Grund-*] or cardinal numbers [*Kardinalzahlen*]—are not founded on mere convention." In his *Numbers in Presence and Absence* (42 n. 11), however, J. Philip Miller rejects 'cardinal number' as a translation of *Anzahl* in Husserl's text. Miller's reasons for rejecting this translation are the following: "*Anzahl* is sometimes translated as 'cardinal number,' *Zahl* simply as 'number.' But in the context of Husserl's philosophy, this seems somewhat inappropriate. It suggests that the mathematical concept of number is the basic one, while *Anzahl* is merely a special case. But Husserl's central point is precisely that the fundamental sense of number is *Anzahl*." While we would agree with Miller that the fundamental sense of number for Husserl is *Anzahl*, we cannot, especially in light of the explicit connection that Husserl makes between *Anzahlen* and *Kardinalzahlen*, follow Miller's argument for rejecting 'cardinal number' as a translation of *Anzahl*.

(*PA*, 116).[2] His critical self-understanding of the failure of this attempt has met with general acceptance in the literature. There is, however, no consensus regarding exactly why Husserl's attempt failed. Gottlob Frege's critical review of *Philosophy of Arithmetic*,[3] in which he took Husserl to task for

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2. That arithmetic, in the sense of a "universal arithmetic," including all the higher operations of arithmetical analysis, has its foundation in cardinal numbers is a view Husserl took over from his mathematics teacher, Karl Weierstrass. See Miller, *Numbers in Presence and Absence*, 1–5; see also Carlo Ierna, "The Beginnings of Husserl's Philosophy, Part 1: From *Über den Begriff der Zahl* to *Philosophie der Arithmetik*," *New Yearbook for Phenomenology and Phenomenological Philosophy* V (2005), 1–56, here 39, 51–52; and "The Beginnings of Husserl's Philosophy, Part 2: Philosophical and Mathematical Background," *New Yearbook for Phenomenology and Phenomenological Philosophy* VI (2006), 23–71, here 38–43. The view that to properly establish this foundation in the logical structure of the concept of cardinal number one must employ psychological analysis can be traced back to Husserl's philosophy teacher, Franz Brentano (see Ierna, "Beginnings of Husserl's Philosophy, Part 2," 20–34).

Husserl came to abandon the view of the foundational status of cardinal numbers for universal arithmetic in the time period between the completion of his *Habilitationsschrift* in 1887 and the publication of *Philosophy of Arithmetic* (which is largely based upon the *Habilitationsschrift*) in 1891. In a letter to the director of his *Habilitationsschrift*, Carl Stumpf, written in 1890, Husserl explicitly states that he has abandoned his commitment to the foundational status of cardinal numbers and gives his reasons for doing so. The letter will be discussed in detail below. The introduction to *Philosophy of Arithmetic* also reflects, though less clearly, Husserl's change of mind, as he characterizes the work's claims regarding the foundational status of cardinal numbers as, in effect, a working hypothesis, which is adopted "in no way to anticipate any definitive resolution of the issue" (*PA*, 12). However, the text of *Philosophy of Arithmetic* itself does not explicitly discuss the philosophical implications of its author's abandonment of his commitment to the foundational role of cardinal numbers for arithmetic, but it rather contains, in some cases literally side by side, analyses that maintain this thesis and others that call it into question.

In 1906, Husserl himself, after rereading *Philosophy of Arithmetic*, remarked "how naive and almost childlike that work appeared to me!" And that "it was not without reason that I was conscience-stricken upon its publication." He goes on to say, "Actually, I had already gone beyond it as I published it. Indeed, it was drawn in essentials from the years 1886 and 1887." See Edmund Husserl, "Personal Notes," in *Early Writings in the Philosophy of Logic and Mathematics*, trans. Dallas Willard (Dordrecht: Kluwer, 1994), 490–500, here 490 (notes dated September 25, 1906); henceforth cited as *Personal Notes*.

Our discussion of *Philosophy of Arithmetic* will be concerned with neither philological questions about its genesis nor historical questions about exactly when Husserl abandoned its underlying mathematical thesis. Rather, our focus in this chapter will be on its—however imperfect—analyses and the conclusions these permit to be drawn regarding Husserl's initial encounter with the problem of the origin of the logic proper to symbolic mathematics. (For an instructive discussion of the philological and historical issues surrounding the composition of *Philosophy of Arithmetic*, see Parts 1 and 2 of Ierna, "The Beginnings of Husserl's Philosophy.")

3. Gottlob Frege, "Rezension von: E. G. Husserl, *Philosophie der Arithmetik*. I," *Zeitschrift für Philosophie und philosophische Kritik* n.s. 103 (1894), 313–32 (reprinted in Gottlob Frege, *Kleine Schriften*, ed. Ignacio Angelelli [Hildesheim: Olms, 2d ed., 1990], 179–92; original pagination is noted in the headers); English translation: "Review of Dr. E. Husserl's *Philosophy of Arithmetic*," trans. E. W. Kluge, in Frederick A. Elliston and Peter McCormick, eds., *Husserl: Expositions and Appraisals* (Notre Dame, Ind.: University of Notre Dame Press, 1977), 314–24; henceforth cited as 'Frege' with German and English page references, respectively.

"the influx of psychology into logic" (Frege, 324/332)—to the mutual detriment of each—along with certain of Husserl's remarks have lent credence to a widely held view that Husserl's main dissatisfaction with *Philosophy of Arithmetic* can be traced to the work's psychologism, that is, to its reduction of both the objects of logical concepts and the objectivity of these concepts themselves to psychological presentations. On this view, Husserl's statement in the Foreword to the first edition of the *Logical Investigations* about his "doubts of principle, as to how to reconcile the objectivity of mathematics, and of all science in general, with a psychological foundation for logic,"[4] along with his remark recorded by W. R. Boyce Gibson "that Frege's criticism of the *Philosophy of Arithmetic* . . . hit the nail on the head,"[5] are interpreted as endorsing Frege's criticism. However, as Dallas Willard has noted, "one searches in vain for passages in [Husserl's] earlier writings where he advocated such a psychologistic logic."[6] In addition, J. N. Mohanty has shown that Husserl did not acquire the distinction between 'object', 'concept', and 'presentation' from Frege.[7]

It is our contention that any interpretation and assessment of *Philosophy of Arithmetic* must keep in mind Husserl's own understanding of the work as a "sequence of 'psychological *and logical* investigations'" (*PA*, 5, my emphasis) that "claims to prepare the scientific foundations for a future construction" of arithmetic. Consequently, the presentation of arithmetic in that work must be kept distinct from its psychological-philosophical account of arithmetic's foundations. Husserl's grasp of the basic concepts and operations of the former should not be equated with his—admittedly "psy-

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4. Edmund Husserl, *Logische Untersuchungen. Erster Band: Prolegomena zur reinen Logik*, ed. Elmar Holenstein, Husserliana XVIII (The Hague: Nijhoff, 1975), 6–7; English translation: *Logical Investigations*, trans. J. N. Findlay, 2 vols. (New York: Humanities Press, 1982), I: 2. Henceforth referred to as *Prolegomena*, whereas the investigations are referred to collectively as *Logical Investigations* or simply *Investigations* or individually by number; quotations are cited as *LI* followed by German and English page references, respectively.

5. W. R. Boyce Gibson, "From Husserl to Heidegger: Excerpts from a 1928 Diary by W. R. Boyce Gibson," ed. Herbert Spiegelberg, *Journal of the British Society for Phenomenology* 2 (1971), 58–81, here 66.

6. Dallas Willard, "Husserl on a Logic that Failed," *Philosophical Review* 89 (1980), 46–64, here 46–47.

7. J. N. Mohanty, "Husserl and Frege: A New Look at their Relationship," *Research in Phenomenology* 4 (1974), 51–62. Consider Miller, who also argues that Husserl's admission in the *Logical Investigations* of the psychologism of his earlier work "does not imply that the analyses which make up the bulk of the published volume of PA were psychologistic in the pernicious sense criticized in the Prolegomena to LU" (Miller, *Numbers in Presence and Absence*, 23). Miller bases his argument on the fact that "[t]he basic thought of PA is that numbers can be presented both authentically and symbolically . . . , [which] would be inconceivable if numbers were regarded as identical with the acts of presentation themselves" (21).

chological"—account of the origination of their foundations in "experience."[8] Only by keeping this distinction in view can clarity be achieved regarding *that* for which Husserl is attempting to provide foundations, namely, both a *specific* understanding of arithmetical concepts and operations and a *specific* understanding of their logic. This is not to say, however, that by maintaining this distinction the shortcomings of Husserl's psychological attempt to found arithmetic and its logic can be obviated. Rather, the claim here is that this attempt—and its failure—can be properly assessed only on the basis of a precise understanding of Husserl's formulation of these concepts and operations together with their logic.

Mohanty and Willard maintain this distinction insofar as they both recognize that what is at issue for Husserl in *Philosophy of Arithmetic* is the attempt to enlist certain concepts from Brentano's psychology in order to ground the *objectivity* of arithmetical concepts and arithmetical knowledge. For this reason, they are able to defend Husserl successfully against Frege's criticism that *Philosophy of Arithmetic* is pervaded by a crude form of psychologism, that is, by the reduction of the objectivity proper to both arithmetical concepts and the logic of arithmetical operations to psychological presentations. They do so by showing how the "psychologism" of *Philosophy of Arithmetic* is much subtler: while Husserl is not guilty therein of *reducing* the objectivity of arithmetic and logic to psychological presentations, he nevertheless attempts to clarify the fundamental genesis and meaning of this objectivity by way of analyses of psychological "acts."[9]

However, Mohanty's and Willard's entirely legitimate concern to show that the development of Husserl's phenomenology—or, at the very least, his phenomenological investigations of the foundations of logic—in effect amounts to his "correction" of this early mistake is not the concern of the present study. Rather, it is to demonstrate the truth of Klein's thesis that "Husserl's logical researches amount in fact to a reproduction and precise understanding of the 'formalization' which took place in mathematics (and philosophy) ever since Vieta and Descartes paved the way for modern science" (*PHS*, 70). Consequently, the present interpretation of Husserl's *log-*

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8. For an excellent account of the historical context (and esp. the contributions of the work of Hermann Lotze and Carl Stumpf to this context) that informed Husserl's understanding of psychology in general and psychological experience in particular, see Dallas Willard, *Logic and the Objectivity of Knowledge* (Athens: Ohio University Press, 1984), 32–33.

9. Miller argues that the problem with Husserl's psychologism in *Philosophy of Arithmetic* is not that he reduces the objectivity "of multitudes and numbers to mental entities, but rather that he construes them as curious halfbreeds, as wholes consisting partly of objects and partly of mental acts" (Miller, *Numbers in Presence and Absence*, 71). See § 45 below, where the exact character of Husserl's understanding of mental acts is discussed in detail.

ical investigations in *Philosophy of Arithmetic* will focus on the issue of precisely *what* the basic concepts of arithmetic and logic are that guide his psychological quest to provide their foundations.

§ 41. Husserl on the Authentic Concepts of Multiplicity and Cardinal Number Concepts, and Inauthentic (Symbolic) Number Concepts

In *Philosophy of Arithmetic* Husserl distinguishes between two kinds of number concepts: authentic and inauthentic (symbolic) (see *PA*, 15–16).[10] By the former is meant a species of the concept of multiplicity, which answers the question of "How many?" (15). The authenticity of the authentic number concept has its basis in the authenticity of that to which the concept of multiplicity refers: "the intuition in consciousness of some concrete multiplicity" (79). "The cardinal number concept thus encompasses, though only indirectly through the extensions of its species concepts, which are the number two, three, four, . . . , the same concrete phenomena as the concept of multiplicity" (15). The inauthenticity of symbolic number concepts is grounded in "a presentation by means of signs. If a content is not given directly to us as what it is, but only indirectly through signs that univocally characterize it, then we have a symbolic presentation of it instead of an authentic one" (193).

For Husserl both the authentic and symbolic number concepts are "empty" (see 49 and 51) in the sense that the multiplicity of "determinate objects" (16) upon which they are based—directly in the former and indirectly in the latter—are "completely arbitrary." Thus, he writes:

> Indeed, for the formation of concrete assemblages there are no restrictions at all with respect to the particular contents concerned. Any imaginable object, whether physical or psychical, abstract or concrete, whether given through sensation or fantasy, can be united with any and arbitrarily many others to form an assemblage, and accordingly can also be counted. For example, several of certain trees, the sun, the moon, Earth, and Mars; a feeling, an angel,

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10. Miller correctly maintains that "the distinction between authentic and symbolic modes of givenness is central to PA as a whole" (Miller, *Numbers in Presence and Absence*, 28). However, his study's focus on the presentation of authentic and symbolic numbers in Husserl's analyses tends to overlook the authentic number concepts and "symbolic formations" (*PA*, 16) that these analyses are intended to clarify.

Willard maintains in connection with *Philosophy of Arithmetic*'s investigation of the distinction between authentic and symbolic presentations and concepts: "It is not a great or a pointless exaggeration to say that the analysis of symbolic representations and knowing is *the* main problem for investigation throughout Husserl's career" (Willard, *Logic and the Objectivity of Knowledge*, 89).

> the moon, and Italy; etc. In these examples we can always speak of an assemblage, a multiplicity, and a determinate number. The nature of the particular contents therefore makes no difference at all.

The emptiness of the authentic number concept shows up in the process of enumeration or counting. This process is directed to the concrete contents that belong to the concept of multiplicity, contents Husserl characterizes "as some content or other, each one as a *certain anything, a certain one*" (79). Consequently, "the concept of multiplicity also contains that of the *anything* [Etwas]" (80). For Husserl, then:

> One can with full justification designate the concepts anything and one, multiplicity and cardinal number—these concepts that are most general and emptiest of content—as form concepts or categories. What characterizes them as such is the circumstance that they are not concepts of contents of a determinate genus, but rather in a certain manner take in any and every content. (84)

Being empty of content yet inseparable from the multiplicity of anythings or ones to which it refers, the generic cardinal number concept is related to its "parts," that is, the determinate species of cardinal numbers, in a manner analogous to "the relationship between the logical part and logical whole (e.g., between color and the difference red)" (82). Therefore, just as there is no "color in general," likewise "[t]here is no cardinal number in general."

Because the authentic meaning belonging to the cardinal number concept is a species of the concept of multiplicity, and because the response to the question of 'how many' with respect to a multiplicity is never zero or one, Husserl flatly denies "that zero and one belong among the number concepts" (130). Moreover: "If we arrange the numbers in the 'natural' sequence, that is, in such a way that each arises following upon the preceding one by the collective addition of one unit, then 1 + 1 is the first number, inasmuch as it has no predecessor" (132). Consequently, 'two' is the first number.

§ 42. The Basic Logical Problem in *Philosophy of Arithmetic*

The last chapter of *Philosophy of Arithmetic* articulates the logical[11] problematic of arithmetic as the clarification of its calculational methods, a clarification that leads to an investigation of the origin of the symbolic methods of arithmetic. Husserl writes, "The individual numbers, considered by them-

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11. Willard identifies the meaning of the term 'logical' "most frequently used by Husserl" (Willard, *Logic and the Objectivity of Knowledge*, 127 n. 1) in *Philosophy of Arithmetic* as "devices which enable us to extend knowledge with absolute assurance . . . , or the needs or conditions of such extension."

selves, give no occasion for treatment with a view to knowledge of them" (256). Thus, "[o]nly out of the relationships of numbers to one another do there arise problems that require a logical treatment." The logical treatment of arithmetic, under the heading of "*universal arithmetic*" (283), has as its task the elaboration of a "universal theory of operations," that is, a theory of calculation and calculational technique. And because the calculational technique is "almost always confined to symbolic presentations of number," the philosophy of arithmetic detects in the latter "the logical origin of universal arithmetic" (287).

On Husserl's view the concept of calculation "admits of various broader and narrower significations" (256). In its broadest sense, it signifies "*any mode of derivation of numbers sought starting from numbers given*." In its narrower sense, calculation signifies "mechanical-exterior sign formation" (257), that is, a calculational technique. With respect to "the *method of derivation* of sought numbers," there are thus two possibilities:

> *Either* this derivation is an *essentially conceptual operation*, in which case the designations play only a subordinate role, *or* it is an *essentially sensible* operation that, utilizing the system of number signs, derives sign from sign according to fixed rules, claiming only the final result as the designation of a certain concept, the one sought.

From Husserl's understanding of calculation and calculational technique, it follows that "[a]ny arithmetical method would *eo ipso* be a calculational method. The calculational technique would be the technique of arithmetical cognitions, and arithmetic only their systematically arranged entirety." The answer to the question of what method is "logically more perfect" is a practical affair, in the sense that it "can be only a question of what they are capable of accomplishing." For Husserl, it is calculational technique "that has the preference in our domain [i.e., arithmetic], and also abundantly deserves it." This is the case because calculation with concepts is "highly abstract, limited, and, even with the most extensive practice, laborious." Calculation with "signs is concrete, sensible, all-inclusive, and . . . already with a modest degree of practice, easy to work with." Moreover, "there is no conceivable [arithmetical] problem that it would not be capable of solving." This means that calculational technique "makes the conceptual method entirely superfluous," and, because of this, calculational technique is "*the* logical method of arithmetic." On account of his insight (in the last chapter of *Philosophy of Arithmetic*) into the superfluity of the conceptual method of calculation for universal arithmetic, Husserl comes to understand its method as one that "encompasses *any symbolic derivation of numbers from numbers that is substantively based on rule-governed operations with sensible signs*" (257–58).

§ 43. The Fundamental Shift in Husserl's Account of Calculational Technique

Husserl's initial investigations in *Philosophy of Arithmetic* into the logical origin of universal arithmetic, in sharp contrast to its last chapter, are informed by the view that the "true basis" (90) of the mechanical operations of calculational technique "lies in the elementary relations between [authentically presented] numbers." Moreover, he maintains, "it is certain" that "authentic presenting [of the items in an assemblage] is present at least in the first stages" (92), for instance, in the "inauthentic (symbolic) presenting" of several assemblages together as unified into *one* assemblage"—"[f]or otherwise the very thought of such a composite structure would be absurd." In the last chapter of *Philosophy of Arithmetic*, however, precisely this view is characterized as "a rigorous consequence of the standpoint we adopted there [i.e., earlier in the book]: namely, the widespread prejudice to the effect that arithmetic has to do with true and authentic number concepts and the laws of their combination or 'operation'" (262). This and other statements in *Philosophy of Arithmetic*, especially in the last chapters, make it clear that Husserl's position on the logical origin of universal arithmetic underwent what can only be characterized as a radical shift regarding the relation of symbolic number concepts and their presentation to authentic number concepts and their presentation.[12] Moreover, it is clear that he was cognizant

12. Husserl's letter to Carl Stumpf written in 1890, before *Philosophy of Arithmetic* was complete, relates that "[t]he view by which I was still guided in the elaboration of my *Habilitationsschrift*, to the effect that the concept of cardinal number forms the foundation of universal arithmetic, soon proved to be false"; see "Husserl an Stumpf, ca. Februar 1890," in Edmund Husserl, *Briefwechsel*, ed. Karl Schuhmann with Elisabeth Schuhmann, 10 vols. (Dordrecht: Kluwer, 1994), I: 157–64, here 158; English translation: "Letter from Edmund Husserl to Carl Stumpf," in *Early Writings*, 12–19, here 13—henceforth cited as 'Stumpf Letter' with German and English pagination, respectively. This means, among other things, that "[b]y no clever devices, by no 'inauthentic presenting,' can one derive negative, rational, irrational, and the various sorts of complex numbers from the concept of cardinal number." Instead, universal arithmetic "finds application to the cardinal numbers (in 'number theory'), as well as to ordinals, to continuous quantities, and to *n*-dimensional manifolds (time, space, color, force, continua, etc.)."

Husserl goes on to relate how, with the realization that "no common concept underlies these various applications of arithmetic," he had to confront the question about its content, about its "conceptual objects," and that he had to reject his original supposition that this content is intrinsically conceptual. That is, he had to reject his initial view that to all of the signs of universal arithmetic there correspond, "at least potentially" (159/15), "designated concepts" (160/15). The view that a "system of signs and operations with signs can replace a system of concepts and operations with judgments, where the two systems run rigorously parallel" (159/14), only holds (within the context of mathematics) in the case of "ordinary arithmetic." Thus, e.g., if arithmetic "deals with discrete magnitudes, then 'fractions,' 'irrational numbers,'

of the fact that, because of this, *Philosophy of Arithmetic* documents this shift, as it were, insofar as the published text contains analyses that reflect both the view that the authentic numbers provide the foundation for the logic of arithmetic and the view that they do not.[13] For instance, in the last section of Chapter X, under the heading "Arithmetic Does Not Operate with 'Authentic' Number Concepts," Husserl writes:

> The presupposition from which we set out at the first, as from something self-evident—namely, that each arithmetical operation is an activity with and upon actual numbers—cannot correspond to the truth. All too hastily we allowed ourselves to be guided by the common and naive view which does not take into account the distinction between *symbolic* and *authentic* presentations of number, and which does not do justice to the fact that all number presentations that we possess, beyond the first few in the number series, are *symbolic*, and can only be symbolic. (190)

Quoting from Eugen Dühring's *Logik und Wissenschaftstheorie*,[14] where the author maintains that "'there would be no distinct arithmetic whatsoever if the various forms that are possible in the grouping of units were not an issue'" (191), Husserl makes it clear that he now considers "such views" unfeasible, and he also indicates why he has arrived at this consideration. The "diverse 'forms in the composition of units,'" which Dühring (and others who hold such views) understand to be "'only more precise determinations of the peculiar combinations in which the *activities* corresponding to the simplest signs, plus and minus, find an application of a specialized nature,'" are, Husserl says, "nothing more than turns and forms of the *symbolism*, grounded in the fact that all operating which reaches beyond the very first

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imaginary numbers and, in the case of cardinal numbers for example, also the negative numbers, lose all sense" (160/15), because the signs for these numbers, in contrast to the signs that refer to the concepts proper to discrete magnitudes, are "representatives of 'impossible' concepts." But rather than have to get clear about "how operations of thought with contradictory concepts could lead to correct theorems," which is what Husserl says he originally tried to do, he came to realize that "through the calculation itself and its rules (as defined for those fictive numbers), the impossible falls away, and a genuine equation remains." Therefore, the system of *arithmetica universalis* "is not a matter of the 'possibility' or 'impossibility' of concepts," but "an accomplishment of the signs and their rules" (160/16). Universal arithmetic is, therefore, "no science, but a part of *formal* logic" (161/17), which Husserl defines here "as a technique of signs (etc., etc.)" and designates "as a special—and one of the most important—chapters of logic as the technology of knowledge." Husserl also adds, very significantly, that "these investigations appear to push toward important reforms in logic, and that he knows "of no logic that would even do justice to the possibility of an ordinary arithmetic."

13. All references to the "shift" in Husserl's view on the matter of the origin of the logic of arithmetic, unless otherwise stated, will be based in what can be discerned in this regard from the content of the analyses in *Philosophy of Arithmetic*.

14. Eugen Dühring, *Logik und Wissenschaftstheorie* (Leipzig: Fues, 1878), 249.

numbers is only a symbolic operating with symbolic presentations." Husserl promises to "supply yet more direct positive proofs" of this fact "in the further course of [his] investigations."

In these subsequent investigations, Husserl shows how his initial supposition—that the calculational technique of universal arithmetic and the calculational technique of arithmetical knowledge (i.e., knowledge in the sense of calculations based on operations with authentic number *concepts*) are identical—gave way to his subsequent view that "obviously the calculational technique is no longer identical with the technique of *arithmetical* knowledge" (259).[15]

Husserl attributes this shift in the understanding of these matters to his having "achieved the insight" (262) that "the pure calculational mechanism that underlies arithmetic and constitutes the technical aspect of its

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15. Walter Biemel was the first to call attention to the shift in Husserl's understanding of the origin of universal arithmetic, though without discussing it in terms of his analyses in *Philosophy of Arithmetic*. See Walter Biemel, "Die entscheidenden Phasen in Husserls Philosophie," *Zeitschrift für philosophische Forschung* 13 (1959), 187–213, here 192 ("The Development of Husserl's Phenomenology," in R. O. Elveton, ed. and trans., *The Phenomenology of Husserl: Selected Critical Readings* (Seattle: Noesis Press, 2d ed., 2000), 140–63, here 145). While Biemel rightly comments that "we can see how the difficulties encountered by Husserl in defining arithmetic as a theory of signs brought him to the study of logic" (146), he does not identify the precise nature of these difficulties.

Willard was the first to call attention to fact that the shift in Husserl's understanding of the origin of universal arithmetic is already evident in *Philosophy of Arithmetic*. He argues that Husserl's rejection of "the dependence of calculation upon the concept of number" (Willard, "Husserl on a Logic that Failed," 62) amounts to his "acknowledgement of the momentous fact that the conceptualization which had guided . . . the *Philosophy of Arithmetic*, is in fact abandoned in the book's final chapter" (63). His explanation of Husserl's reason for abandoning this conceptualization focuses on Husserl's coming to view "the numeral system itself" as something that is "presupposed in, and cannot be explained by, symbolic (or inauthentic) representing and knowing within the domain of mathematics" (64). Willard's explanation is based on the observation (with which we agree) that Husserl came to the realization that the conceptualization of symbols as inauthentic presentations of the same contents as authentic presentations (which is basically the view he adopted from Brentano) cannot provide an adequate account of the numeral system operative in *arithmetica universalis*. However, we shall argue in detail below (see §§ 48–50) that what Husserl abandoned was the appeal to *this* conceptualization of "symbols" and not the appeal to symbols per se to account for the number system.

Miller also calls attention to the shift in Husserl's understanding of the origin of universal arithmetic, noting that due to the shift *Philosophy of Arithmetic* "poses some extremely difficult questions of interpretation, questions which have almost never been addressed or even acknowledged" (Miller, *Numbers in Presence and Absence*, 10). For example, he quotes Marvin Farber's view that *Philosophy of Arithmetic* is "'a well-organized and, in the main, remarkably clear treatise'" (25). However, beyond arguing that "[b]y the time Husserl wrote the Introduction (presumably in 1890 or 1891), he had *already* come to realize analysis [i.e., universal arithmetic] was in fact *not* based on the concept of number" (12), Miller does not draw any connection between this issue and Husserl's analyses of the relation between symbolic and cardinal numbers in *Philosophy of Arithmetic*.

methodology" (259) has, as its "true underlying substance" (262), "symbolic number formations."[16] He traces this insight, in turn, to his taking "into account that the mechanism of the symbolic methodology can break completely free of the conceptual substrates of its employment" (258). With the detachment of the symbolic methodology from its conceptual basis, calculation can be conceived of as "*any rule-governed mode of derivation of signs from signs within any algorithmic sign-system according to 'laws'—or better: the conventions—for combination, separation, and transformation peculiar to that system*."[17] Husserl notes that with this algorithmic conception of calculation, "The relationship between arithmetic and calculational technique now, with this new concept of calculating (which from now on is the only one we wish to use), certainly has changed. If we disengage the number signs from their conceptual correlates and work out, totally unconcerned with conceptual application, the technical methods that the sign system permits" (259), we have the "*calculational methods in arithmetic* . . . already given to us in the systematic forms of the sequence of natural numbers,[18] or in the higher forms of the 'system' of numbers (in the specific sense of the term)."[19]

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16. See n. 12 above.

17. Strictly speaking, for Husserl this conception of calculation is "more restricted in one sense" (*PA*, 258) than the derivation of *numbers* based on rule-governed operations with sensible signs (symbols), "but in another, by contrast, [it] is broader." It is more restricted because, not being guided by the "logical interests" of the *arithmetica numerosa* of concern to Husserl in *Philosophy of Arithmetic*, it is not numerical. However, it is broader because the "higher level logical interests" that "require this delimitation of the concept" are "significant for the deeper understanding of mathematics," insofar as "one and the same system of symbols can serve in *several* conceptual systems that, different as to their content, exhibit analogies solely in their structural form." According to Husserl, this "new formulation of the concept of calculating"—which, we should mention even though Husserl does not, is tantamount to its "formalization"—"recommends itself still further in that it places in our hands a logically clear separation of the different stages required by problem-solving in those domains that are susceptible to treatment by technique." In the case at hand, that is, the "domain of numbers," the broader sense of calculation permits clarity to be achieved with respect to what is involved in the first stage of their symbolic derivation, namely, "that in the complexes of concepts and names [i.e., the symbolic number signs] given in each case, one abstracts from the former and only holds to the latter."

18. That is, in the forms generative of the symbolic number signs that are formed by the concepts proper to the idealizing method of expanding numbers beyond their authentic presentation, concepts that indirectly refer to the latter. See §§ 48–49 below.

19. That is, in the forms generative of the symbolic number signs that are formed by the calculational algorithm, which, being unrealizable in terms of the conceptual content of symbolic numbers that are characterized as idealizations of authentically presented cardinal numbers, can only be signitively symbolized. In his "Double Lecture: On the Transition through the Impossible ('Imaginary') and the Completeness of an Axiom System" from 1901 (Essay III in Willard's translation of *Philosophy of Arithmetic*, 409–73), Husserl makes clear what his analysis in *Philosophy of Arithmetic* of numbers that can only be "signitively symbolized" (see § 49 below) does not, namely, that numbers belonging to the "imaginary," which he takes "in the

Husserl's account of the shift in his understanding of the calculational methods in arithmetic is thus tied to a shift in his understanding of the status of the numbers that function as the substrate for arithmetical calculation. More precisely, the latter shift has its basis in a gradual transformation of his understanding of the logic proper to symbolic numbers themselves. Husserl initially speaks about symbolic numbers "as a matter of symbolic formations for those species of the number concept that are not accessible to us in the authentic sense" (234). Accordingly, the signs involved in the symbolic number formations were conceived of as making "possible, through a complex of indirectly characterizing (but themselves authentically presented) relative determinations, an unrestricted expansion of the domain of number" (240). Husserl characterized this expansion in terms of "an idealization of our finite mental capacities," whereby the symbolic number formations were understood to be surrogates for actual cardinal numbers that are incapable of being authentically presented, that is, of being presented as concrete multiplicities "in which each of their terms as something separately and specifically noticed" (192) is grasped "together with all of the others, in one act." Husserl therefore initially considers "[t]he sensible signs involved here . . . in the manner of language signs, mere accompaniments of concepts" (241). As such, he understood these signs, the numerals composing symbolic number formations, to be general names that refer to the completely arbitrary objects that happen to fall under whatever inauthentic concept of number with which each such sign is associated. Moreover, Husserl understood these objects, being completely arbitrary, to fall under the generically empty concept of 'anything' (*Etwas*), and therefore to compose concrete multiplicities of units that, because of the finitude of the human mind, are capable of being accessed only by arithmetic in an indirect manner, through the expedient of its systematic expansion of the few cardinal numbers that the human mind is capable of properly intuiting.[20]

Husserl eventually arrived at the view, however, that this idealized account of the formation of symbolic numbers, and therewith of their logic, "was an incorrect way of speaking, adopted for the sake of a simplified presentation" (240). The correct way of considering the sensible signs involved in symbolic number formations and therefore of understanding the symbolic numbers is to recognize that sensible signs

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widest possible sense, according to which the negative, indeed, even the fraction, the irrational number, and so forth, can be regarded as imaginary" (416), are not "ever in any way represented in cardinal arithmetic."

20. Husserl's account of the size of such numbers varies in *Philosophy of Arithmetic*, from 3 to 12.

> participate in a far more striking manner in our symbolic formations than we have asserted to this point; indeed, so much so that they ultimately dominate nearly the entire field. In fact, the rigorous parallelism between the system of the number concepts and that of the number signs makes it possible to regard the systematic continuations of the sequence of signs as representatives of the (inauthentically presented) systematic continuations in the sequence of concepts. (241)

It will be the concern of the remainder of this chapter to establish what, exactly, Husserl means when he characterizes number signs as "representatives" of the inauthentically presented systematic sequence of concepts. At this point, however, it should be clear that he does not understand such signs as "language signs, mere accompaniments of concepts," because he has clearly labeled such an understanding a mistake. The "representative" status of the signs that compose the symbolic number formations proper to arithmetic, therefore, has to be differentiated from what is now viewed as the incorrect "linguistic" understanding of such signs. Symbolic number signs, as "representatives" of inauthentically presented number concepts, thus are *not* general names accompanying such concepts, concepts that, as the result of a process of idealization tied to the mind's limited capacity to present numbers authentically, function as the surrogates for their unrealizable—but nevertheless signified—presentation. Husserl therefore makes the distinction, within the very domain of symbolic number formations themselves, between "the numbers it is still possible to symbolize conceptually" (242), numbers whose "conceptual content could still be brought before the mind," "and those that can *only* be *signitively* symbolized."[21]

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21. The fact that, in *Philosophy of Arithmetic*'s analyses of the calculational technique proper to universal arithmetic, Husserl comes to distinguish between these two distinct "concepts" of symbolic numbers, and, therefore, the fact that correlative to this distinction, *two* distinct accounts of the symbolic presentation of number can be found to be operative in these analyses, seems to have been entirely overlooked in the literature on his philosophy of arithmetic. The results of this oversight present significant obstacles to a proper grasp of the failure of Husserl's attempt to account for the origin of universal arithmetic in the authentic concept of number and to a proper understanding of the development of his thought subsequent to this failure. For instance, Miller argues that the distinction between 'authentic' and 'symbolic' presentation in *Philosophy of Arithmetic* is equivalent to the distinction Husserl introduced in the *Logical Investigations* between, respectively, 'filled' and 'empty' acts (see Miller, *Numbers in Presence and Absence*, 38). Further, following Robert Sokolowski, he uses "the terms 'absence' and 'presence' as alternatives to Husserl's own talk of 'empty' and 'filled' intentions" (42 n. 21). Although Miller is aware "that we should not exaggerate the extent to which the distinctions Husserl worked out only later are found already in PA"—"in particular that there is no explicit discussion in this work of the 'fulfillment' of an empty (or 'symbolic') act by an intuitive (or 'authentic') one" (39), he nevertheless justifies his employment of these terms for his interpretation of Husserl's philosophy of arithmetic by stating, "[n]onetheless Husserl does regard authentic and symbolic acts as capable of being directed toward the same object, even in this

§ 44. Husserl's Account of the Logical Requirements behind Both Calculational Technique and Symbolic Numbers

Corresponding to each of these different symbolic number formations is a distinction in the logic of their formation, in the sense that the "logical requirements imposed" (232–33) on their formation is different. To appreciate better the basis for this radical shift in Husserl's understanding of the logic that characterizes the symbolic status of the numbers that function as the substrate for arithmetical calculation, it is necessary to consider in more detail Husserl's understanding of the logical requirements that give rise to the necessity of both a calculational technique and symbolic numbers. The logical considerations of *Philosophy of Arithmetic* stem from two sources. The first concerns the status of the unity proper to the multiplicity that forms the basis for authentic number concepts. The second concerns the "finitude of human nature" (191). At issue in the first consideration is the peculiar character of the "whole" (76) proper to the concept of multiplicity that forms the basis of the authentic number concept. Owing to its categorial status as one among the "concepts that are most general and emptiest of content" (84), for Husserl the logical status proper to the unity of this whole cannot be derived from the "physical" or "metaphysical"[22] combination of the elements (and thus their relations) that make up its contents. At issue in the second

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early work." However, as we shall show (see §§ 49–50 below), because Husserl's analyses of the calculational technique operative in universal arithmetic led him to make the distinction between numbers that are "conceptually" symbolic (i.e., numbers that indirectly refer to authentic numbers) and numbers that are "signitively" symbolic (i.e., numbers that, as a result of their representation of *inauthentic* numbers, lack any reference to *authentic* numbers), he in effect abandons (in the last two chapters of *Philosophy of Arithmetic*) his earlier view of authentic and symbolic presentations of numbers "as capable of being directed toward the same object" (see § 46 below). One of the consequences of Miller's inattentiveness to this state of affairs is that his interpretation of the symbolic presentation of number in general (in the *PA*) as a mode of absence relative to the authentic presentation of number (69) misses Husserl's characterization (in *Philosophy of Arithmetic*'s final chapters, XII and XIII) of the mode of symbolic number and its presentation that is distinguished by its *lack* of even an indirect reference to authentic numbers and their presentation. Moreover, the failure to see this renders untenable the thesis that guides Miller's attempt to develop a Husserlian phenomenology of number, namely, "that numbers are available to us as identities in presence and absence" (65).

22. Husserl indicates that he has in mind here Franz Brentano's notion of "'metaphysical' combination" (*PA*, 19 n. 1), which he characterizes as a whole whose parts are united insofar as they "reciprocally penetrate and connect with each other" (19); e.g., the manner in which, "in the case of any arbitrary visual object, spatial extension and color (and color, in turn, and intensity)" combine to form a whole. Thus, in the case at hand, Husserl's point is that the status of the unity proper to the "whole" peculiar to the concept of multiplicity (and therefore the status of the unity proper to the "wholes" of its species, the authentic number concepts) *cannot* be derived from either the physical or the metaphysical combination of the parts of the multiplicity which it both refers to and unifies.

consideration are "the limits imposed upon us by the *de facto* weakness of our capacities for presentation" (228), limits that impact upon our ability both to count and to calculate with authentic numbers.

§ 45. Husserl's Psychological Account of the Logical Whole Proper to the Concept of Multiplicity and Authentic Cardinal Number Concepts

Husserl's original conviction that "not only is psychology indispensable for the analysis of the concept of number, but this analysis even belongs within psychology,"[23] is a conviction that, paradoxically, is tied to a specifically logical concern. The *logical* unity of the whole that characterizes the concept of multiplicity for Husserl is empty of physical and metaphysical contents, and, therefore, the composition of this unity cannot be properly understood as a logical *part* of such contents. In order to account for the logical unity of both the concept of multiplicity and the authentic number concepts (which, as the species of the former, are based upon it), then, it is "necessary to obtain a precise view of the concrete phenomena from which they are abstracted" (64). As Husserl sees it, the phenomena at issue here are "psychological," and as such they are manifest in "inner experience itself" (73). He maintains that the latter presents evidence belonging to acts of "collective combination" (75) wherein originates the "assemblage" (83) united in a multitude. Husserl describes the acts in which "[*a*]*n assemblage originates*" (74) in terms of "*a unitary interest—and simultaneously in and with it, a unitary noticing*—[that] *picks out and encompasses various contents*." Such acts of collective combination provide the basis for "the peculiar abstraction process" (79) that yields both the concept of multiplicity and the authentic number concepts. Husserl describes

> the abstraction to be carried out . . . in the following manner: Individual contents that are in some way determinate are given in collective combination. In abstractively passing over to the general concept, then, we do not attend to them as contents determined thus and so. Rather, the main interest is concentrated upon their collective combination, whereas they themselves are considered and attended to only as some content or other, each one as *a certain anything*, *a certain one*.

Husserl holds that "Collective combination plays a highly significant

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23. Edmund Husserl, *Über den Begriff der Zahl. Psychologische Analysen* (Habilitationsschrift) (Halle a. d. Saale: Heynemann'sche Buckdruckerei, 1887; reprinted in *Hua* XII, 289–338, here 295); English translation: "On the Concept of Number: Psychological Analysis," in *Philosophy of Arithmetic*, 305–56, here 295.

role in our mental life as whole" (75). For "Every complex phenomenon which presupposes parts that are separately and specifically noticed, every higher mental and emotional activity, requires, in order to be able to arise at all, collective combinations of partial phenomena." Indeed, not only complex phenomena but likewise "simple relations (e.g., identity, similarity, etc.)" could not be presented "if a unitary interest and, simultaneously with it, an act of noticing did not pick out the terms and hold them together as unified." Moreover, Husserl maintains that notwithstanding "the elemental character of the collective combination," "ordinary language possesses no independent name" for it, but rather gets by with "the little syncategorematic word 'and'." This word, "[i]n and of itself . . . without signification," nevertheless represents the "imprint in ordinary language" of the "collective combination," such that "where it links two or more names, it indicates the collective combination of the contents named."

Husserl "capitalizes" (79) on the result of the abstraction process that yields the concept of multiplicity from the collectively combined assemblage "by combining" the result with the observation about the indication of the act of collective combination "in language by the conjunction 'and'" to express the content of the concept of "multiplicity in general" (80). Hence, on his view, what is meant by this content "is nothing other than: a certain anything and a certain anything and a certain anything, etc.; or, some one and some one and some one thing, etc.; or, more briefly, *one and one and one*, etc." Because this expression "refers by means of the 'etc.' to a certain indeterminateness" (81), by which "nothing else is meant than that no determination is set with respect to an upper bound, or else that the actually present limitation is something that should be seen as something of no consequence," Husserl maintains that "there are many possibilities" "to do away with this indeterminateness"—"if we want to." More precisely, on Husserl's view, "corresponding to these [possibilities], the concept of multiplicity immediately divides up into a manifold of determinate concepts that are most sharply delimited from one another: the *numbers*." The number concepts thus arise, according to Husserl, as: "*one and one; one*, *one and one; one*, *one*, *one and one*, and so forth." It is important to note, however, that for him, "It is of course not necessary to assume the universal and indeterminate concept of multiplicity as mediator in the derivation of the number concepts." This holds because "We come by them directly, setting out from arbitrary concrete multiplicities; for each of these falls under one, and indeed a determinate one, of those concepts." These concepts, in turn, designated by the "*names* 'two', 'three', etc." (82 n. 3), are what the term 'cardinal number' (*Anzhal*) arose "as a general name for."

Husserl's analysis of "[t]he abstraction process which yields the determinate number accruing to a given concrete multiplicity" (81), in the case where this process does not involve the mediation of the concept of multiplicity, nevertheless builds upon his analysis of the collective combination that yields the latter. He writes:

> Disregarding the specific character of the particular contents grasped together, one considers and retains each content only insofar as it is an 'anything' or 'one'. And thus one obtains, in taking account of the collective combination of them, the universal form of the multiplicity appertaining to the multiplicity at hand: one and one, . . . and one—a form with which a definite number term is associated. (82)

And he concludes that this analysis of the process "is completely clear."

Thus, while "[t]he enumerated contents certainly can be physical as well as psychical . . . the number concepts and the one belong exclusively to the domain of reflexion" (85). Husserl's appeal here to the "domain of reflexion" makes it clear that he is under the influence of "the influx of logic into psychology" on which Frege's critique of *Philosophy of Arithmetic* focused and from which Husserl later distanced himself in the *Logical Investigations*, when he characterized the error of the "doctrine put about since the time of Locke, that . . . the *logical categories*, such as . . . unity . . . or cardinal number arise through *reflexion upon certain psychical acts, and so fall in the sphere of 'inner sense,' of 'inner perception*'" (*LI*, 668/278). Notwithstanding the influence in *Philosophy of Arithmetic* of this subsequently discredited view of the origin of these concepts, it is important for our purposes that we not lose sight of precisely what it is that Husserl was attempting to clarify in his first work by enlisting psychology in the service of a philosophical analysis of the authentic concept of number: the peculiar logical unity proper to the whole characteristic of the *logical* concept of multiplicity and therewith the logical unity proper to the whole belonging to each of the cardinal numbers that are the *logical* parts of this concept. And, again, what is peculiar about the logical status of this unity in each case is that its basis can be derived *neither* from the qualities of the unified objects nor from their relations. Thus, even though Husserl will come to see in his logical investigations subsequent to *Philosophy of Arithmetic* the difficulties in defending the position that the view expressed here does not "maintain that the act involved creatively produces its content" (43),[24] the problem of philosophically accounting for the

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24. Husserl had maintained this in *Philosophy of Arithmetic*, because "[c]ertainly one distinguishes in complete universality the relational psychical activity from the relation itself" (*PA*, 43).

origin of the logical unity proper to the empty form concepts of the collective wholes in question nevertheless remains for his latter investigations.

§ 46. Husserl on the Psychological Basis for Symbolic Numbers and Logical Technique

The second source of Husserl's logical considerations in *Philosophy of Arithmetic* also derives from what he will later characterize as his attempt "to reconcile the objectivity of mathematics . . . with a psychological foundation for logic" (*LI*, 6–7/2). His initial account of the "logical requirements" (*PA*, 237) for symbols and calculational technique in arithmetic is based on what he considers to be a fact, namely, that "In the purposeful carrying out of a collective combination under the most favorable of conditions (i.e., with exertion of all our psychic power, presupposing contents that are especially easy to perceive and that present themselves for apprehension in a succession that does not move too quickly), we take in not more than a dozen elements" (196–97).[25] This psychological condition is the basis for the logical requirement that the presentation of multiplicities beyond the scope of their authentic presentation must be a symbolic presentation. "The symbolic presentation of multitudes" (222), in turn, composes "the foundation for the symbolic presentations of *numbers*" (likewise) beyond their authentic presentation. This state of affairs also forms the basis of the logical requirement for logical technique, because "All logical technique is aimed at the overcoming of the original limits of our natural mental abilities" (234). Indeed, "If we had authentic presentations of all numbers, as we do of the first ones in the sequence, there would be no arithmetic, since it would then be completely superfluous" (191). Moreover, through the "idealization of our finite mental capacities" (240), the symbolization of multitudes and numbers can be expanded beyond its initial reference to determinate multitudes and numbers, such that eventually the "idea" (221) of the concept of infinite multitude and infinite number becomes "actually admissible logically."

§ 47. Husserl on the Symbolic Presentation of Multitudes

Husserl's initial account of symbolic presentation is guided by his view

> that the authentic presentation and a symbolic presentation correlative to it stand in the relationship of logical equivalence. Two *concepts* are logically

25. See n. 20 above.

> equivalent when each object of the one is also an object of the other, and conversely. That, for the purposes of our interests in forming judgments, symbolic presentations can surrogate, to the furthest extent, for the corresponding authentic presentations rests upon this circumstance. (194, my emphasis)

The object that characterizes the logical equivalence of authentic and symbolic presentations of numbers is a multiplicity of units. Authentic number concepts, as a result of their basis in the authentic presentations of numbers, are directly related to this "object," while the symbolic number concepts, because they have their basis in the symbolic presentation of number, are indirectly related to it. That the object that establishes the logical equivalence of authentic and symbolic concepts in the case of numbers is indeed the determinate collection of separate contents and *not* the *concept* of such contents is clear from Husserl's discussion of the "logical content" (218) proper to the symbolization of the presentation of multitudes. He writes, "the symbolization involved does not effect its logical content. Multiplicity remains the concept of a totality, of a determinate collection of separate contents. Only, in the cases now considered [i.e., symbolic presentations], the segregation of contents and their collection, instead of coming to actual realization, remains either wholly or largely a mere intention." As our preceding discussion of the transformation in Husserl's understanding of calculational technique has indicated, however, the importance for universal arithmetic of numbers that "can *only* be *signitively* symbolized" (242) comes to be stressed in *Philosophy of Arithmetic*'s last chapters. That is, what is stressed are the symbolic numbers whose status as "symbolic" is fundamentally different from numbers that are "symbolic" by virtue of their inauthentic presentation of multitudes. Whereas the latter type of symbolic numbers are properly understood as general names that refer, albeit indirectly, to the segregation of contents and their collection, symbolic numbers that are symbolic owing to their function as the signitive "representatives" of precisely these symbolic numbers, namely, the symbolic numbers that are characterized by their inauthentic presentation of multitudes, do not refer to multitudes at all. As we shall see below, this recognition amounts to the abandonment of the view expressed here that authentic and symbolic presentations are logically equivalent, which is the view that formed the point of departure for Husserl's investigation of the symbolic presentation of multitudes and numbers. In the remainder of this chapter, we shall retrace the key steps in Husserl's analyses that present this transformed understanding of the relationship between the authentic and symbolic numbers and thus to his transformed understanding of their concepts.

The symbolic presentation of a multitude for Husserl has its basis in acts of collective combination that confirm "the *existence of quasi-qualitative moments*" (203) in the experience of multitudes, moments that "can be grasped at one glance" (204) to present what he calls a "*figural* moment" (209).[26] By this he understands instances of "unified intuitions analogous to sense qualities,"[27] which are expressed by "such names as 'file', 'gaggle', 'covey', 'heap', etc." (210). These arise when, after "partial intuitions" (204) of members of a multitude, "the peculiarities of the contents or their primary relations fuse with one another such that the unified moments are something more than mere sums." Moreover, "[t]he general concept of the configura-

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26. Husserl's account of the symbolic presentation of a multitude is guided by the thesis—which, as we have already suggested and will show in detail below in §§ 49–50, is eventually abandoned in the last two chapters of *Philosophy of Arithmetic*—that the logical function of symbolic numbers is to act as surrogates for the authentic presentation of multitudes and thus that *all* symbolic numbers are inauthentic presentations of multitudes. His interest here, therefore, is to trace the origin of this surrogate (i.e., "symbolic") function from the presentation of multitudes grasped as "quasi-qualitative" or "figural" moments in order to provide the basis for his investigation of the symbolic presentation proper to the symbolic concept of number. Their failure to attend properly to this context of Husserl's discussion of the collective combination proper to multitudes has led commentators to overlook the fact that at issue in the analysis of the presentation of multitudes in terms of "figural moments" is not the origin of number *per se* but rather the origin of *symbolic* numbers. For instance, Miller claims: "The importance of sensuous multitudes is obscured somewhat by the organization of PA. Instead of beginning his investigation of the origin of number with a discussion of sensuous multitudes, as we have done in the present study, Husserl introduces this topic only in Part Two, in connection with the phenomenon of 'symbolic presentation'" (Miller, *Numbers in Presence and Absence*, 61 n. 14). Miller's need to speculate about Husserl's motivations for this arrangement, evident in his suggestion that "[p]erhaps Husserl's aim in selecting this arrangement was to make vivid the point that number is not itself a 'visible and tangible phenomenon,' as Mill, for example, had claimed numbers themselves are visible or tangible," highlights his confusion on this point.

27. Although Husserl's examples of the figural moments of multitudes in *Philosophy of Arithmetic* are predominately based on multitudes of visible sensible objects, it is clearly not his view that they are limited to either visible or sensible objects. Thus, he writes: "All of what we here have stated for multitudes within the field of vision can obviously be carried over to sensible multitudes of every type, likewise to multitudes in general, whether multitudes of sensible objects presented in fantasy, or multitudes of *psychical* acts. In the latter case, for example, temporal succession, and, in general, temporal configuration (the exact analogue of the spatial), forms a moment of this kind" (*PA*, 209, my emphasis). Most commentators miss Husserl's important qualification of this point. Miller, e.g., not only equates Husserl's concern with multitudes in *Philosophy of Arithmetic* with a concern for sensuous multitudes (see Miller, *Numbers in Presence and Absence*, chap. III), but also argues that "[s]uch multitudes provide the sensuous foundation for the 'authentic counting' which generates an original presence of number. They underly [*sic*] the original determination of 'how many' that leads to a constitution of number" (50). The latter claim, of course, misses the importance of Husserl's claim that the multiplicity of objects that make up the concrete totalities that are counted is "'completely arbitrary'" (*PA*, 16; see also § 41 above) and therefore cannot be reduced to a single genus—here, sensible—of objects.

tion [i.e., the figural moment] is the exact analogue of the concept of a genus of sensible qualities" (207), and it is "that from which there develops a genus concept in the rigorous Aristotelian signification of the term."

Husserl characterizes the way in which the collective combination of an intuitively collocated multitude comes to be indirectly signified and thus "symbolized" as follows:

> The rudimentary process [of "actually carrying out the requisite psychical activities on at least a few members selected"] then serves as the sign for the full process intended [of actually carrying out the psychical activities of the collective combination of multitude members], whereby the unified figural quality of the intuition of the multitude guarantees us that the process begun can be continued—especially since the intuitive multitude-unity of the members picked out is recognized as part of the total intuition of the multitude. (213)

From this account, it can be seen that the sign involved in the symbolization of the multitude designates both the intuitive collective combination of individual members of the multitude and the multitude as a whole. This dual designation comes about because

> from early life on we have brought transversive apprehensions of individuals into play with the most heterogeneous types of sensible multitudes, those characteristics (or else their various generic types) had to become associated with the concept of such processes, and, in further consequence, with the concept multitude, and thus produce in each case bridges to the immediate recognition of what is at first a unified sensible intuition, of the type here considered as a multitude. (203)

The sign and what it designates (or symbolizes) therefore enter into a mediating relation with one another that is founded in the associative connection between the actual, "term-by-term" (211) collective combination of the contents of the multiplicity and the presentation of the figural moment of the multitude.[28] The latter "is separated out only by abstraction" (209), with the result that the inauthentic presentation of the multitude signified by the figural moment becomes the "symbolic concept of multitude" (211) and as

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28. Husserl's point here is not that "the experience of sensuous groups involves . . . the taking of one thing (the figural moment) as a 'sign' of another (the sensuous group as such)" (Miller, *Numbers in Presence and Absence*, 50), since on his view *not all sensuous multitudes* (*groups*) *are grasped in terms of figural moments*. Moreover, Husserl does not explain the latter's signitive function *solely* by "taking such a moment as a 'sign' (*Anzeichen*) of the group as such" (48), because, in addition to designating the multitude as a whole, the figural moment *designates* the intuitive collective combination of its members (see Willard, *Logic and the Objectivity of Knowledge*, 97, 99). The latter point is important because it is crucial to Husserl's account in *Philosophy of Arithmetic* of the emergence (and role) of the signs that figure in the symbolic presentation of multitudes.

such the replacement for the authentic presentation and concept of the multitude.[29]

According to Husserl this process "can also extend the concepts of the elementary operations and relations to symbolically presented multiplicities, in which once again the figural moments will often serve as mediators" (217). Thus, when "several sensible multitudes are simultaneously given to us, marked as such by the familiar symbolizing moments, then there belongs to them also a total intuitive unity. They have it in virtue of a figural moment encompassing them all, which in turn characterizes the whole as a multitude." Finally, Husserl stresses the logical equivalence of the authentically presented multitude and its symbolic presentation:

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29. Frege finds in Husserl's account of the inauthentic presentation of a multitude an inconsistency with Husserl's view "that the number belongs to the extension of the concept, i.e., to the totality" of units that comprise the content of the concept of multiplicity (Frege, 318/322). Frege pinpoints the inconsistency in Husserl's claim that number conceived as a "figural moment" is "something predicated of a concept" and therefore precisely *not* something predicated of its extension. Thus, for Frege, "This much is certain, that neither the extension of a concept nor a totality are designated directly, but only a concept" (319/322). Frege concludes from this, "Herewith everything I maintain [i.e., that predications about numbers are statements about a concept] has really been admitted" (318/322). However, Husserl's alleged inconsistency dissolves when one realizes that his account of figural moments is the basis for a concept of number that is fundamentally *different* from his account of number as the "extension" proper to species of the concept of multiplicity. In the latter case, what is at stake for Husserl is the authentic content of the concept proper to number, i.e., number as the determination of a multiplicity of units, of completely arbitrary "anythings." In the former case, his concern lies with the basis for the *inauthentic* concept of number, i.e., the symbolic concept of number. Consequently, Frege's critique of Husserl on the point is based on an equivocal use the term 'number', of which he but not Husserl is guilty. David Bell accepts this equivocation when he writes: "As Frege himself noted, this [i.e., Husserl's notion of 'figural moment'] makes Husserl's theory, in spite of initial appearance to the contrary, not dissimilar to that according to which an ascription of number involves an assertion about a concept." See David Bell, *Husserl* (London: Routledge, 1991), 56.

Frege also identifies what he maintains is yet another inconsistency in Husserl's account of figural moments insofar as instances of the latter, e.g., "the presentation of a swarm" (Frege, 319/323), would seem to present the unity of the multiplicity in question in terms of the homogeneous qualities of its contents. On Frege's view, this is in direct opposition to Husserl's earlier view in *Philosophy of Arithmetic* that a multiplicity is "a presentation of parts whose union, though present, is not presented with them." In other words, Frege considers Husserl's appeal to the figural moment as confirming "*the existence of quasi-qualitative moments*" (*PA*, 203) to be at odds with Husserl's account of the emptiness of the concept of multiplicity, such that the genus of its contents are "completely arbitrary" (16). However, what Frege's critique overlooks here is the "signitive" function of the figural moment in Husserl's account. For Husserl, the figural moment functions as a *sign* that designates the psychical capacity to continue the collective unification of this (e.g., the multiplicity of bees in a swarm) *or any other multiplicity whatever*. Thus, the unification at issue for Husserl has its basis not in the genus of the contents that are unified by the figural moment in question but in the—however problematic (see § 45 above)—psychical process of "collective combination."

> Lastly, I emphasize that the modifications that the multiplicity-presentation undergoes through all the symbolizations described do not affect its logical content. Multiplicity remains the concept of a totality, of a determinate collection of separate contents. Only, in the cases now considered, the segregation of contents and their collection, instead of coming to actual realization, remains either wholly or largely a mere intention. (218)

The process of the symbolic presentation of a multitude considered thus far concerns the symbolization of finite multitudes. However, Husserl maintains that it is also possible to extend "the original concept in such a manner that it surpasses not merely the, so to speak, accidental limits, but also those necessary for the essence of all knowledge, and thereby also attains what is basically an essentially new content." He does so because, in his words, "We speak of *infinite multitudes*" (219). For Husserl, "whenever the talk is of an infinite multitude, we come upon the symbolic presentation of a process of concept formation that can be continued without limit." He describes this process as follows:

> Already in the symbolic presentation of multitudes in the ordinary sense there often surrogates, as we saw earlier, the idea of a process whose unity receives its determinacy through some figural moment of intuition. It is similar here; only now it is a more removed conceptual principle that confers upon the process its determinacy, and that gives the presentation a sure grasp on all that is attainable through it, which it "includes." But whereas in the first case its finitude belonged to the concept of the process, in such a way that in the succession of steps one must be its last, here, to the contrary, what belongs to the concept [of the process] is its being unlimited: The concept of a last step, and thus a last reached member of the multitude, becomes senseless. (220–21)

Although the two processes are "essentially distinct logically . . . the analogy . . . awakens a natural inclination to insert into the presentation of the infinite multitude the intention toward the formation of the corresponding actual collection—in spite of the absurdity of the thought" (221). As a consequence, "there arises a concept that is, as it were, imaginary." While for Husserl it is clear that what this process yields is "an essentially new concept that is no longer a concept of a multitude, in the true [finite] sense of the word," it is also clear that "The presentation of a determinate, unrestricted process is logically irreproachable, as is the idea of all that which falls within its range, which it encompasses by means of its own conceptual unity."

§ 48. Husserl on the Psychological Presentation of Symbolic Numbers

Husserl's account of the presentation of "symbolic number concepts and their infinite multiplicity" (222) begins with the consideration that "the symbolic

presentations of multitudes form the foundation for the symbolic presentations of numbers." Thus, the "obvious lack of restriction on the symbolic expansion of multitudes . . . is also given for numbers." Husserl's account also begins with the conviction that numbers, as "the distinct species of the general concept of multiplicity," have their basis in "a determinate multiplicity of units" that corresponds to "each concrete multiplicity" and as such presents their "number." Therefore, whether this multiplicity is presented authentically or symbolically, "the concept of the collection of all these units is indeed a completely determinate one." As a consequence, "In the symbolic sense we . . . can say of any arbitrary multitude that a determinate number belongs to it even before we have formed that number itself; indeed, also when we are not in position to undertake the actual formation of it."

However, as Husserl's analyses unfold, he abandons his initial view that the symbolic formation of numbers is "coordinated in their rigorous distinctiveness with the true—but to us [authentically] inaccessible—number concepts 'in themselves'" (223). This is clear from Husserl's analyses, which show that that to which the symbolic concept of number refers, or, more precisely, that which the number sign and sign system that comes to express the symbolic concept of number designates, is neither "the number sequence . . . already given beyond any specifiable limit" (229) nor the "step-by-step formation of the (authentic or symbolic) number concepts" (239). Thereby, Husserl brings to its logical conclusion what he noted earlier in his investigations (without developing there its full implications) about the role of external signs in the symbolization of multitudes. Specifically, that "in logical meanings such signs come into consideration only in cases where the concept of what is to be designated by an external sign belongs, as such, to the essential content of the symbolic presentation" (194). In other words, as we have already suggested and will demonstrate below, Husserl's analysis of symbolic numbers arrives at the conclusion that their logical content is inseparable from the external signs and the sign system by which they are designated. Consequently, the logical status of their symbolic content is now *signitive*, which means that they refer neither to the *concept* of a determinate amount of units nor to their symbolic idealization, that is, to the *conceptually* symbolic system of number formation that expands numbers beyond the psychological limits of their authentic presentation. Thus, as we shall see, it is the number signs themselves that function as the "numerical" reference in signitively symbolic number formations, because it is their sensible character that forms the basis for a "numerical interpretation" in accord with the (conventional) rules that govern their combination and transformation. And, as we shall also see, Husserl does not understand these rules to present *concepts*.

On the contrary, he takes these rules to present just that, rules that are "parallel" to the *inauthentic* concepts proper to the ideal concept formation and designation of the symbolic numbers through which the authentic number sequence (i.e., the so-called natural numbers) is expanded beyond its authentic presentation. To the extent that it still makes sense to understand signitively symbolic number formations as having a "logical" *reference* to anything, we will see that they refer—indirectly, via the mediation of the rules for their combination and transformation—to the *inauthentic* concepts of number formation and number designation, concepts in relation to which these rules are established as the signitive (*and thus emphatically not conceptual*) parallels.

It is clear, then, that for Husserl the "external signs" that compose signitively symbolic number formations are not understood as "general names," that is, as names that refer to the determinate, but non-intuitable collections of arbitrary items that happen to fall under inauthentic number *concepts*. Consequently, these signs achieve a "numerical" status only insofar as they are "interpreted" by means of the rules that govern their combination and transformation. The logical status of symbolic numbers conceived in this way is therefore at odds with Husserl's initial position that symbolic and authentic presentations (and concepts) are logically equivalent because of the identity of their object. For just this reason, Husserl understood idealized symbolic numbers, as surrogates for the actual presentation of multitudes of units, to retain a *logical* connection with the authentic cardinal number concept. In other words, as the *ideal* surrogate for the presentation of multitudes of units that exceeds the finite capacity of the human mind to grasp them intuitively (and, therefore, authentically), symbolic numbers in *this* sense retain a tie with what Husserl characterizes as the authentic meaning of the concept of cardinal number as such, namely, a multiplicity of units.

However, the supposition that initially guided Husserl's account of the systematic symbolization of numbers that yields the number *system* was that the foundation of this systematic *is* precisely the authentic presentation of the first few natural numbers. Moreover, the intuitional foundation provided by their presentation is, at first, said to function as the basis of the number system on two closely related yet fundamentally different levels. Thus, on the one hand, Husserl initially understood the number system to be "a matter of symbolic formations for those species of the concept of number that are not accessible to us in the authentic sense" (234). On the other hand, he characterized the authentically acquired concepts of the "more and less" (226) as what permits the determination "as to which of two [symbolically formulated] numbers would be the greater, and which the smaller, di-

rectly from their mere position [as a predecessor or successor in a sequence of number signs] in the system."

Therefore, according to Husserl's statement of the matter prior to *Philosophy of Arithmetic*'s last chapter, "The essence of the systematic number formation consists in this: that it constructs all other number concepts by means of some few elementary concepts and propositions (numerical formulas and rules of operation)" (238). The elementary concepts involved are the number signs (numerals) 1 through 9, "structured in conformity with the principle of the natural number sequence" (236),[30] and the correlative concepts of the 'more' and the 'less'. The rules of operation involved concern the "constructing [of] new concepts and simultaneously designating them along with their construction" (234). As such, these rules present a system that has "a two-fold aspect" (237). Husserl characterizes this as follows:

> On the one hand, it provides for each number a systematic mode of formation (as symbolic stand-in for the missing authentic number concept) utilizing certain elemental numbers, 1, 2, . . . , X, that are given. And, on the other hand, it provides, starting from the number names 1, 2, . . . , X, a systematic mode of formation for the number *names* appertaining to each one of the numbers. A rigorous parallelism governs here between the method for the continuation of the sequence of number *concepts*, and the method for the continuation of the sequence of number *signs*—and this not merely in general, but rather for each individual step, one after the other.[31]

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30. That is, the "principle" that governs the sequence proper to "the domain of numbers that are authentically presentable" (236), i.e., the principle of successive increase on the basis of the collective addition of 'one'.

31. Husserl's letter to Stumpf (see also n. 12) makes it clear that with his discovery that the concept of cardinal number does not form the foundation of universal arithmetic, he also rejected the view that the latter's sign system runs rigorously parallel to any common concept. Thus, the view expressed here of precisely such a parallelism must reflect his view of the matter prior to that discovery. Indeed, Husserl writes to Stumpf that "There is nothing to be wondered at in the fact that a system of signs and operations with signs can replace a system of concepts and operations with judgments, where the two systems run rigorously parallel (*Stumpf Letter*, 159/14). But he then goes on to make it clear that this is *not* how he understands "things to stand with respect to the system of signs in arithmetic" (160/15). He writes: "The sign system of *arithmetica universalis* divides into a certain sequence of levels, comparable to that of a system of concentric circles. The lowest level (the innermost circle) is occupied by the signs 1, 2 = 1 + 1, 3 = 2 + 1, etc.; the next level by fractional signs; and so on. The signs of the lowest level, and they only, are independent. . . . The rules of calculation are, then, so formed that each 'equation' (in whatever way it is set up, i.e., by means of whatever domain levels) is satisfied as an identity with reference to *the* signs and *the* domain of rules which it actually involves" (161/16).

Husserl characterizes the lowest level of signs as belonging to "the totality of whole numbers" and appears to equate these numbers with their signs. This view of the foundational role of signs equated with numbers for arithmetic is also reflected in his text, "On the Logic of Signs (Semiotic)," dated 1890 by the editor and late 1890 or 1891 by the translator—see Ed-

As we have seen (§ 42 above), on Husserl's account, the authentic number concepts for which the systematic numbers function as symbolic stand-ins emerge with the doing away of the "indeterminacy that is essential to the concept [of multiplicity] in its broad sense," whose content is indicated in the "expression 'one and one and one, etc.'" The natural number sequence arises when "a definite number term is associated" with each determinate number concept (one and one; one, one, and one; etc.). For Husserl, "The Numbers in Arithmetic Are Not Abstracta" (or concepts), as the title of the first subsection of Chapter XIII reads (181). For example, "5 does not signify the concept (the abstractum) *five*; but rather 5 is a general name (or else calculational sign) for any arbitrary multitude as one falling under the concept *five*" (181–82).[32] As a consequence, Husserl initially maintained that

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mund Husserl, "Zur Logik der Zeichen (Semiotik)," in *Hua* XII, 340–73; English translation: "On the Logic of Signs (Semiotic)," trans. Dallas Willard, in *Early Writings*, 20–51; henceforth cited as *Semiotic* with German and English page references, respectively. In that early text, Husserl says of systems of signs in general that "In every system of signs we distinguish the basic signs from the derived or composite signs" (368/46), and he says of the arithmetical sign system in particular that "the arithmetical operations, insofar as they are formative of numbers, are rule-governed methods for the production of inauthentic presentations"—with "[t]he basic signs of number theory" being "the signs 0, 1, . . . , 9."

32. Husserl's characterization of the authentic presentation of number as a "determinate multiplicity," i.e., as the amount of any arbitrarily collected multitude of objects whatsoever, is extremely hard to follow if one attempts to get at what he means by beginning on the conceptual level proper to symbolic mathematics, which presupposes that numbers are in fact "abstract concepts." Miller illustrates this difficulty when he avers that "The determinateness which is characteristic of numbers in contrast to mere multitudes is not explained with reference to our experience of concrete multitudes. It is held rather"—by whom, we wonder? Clearly *not* by Husserl—"to arise in a kind of thinking which is directed toward abstract concepts" (Miller, *Numbers in Presence and Absence*, 40). Whatever obscurities are present in Husserl's analyses (in *Philosophy of Arithmetic*) of the psychological origin of the collective combination characteristic of the authentic number concept, it is clear that "[t]he abstraction process that yields the determinate number belonging to a given concrete multiplicity" involves nothing more than "[d]isregarding the specific character of the particular contents grasped together, [such that] one considers and retains each content only insofar as it is an 'anything' or 'one'" (*PA*, 81–82). Rather than yielding the various species of numbers as "abstract" concepts over and above the unity of the collectively combined "anythings" or "ones" that comprise "the universal form appertaining to the multiplicity at hand: one and one, etc." (82), Husserl is very clear that the "similarity [of the universal forms of the species of numbers] rests upon the equivalence of the partial presentations that make them up (the 'ones' or units), as well as the elemental similarity of the psychical acts unifying those presentations." Consequently, apart from the "most universal and empty" (84)—and therefore in precisely *this* sense "abstract"—status of the units rendered determinate by the various "species" of collective combination proper to numbers (e.g., one and one; one, one, and one; etc.), *no other abstractum* is at issue for Husserl in his analysis of the determinateness characteristic of numbers.

This last point is especially clear from Husserl's analysis of the question of how "the concepts of *cardinal number* and *multiplicity* are related to one another" (82). He maintains that "The distinction consists only in this, that the concept of the cardinal number already pre-

supposes a differentiation of the abstract forms of multiplicity from each other, but that of multiplicity does not" (83). It is important to note that what is at issue for Husserl in this quote is the "generic concept" of cardinal number *and not* the concepts of the species of numbers themselves. Thus, "The former is to be taken as the generic concept that originates from the comparison of the determinate multiplicity forms or numbers as species concepts, already differentiated from each other." As a consequence, as we noted in § 41 above, "There is no cardinal number in general" (82). Moreover, as we have seen, Husserl is very clear that we come by the derivation of the number concepts "directly, setting out from arbitrary concrete multiplicities; for each of these falls under one, and indeed a determinate one, of those concepts" (81). It follows from this that because there is no cardinal number in general, the concepts of the species of number cannot arise on the basis of its mediation. And it also follows that because each species of number does not comprise anything beyond a determinate collective combination and the units so combined, numbers for Husserl are not concepts but rather the "universal form appertaining to the multiplicity at hand" (82). Miller's contention, that "the collective combination alone" cannot account for this, "for this connection is common to all multitudes and is in no way distinctive of [specific] multitudes" (Miller, *Numbers in Presence and Absence*, 92), has been rightly criticized by Willard as indicating that Miller "has not got to what Husserl is saying about collective combinations—and the concept of number and number itself" (Dallas Willard, "Review of J. P. Miller, *Numbers in Presence and Absence*," *Husserl Studies* 1 [1984], 124–30, here 127). Willard goes on to say, "Husserl makes boringly clear that there is a different structure of collective combinations of 'anythings' for each number." In connection with Willard's observation here, however, it is curious that in his own account of the authentic concept of number presented by Husserl in *Philosophy of Arithmetic* Willard argues that the "species" at issue in this work refer to "abstract entities that may be contained in other (relatively concrete) entities" (Willard, *Logic and the Objectivity of Knowledge*, 64). As evidence for this, he cites Husserl's discussion of figural moments as producing a "genus concept in the rigorous *Aristotelian* signification of the term." Yet, as we have shown, Husserl draws this connection in his analysis of the symbolic presentation of multitudes, which forms the basis of the "symbolic concept of multitude" (*PA*, 61), and thus with respect to neither the authentic concept of multitude nor the authentic concept of number. It is the latter that is in question in the discussion of the status of the species proper to authentic numbers.

To be fair to Miller, it has to be pointed out that he claims that Husserl's recognition of the categorial unity of number came later, in his *Logical Investigations*. Thus, in the case of the "collective presentation of this or that set of five objects . . . [an] act . . . is intuitively given in a certain formal articulation, and so as an instance of the number species in question. Looking at this intuited individual [act of collection], we perform an 'abstraction,' i.e., we not only isolate the non-independent moment of collective form in what is before us, but we apprehend the idea in it: the number Five as the species of the form swims into our conscious sphere of reference. What we are now meaning is not this individual instance [of the concrete collection of this or that set of five objects], not the intuited object as a whole, not the form immanent in it, but still inseparable from it: what we mean is rather the *ideal form*-species" (*LI* I, 171 [180], cited in Miller, *Numbers in Presence and Absence*, 93). Two comments are in order here. First, contra Miller, in this passage Husserl clearly understands the original, non-abstracted act of collective combination to be what is responsible for the initial *determination* of the number species (in this case, five), whose ideal form is then yielded on the basis of "an abstraction." And, second, Husserl's view that "there must be a form of 'categorial perception' or categorial intuition in which objects such as numbers and multitudes are given as unities" (ibid.) is something that is totally absent from his analyses in *Philosophy of Arithmetic*. (See Part IV, § 168 below for a detailed comparison of the account of number in the *Logical Investigations* with that in *Philosophy of Arithmetic*.)

"The arithmetician absolutely does not operate with the number concepts as such, but rather with the universally presented objects of those concepts" (181). For example, in adding 5 and 5, it is not the concepts that are conjoined, "since each remains identically what it is [i.e., the concept '5']; and since each concept, in itself, is only a single one," which raises a "logical difficulty" of "how are we ever to conjoin the *identical* concepts." According to Husserl, this difficulty is resolved when we realize the following: "5 + 5 = 10 means the same as: a multitude—any one, whichever it may be—falling under the concept *five*, and any *other* multitude falling under that same concept, when united yield a multitude falling under the concept of *ten*" (182).

In line with this non-conceptual character of the content of the authentic concept of number, Husserl characterized the concepts of the more and the less as being based in "an elemental fact, to be described in no other way than by reference to the phenomena" (91), namely, to the collectively thinking of aggregates in which "adding to and taking away is present." Thus, the relation of the more and the less "presupposes for its realization that the original and the augmented assemblages be present to us simultaneously," by which Husserl means "as *one* assemblage without their separate unifications being lost" (92).

It is on the basis, then, of this account of the authentic number concept and the concepts of the 'more' and the 'less' that Husserl initially—that is, before the "shift" in the account of both symbolic numbers and symbolic

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The upshot of all this is that for Husserl (in *Philosophy of Arithmetic*) the "concept" of authentic number derives its status from the peculiar unity of the whole of a determinate collective combination of units, the unity and therefore "wholeness" of which is *not* present in the abstracta (i.e., the units) proper to the latter, *except* insofar as they are combined in accordance with the universal form of however many ones (one and one; one, one, and one; etc.) they happen to be. In other words, apart from the latter and their collective combination, which arises "directly, setting out from arbitrary concrete multiplicities," the number species have no other "contents." (In answer to the question "What . . . distinguishes a case of physical [or metaphysical] combination from a case of collective combination?" [*PA*, 332], Husserl writes: "in the first case a unification is noticeable within the contents of the presentation, while this is not so in the latter case. . . . And this is so even though a certain unity is present in the totality and is perceivable with evidence.") Thus, even if we admit into our discussion Husserl's account of the "ideal form-species" in the *Logical Investigations*, nothing is changed with respect to his account in *Philosophy of Arithmetic* of the origination of the unity and wholeness of authentic numbers. This holds inasmuch as by 'ideal form-species' Husserl clearly does not understand free-floating Platonic forms or 'universal concepts' (e.g., the universal concept of Five), but the abstractively isolated unities of that which is necessarily first given in concrete acts, which in the *Investigations* as in *Philosophy of Arithmetic* do not arise through the mediation (or application) of "abstract" concepts or forms in relation to concrete individuals (i.e., to individual acts of collective combination). For it is the reverse: what is initially given are the concrete individuals and then, only on the basis of this, the *abstracted* forms that they (*subsequently*) are maintained to instantiate.

calculation in *Philosophy of Arithmetic*'s last chapters—articulates the rules for constructing symbolic number formations. This articulation presents the root of these rules in an "idealization of our finite mental capacities" (240), an idealization that manifests an "analogy" (221) with the "idealization" at work in the symbolic presentation of the universal concept of multiplicity. Specifically, "if we start with any arbitrarily symbolic multitude presentation, then we possess (at least ideally) the capability of expanding it without limits by continuously adding new and ever newer members" (223). Thus, we "can think of the members proper to the multitude as mirrored in a constant reiteration and accordingly form the concept of the continuing expansion of the multitude by means of the members of its mirrorings [*Spiegelungen*]." Although "we cannot in fact form the required reiterations *in infinitum* and arrange them in sequence . . . , we can idealizingly disregard these limitations on our abilities and conceive the concepts, which are symbolic also in this respect." Thus, to each symbolic multitude symbolically expanded by "such means . . . there belongs—again, in symbolic presentation—to each level a determinate cardinal number, different for each level." Consequently, "[t]he manifold of conceivable number specifications is . . . infinite, like the manifold of conceivable multitude levels."

In line with this view, the coordination of determinate symbolic number formations "with the true—but to us inaccessible—[authentic] number concepts in 'themselves'" (223) requires something more than the "remote symbolizations" achieved in the idealizing expansion of multitudes. What is required is the invention of a system of signs with the following characteristics: 1) that a few basic signs "correspond, in rigorous parallelism, to a system of conceptual formations grounded upon certain *basic* concepts" (228); 2) that the "unsurpassable simplicity in the formation of natural numbers" (230), that is, "the procedure of *successive number formation through the addition, in each case, of one unit to the number already formed*" (226), be preserved "to the extent feasible" (230); and 3) that "the number system arrived at . . . is not . . . a mere method of symbolizing concepts that are given, but rather one of constructing new concepts and simultaneously designating them along with their construction" (234).

According to Husserl, ordering the sequence of symbolic numbers in terms of levels that repeat a determinate sequence of elementary numbers is the concept of number formation most desirable for rendering symbolically the authentically inaccessible determinate multitudes of ones that compose any arbitrarily given multiplicity. The "sequence must be structured in conformity with the principle of the natural number sequence" (236), such that "The numbers within each level form a sequence ordered according to mag-

nitude. Each number is greater by one than the previous one, but the first (of each level) is greater by one than the last one from the previous level" (232). Thus, "all levels link together and form a unique and endlessly continuing sequence of numbers which corresponds exactly to the natural, primitive number sequence." Such systematically constructed numbers encompass, "in concept, the entire domain of number: that is, there is no actual number to which there would not correspond, as its symbolic correlate, a wholly determinate systematic formation equivalent to it" (233).

The choice of the magnitude of the elementary sequence of numbers for the first level, which comprises the "base number" (235) for the system, is governed by the criterion that it "must be accessible to us without remote systematic expedients" (236). Although number concepts that are authentically presentable (e.g., the first few cardinal numbers) would meet this criterion, Husserl explicitly rejects "the requirement that the elemental numbers must still fall within the domain of numbers that are authentically presentable."[33] And he does so on the grounds that "Such a far-reaching restriction would . . . be unnecessary." With this rejection, Husserl's analyses begin to move beyond their initial supposition of the logical equivalence of symbolic and authentic presentations and therewith of the identity of the object to which both symbolic and authentic number concepts refer. This can be seen when—after expressing the view that "we can form the ideal of an unrestricted continuation of the simple number sequence by correspondingly idealizing our mental capacity" (234) and that, further, "we can think of the sign-formations of the number system also as marks for the parallel

33. Husserl's explicit rejection of authentically presentable numbers, and therefore of authentic number concepts per se, as the basis of the system of symbolic numbers is curiously overlooked by a number of commentators who no doubt miss the fact that Husserl's analyses eventually abandon his initial supposition of the correlation and therefore logical equivalence (based on the identity of their content) of authentic and symbolic presentations and thus of the correlation and logical equivalence (likewise based on the identity of their content) of the authentic and symbolic concepts themselves. Take, e.g., Bell, who writes in connection with "Husserl's theory" of "the symbolic presentations of determinate numbers": "One or more of the earliest signs in the series must be correlated with an *authentic* concept of number" (Bell, *Husserl*, 56). Likewise, Rudolf Bernet maintains regarding the construction of "a system of numerical symbols and inauthentic numeral concepts" that "There is here a possible principle of construction for designating the (intuitionally formed) numbers 1 through 9 as elementary numbers and forming the further numbers by way of a repetition of the series of elementary numbers." (See Rudolf Bernet, Iso Kern, and Eduard Marbach, *An Introduction to Husserlian Phenomenology* [Evanston, Ill.: Northwestern University Press, 1993], 26.) By contrast, Willard correctly observes that in calculation by means of symbols "the intuition of numbers is thus simply by-passed, as are the complicated articulations in the *explicit* conceptualization of numbers as correspondents to numerals in virtue of an analogy between number series and numeral series" (Willard, *Logic and the Objectivity of Knowledge*, 127).

members of the (ideally expanded) number sequence"—Husserl warns that "one must consider well the fact that these all are only modes of presentation and language that are inauthentic in the highest degree and have their source in the idealizations mentioned." With these words Husserl indicates, remarkably, that he no longer holds to the account and language of his earlier analyses in *Philosophy of Arithmetic* regarding the sign formations of the number system, that is, the symbolization that yields the symbolic numbers with which arithmetic operates. Husserl no longer considers such sign formations to be marks for an ideally expanded number sequence putatively running parallel to these formations, because, as he puts it, this way of presenting and talking about these sign formations stems from these very "idealizations" themselves. In other words, the very idealization of natural numbers that yields symbolic numbers is what is responsible for rendering problematic the characterization of the sign formations produced by this expansion as having a reference to supposed ideal natural numbers, understood as actual multiplicities of units that are nevertheless incapable of authentic presentation due to the limits for such presentation that are imposed by the human mind's finitude. Thus, Husserl maintains, "to interpret" the symbolic sign formations "in another, more authentic sense would be to distort the entire sense and purpose of the systematic formation of numbers." Husserl now rejects the view that "the systematic number concept would be the mediator between the natural number and the systematic designation" (233) because "the situation is not one in which the sequence of natural numbers was first given to us, and we then subsequently sought for a symbolization adequate to its conceptual formations." Rather, as already mentioned, what is at stake in the number system so characterized is a method of "constructing new concepts and simultaneously designating them along with their construction" (234), a method whose foundation, according to the analysis in *Philosophy of Arithmetic*'s last two chapters, cannot be characterized appropriately in terms of sign formations that are presentable in the step-by-step formation of either authentic or inauthentic number *concepts*, that is, in terms of concepts that have as their content—either directly (authentic) or indirectly (inauthentic)—multiplicities of units.

§ 49. Husserl on the Symbolic Presentation of the Systematic Construction of New Number Concepts and Their Designation

Based on his analysis of the decadal number system, wherein he aims first of all to reconstruct this system's "method" for the construction of new number "concepts," Husserl therefore comes to realize that the level-by-level re-

iteration of the magnitude of the numbers that form the base, elementary level is *not* accomplished by number concepts at all but by the system of number signs. This is because, on his view, "it proves in general feasible and preferable to substitute certain symbolizations even for the number concepts accessible to us in the form of authentic presentations—indeed, to put it plainly, to substitute external signs" (236). As a consequence of this substitution, there prevails a "rigorous parallelism . . . between the method for the continuation of the sequence of number *concepts* and the method for the continuation of the sequence of number *signs*—and this not merely in general, but rather for each individual step, one after the other" (237). Thus, "number definitions and operation rules, which are the regular medium of systematic processes," are replaced "with corresponding, conventionally fixed formulas expressing equivalencies of sign combinations" (237–38). In connection with this, Husserl writes:

> One will recognize that, in this way, there actually originates an independent system of symbols that permits the derivation of sign after sign in a uniform pattern without there ever turning up—nor could there ever, as such, turn up—other sign formations that appear in other circumstances, accompanying a conceptual process, as designations of the concepts here formed. (238)

The "evidence" for this "peculiar relationship" between concepts and signs has its "inner ground" in the fact that "the system of designations faithfully reflects these conceptual formulations," such that "each number designation will also have to be one formed, in exact correspondence, from the designations of the elements and operations in which formal rules for the replacement of sign compounds by others will be used—rules that correspond to those propositions concerning the relation of concepts."

Despite the correspondence between concepts and signs that results from the method of substituting the latter for the former, Husserl maintains that there is a fundamental difference between the two systems. In the case of number formations that "are systematically rigorous consequences of the elementary concepts along with their forms of combination and transformation . . . the transformations unfold on the basis of knowledge proceeding with necessity from the relevant concepts," whereas in the case of parallel sign formations, "the transformations of signs will indeed proceed according to certain types, but in a wholly external and mechanical manner." Husserl maintains, moreover:

> If, now, we detach these types from their conceptual supports, and if we fix them once and for all in the form of conventional sign equivalencies (in the manner of the rules of a game), then it is clear a priori that we now possess all that is necessary for the independent development of the system of signs,

> and that no result can come about that would not find its correlate upon the side of the conceptual systematic. (238–39)

The consequence of this independent development for Husserl is that "from the logical perspective . . . , both in the tasks of practical enumeration of a given multitude, as well as in those of derivation through calculation of number from number, the solution can be obtained in a purely mechanical fashion." Finally, he considers this consequence to be a "perfection of the number system" (240).

§ 50. The Fundamental Shift in the Logic of Symbolic Numbers Brought about by the Independence of Signitively Symbolic Numbers

It is precisely in Husserl's analysis of the perfection of the number system that the shift in his understanding of the calculational methods in arithmetic noted above (§ 40) comes to the fore and that, corresponding to this, the fundamental shift in his characterization of the logic of symbolic numbers comes about. Husserl now realizes that the independent status of the sign system that characterizes symbolically formed numbers means that the "referent" of their sign formations cannot be properly understood as the logical content of the idealized concepts of the number, a content that he had characterized as the determinate collection of units that belongs to each inauthentically presented multitude. So, instead of referring indirectly, via symbolic concepts whose idealized mode of formation eventually leads back to a determinate collection of units, the "referent" of each sign formation belonging to the conceptually independent sign system is precisely itself a *sign*. That is, the signitive status of the sign is exhausted in its function to "call attention" to itself as a sense-perceptible object. This has two consequences. The first is that "the symbolic number formations of the system are precisely not thought of as compositions of purely abstract [i.e., conceptual] determinations" (241). The second consequence is that "Each such sign complex supplies, in its intuitive unity and typical form, the stable substrate for that chain of conceptual transformations that make up the 'interpretation' of the compound sign." Moreover, the "sensible signs" (242) proper to "each such large sign complex" (241) involved "*in praxi* in all counting and calculating" (242) are understood to be grasped by "the figural moments that impart a unified character to even very large sign complexes" and that, as a result, "extraordinarily facilitate their comprehension in one's grasp" (241).

The major implication of the first consequence is the fundamental change in Husserl's account of the logic of symbolic numbers. The condition

for their newly recognized status as *signitively* symbolic formations is the recognition that they do not refer, even indirectly, to arithmetical operations (enumeration and calculation) that are determined conceptually. Husserl's analyses had initially characterized the indirect reference to operations performed on authentic cardinal numbers as being essential to symbolic numbers, as a consequence of the logical equivalence of their presentation and the authentic presentation of cardinal numbers. But he now comes to realize that "Already with the expansion of the number sequence in the more primitive form first discussed, it is clear that the enumeration of a multitude by means of successive steps along the sequence in no way requires the step-by-step formation of the (authentic or symbolic) number concept" (239). Once this requirement falls away, Husserl explicitly realizes that "Even the successive subsumption of the individual multitude members under the concept of unit becomes superfluous." And this means that Husserl now understands symbolic numbers to be logically independent of the concept of number defined as a determinate multiplicity of units, that is, of the concept of cardinal number.[34]

The major implication of the second consequence concerns the role that Husserl now recognizes the *perception* or *intuition* of number signs to play in the formation of symbolic numbers. Because "a number is defined through such a systemic complex of sensible signs" (241–42), it follows that "the uniformity in this complex forms the means of symbolization for the sequence of conceptual steps that, otherwise, does not hold together."[35] The "conceptual

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34. In his otherwise excellent study, "The Beginnings of Husserl's Philosophy," Ierna apparently misses Husserl's account here of the logical independence of symbolic number formations from the proper conception of numbers, i.e., numbers conceived of as determinate multitudes of units, when he writes (Part 1, 34): "Ultimately it appears that symbolic numbers have a sense only because they could in the end be reduced to 'real,' i.e., properly conceived, numbers. Even though the symbolic presentations can become very complex constructions of signs upon signs, ultimately they obtain sense and validity due to the small but indispensable field of properly conceived numbers. This is what appears to warrant the truthfulness of symbolic numbers and symbolic counting as logically equivalent to proper conceptions."

35. Willard argues that Husserl "believed, that in most arithmetical operations we are not thinking of or representing [*Vorstellung*] numbers at all—whether authentically *or symbolically*" (Willard, *Logic and the Objectivity of Knowledge*, 108). He goes on to state, "This is because what we do to or with symbols in calculation—even within arithmetic—simply has no essential conscious bearing upon numbers and their relations at all" (108–9). Willard bases his argument on Husserl's coming to recognize "that calculation in arithmetic is a mainly nonconceptual manipulation of sensible signs" (108). While in agreement with this last statement, our argument does not draw the same conclusion from it that Willard does, namely, that for Husserl arithmetical calculation with "sensible symbols and their sensible relations" does not involve—*in any sense*—numerical thinking or *Vorstellung*. Clearly, on Husserl's view, what is being thought of or presented in such thinking are not *authentic* numbers. However, it does not follow from this that arithmetical calculation with sensible signs lacks any "numerical"

steps" at issue here concern, of course, the "conventionally fixed formulas" (237) for combining and transforming the sensible complexes of signs, and *not* the conceptual steps that are designated by their sign equivalencies.

§ 51. The Unresolved Question of the Logical Foundation of Signitively Symbolic Numbers

The shift in Husserl's account of the referent proper to signs in the formations composing symbolic numbers and the corresponding shift in his account of arithmetical calculation bring to the fore the question of the extent to which, in the remainder of *Philosophy of Arithmetic*, he addresses the problem of the logical foundation of conceptually independent symbolic numbers and, corresponding to this, conceptually independent symbolic calculation. Despite the new analysis of both, the last chapter contains the following statement, which seems inconsistent with the new analysis:

> Only the systematic combination of the concepts and their interrelationships, which underlie the calculation, can account for the fact that the correspond-

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sense whatsoever. We draw this conclusion for two reasons. First, we argued (in § 46 above) that as a consequence of the same shift (in the course of Husserl's analyses in *Philosophy of Arithmetic*) in his understanding of calculation—a shift that Willard was the first to note—Husserl abandoned his earlier (essentially Brentanian) view of the logical equivalence of the authentic and the symbolic presentation of numbers and therefore the authentic and the symbolic number concepts. Moreover, we also argued (in §48 above) that the consequence of this is Husserl's *non-conceptual* extension of symbolic numbers, an extension that recognizes "numbers" that are *signitively* symbolic, in the sense proper to their status as functional sign equivalencies of inauthentic (i.e., *conceptually* symbolic) number concepts. Second, we understand Husserl's "strictly formal" (Willard, *Logic and the Objectivity of Knowledge*, 109) extension of the concept of calculation beyond the arithmetical domain, to which Willard appeals in support of his claim about the non-numerical status of calculation with symbols, to be something that Husserl recognized as stemming from "higher logical interests than those of *arithmetica numerosa* (with which we currently [i.e., in *Philosophy of Arithmetic*] have to do)" (*PA*, 258). Thus, rather than conclude that the shift in Husserl's understanding of calculation in *Philosophy of Arithmetic* results in a non-numerical understanding of the symbols operative in arithmetical calculation, we argue that attendant to this shift is a transformed understanding of the logical status proper to the numerical significance of "a number [when it is] defined through such a systematic complex of sensible signs" (*PA*, 241–42). As we argue in § 51 below, however, despite this documentable transformation in Husserl's analyses of the numerical significance proper to the "numbers . . . that can be symbolized only signitively" (*PA*, 242), his analyses neither thematize nor analyze the *logical* significance of this transformation. Thus, we are in agreement with Willard that *Philosophy of Arithmetic* "does not provide any analysis of the logic of arithmetic in the formation of judgments through procedures that utilize symbols without conceptualization" (Willard, *Logic and the Objectivity of Knowledge*, 127). Nevertheless, our argument diverges from Willard's insofar as it is able to provide an interpretation of Husserl's consistent appeals to 'number' in his discussions of symbolic calculation, which for the reasons just given is something that Willard's argument for the non-numerical status of such calculation has to overlook if it is to remain consistent.

> ing *designations* come together to form a coherently developed system and that thereby we have certainty that to any derivation of the signs and sign-relations from given ones, which is valid in the sense prescribed by the rules for the signs, there must correspond a derivation of concepts and conceptual relations from *concepts* given, valid in the sense that *thoughts* are. Accordingly, for the grounding of the *calculational methods in arithmetic* we will also have to go back to the number concepts and to their *forms of combination*. (259)

However, in light of our presentation of the new analysis, it is not clear which "number concepts" Husserl has in mind in this passage that we have to return to in order to ground the calculational methods in arithmetic. On the one hand, in accord with his original supposition regarding the logical equivalence of the presentations proper to the authentic and the symbolic number concepts, he says:

> A number system (as, for example, our decadal system) can accordingly be regarded as the most perfect mirror reflection of the realm of the numbers in themselves, that is, of the actual numbers that are in general inaccessible to us. And this is also true with respect to their order, which—with the symbolic as with the authentic concepts—is that of a simple sequence. Thus we may justifiably regard the indirect formations of the system as the symbolic surrogates of the numbers in themselves. (260)

On the other hand, in keeping with his analysis of the conceptual independence of signitive symbolic numbers, Husserl maintains that although the systematic status of these numbers is such that they are "themselves only symbolic surrogates for other concepts that are inaccessible to us" (261), nevertheless, they "are regarded in arithmetic as the ultimate number concepts." As such, they are precisely *not* regarded as the "symbolic stand-ins for other concepts that are inaccessible to us," but rather as the number concepts to which "all other number forms only lead back to and therefore can, in addition, be reconstructed starting from." In contrast, then, to the view that symbolic numbers, being the result of idealizations that can be traced back to authentically presented numbers, are grounded in such numbers and therefore have their logical foundation in the concept of cardinal number (understood as a determinate collection of units), a very different view of the logical status of symbolic numbers can be seen to emerge when Husserl writes:

> In truth they [i.e., systematic symbolic numbers] only function as *normative numbers*—fixed standards, as it were—which all other number forms are compared to for the purposes of an exact comparison among themselves with respect to more or less. And thus we see that *by means of the number sequence the ideal of a universal and exact classification of numbers* is realized in the most perfect way. But that it accomplishes this entirely corresponds to our original intentions. The need for order and classificatory distinction in the jumble of symbolic number forms was in fact the original impulse

> that forced our logical development to expand the domain of authentic numbers given at the outset precisely in its systematic forms.

In contrast to the view guiding his analyses in the first part of *Philosophy of Arithmetic* that it is the finitude of the human mind, reflected in the de facto weakness of its presentational capacities, that is the source of the logical expansion of authentic numbers, here Husserl's identification of this impulse with the need to order and classify symbolic number forms clearly represents another view of the matter.

Thus, when Husserl concludes—with respect to the means of mechanical calculation that characterizes the universal arithmetic involved in the "cases where a number is defined symbolically by means of a structure of numbers that are, without exception, known" (281)—that "[t]hey represent logical methods for evaluating symbolic number compositions . . . , that is, for determining the symbolic normative-formations corresponding to those compositions as the logically qualified stand-ins for the actual number concepts" (271–72), it is no longer clear exactly *what* concept of number he is referring to as 'actual'. Husserl does not tell us which of the two "actual number concepts" that are operative in his analyses of the logical sources of universal arithmetic he has in mind here. Is it the number concept that is "actual" in the sense of rendering determinate the answer to the question of 'how many' with respect to a concrete multiplicity, that is, the multiplicity proper to a multitude whose individual members are subsumed under the concept of unit? That is, is Husserl still referring here to the concept of cardinal number with which he began *Philosophy of Arithmetic*? Or is the number concept that is "actual" the number concept employed by the calculational technique of arithmetical praxis, namely, the normative number concept[36] of the rule-governed, signitively symbolic expansion of the domain of authentic numbers? This concept of number, as we have seen, no longer refers to multitudes whose members are subsumed under the concept of unit,[37] because, being inseparable from the external sign that "belongs, as such, to the essential content of the symbolic presentation" (194), it is *already* symbolic in its essence.

Husserl's concluding analyses in *Philosophy of Arithmetic* of the "so-called 'four species,' the most elemental of arithmetical operations" (262)—that is, addition, subtraction, multiplication, and division, along with analyses of the "higher operations" (276) (e.g., "multiplying a sum by a number, di-

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36. Strictly speaking (as we have seen), signitively symbolic numbers are not number concepts, according to Husserl, because their meaning as signs does not—as in the case of systematically symbolic numbers—accompany the concepts and conceptual operations of number formation "whose conceptual content could still be brought before the mind" (242).

37. See § 50 above.

viding a product by a sum, raising a quotient to a power, etc." [278])—make it clear that "it is . . . a matter of indifference whether we take as the basis of arithmetic merely the sequence of natural numbers or the system numbers" (264). Consequently, the question of which are the "actual" numbers that Husserl understands to underlie the operations of arithmetic remains unclarified—or perhaps better: ambiguous—at the conclusion of this work. This is the case because, on the one hand, Husserl arrives at the view that the "scientific deliberations"[38] that "form the *domain of universal arithmetic*" (280) and that prove the truth value of mechanical calculation *are able to do so without any appeal to authentically given numbers*, because this truth value "is necessarily independent of whether we operate with the concepts or simply with their signs." In other words, because "the logical validity" of mechanical calculation "is guaranteed by means of the rigorous parallelism between the systematic of the numbers and the number relations, on the one hand, and that of the number signs and the relations of number signs (equivalencies of symbols), on the other" (271), Husserl comes to realize that logically valid results can be achieved without any appeal to authentic numbers or to calculational operations with such numbers. That is because the "small numbers" (263), which "can be given to us as simple and non-systematic," nevertheless lend themselves to being easily positioned "within the system" of systematic or symbolic numbers, such that "we can regard it in each case as already accomplished [i.e., that the small numbers have been given to us as simple and non-systematic] and therefore can also take combinations of such small numbers to be as such already combinations of systematic numbers" (263–64). On the other hand, however, Husserl continues to hold that "just as the individual symbolic number stands in for a definite authentic one, so also each symbolic operation of combination stands in for a definite (although not actually executable) authentic one" (263).

Husserl's recognition of the "authentic" inaccessibility of most numbers, together with the fact that now, in *Philosophy of Arithmetic*'s last chapter, he understands the "symbolic surrogate concepts" to be something that we "classify by taking as our basis a sequence of normal-form concepts," gives rise to the following question: can his overall account of symbolic numbers remain coherent while continuing to maintain what he maintained prior to *Philosophy of Arithmetic*'s last chapters, namely, that the symbolic numbers and symbolic operations proper to their combination (calculation) "stand in for" the authentic number concepts and their combination? Once Husserl's analyses are guided by

38. It is interesting to note that the view expressed here, that the truth value of mechanical calculation is the result of "scientific deliberations," is contradicted when Husserl writes to Stumpf that "[t]he *arithmetica universalis* is no science" (*Stumpf Letter*, 161/17).

the view that the symbolically systematic numbers in arithmetic are no longer understood properly when one maintains that, "in truth" (261), they lead back to number concepts that are "authentic," this question becomes unavoidable.

Yet Husserl does not raise this question, let alone attempt to answer it, at the conclusion of *Philosophy of Arithmetic*.[39] Rather, after noting that his analyses of arithmetic "have all along had in mind only the cases where a number is defined symbolically by means of a structure of numbers that are, without exception, known" (281), he calls attention to another "great" (282) group of problems. This group deals with "a special branch of number theory of the highest importance, that of algebra." It has to do with the problems involved in symbolic numbers "defined by *equations*" (281), in which "[a] number can also be symbolically defined as an unknown constituent of such a precisely characterized structure of numbers whose value is already known or is to be calculated from an operation structure built up completely out of known numbers." Just as the first group of problems, that is, those taken up in *Philosophy of Arithmetic*, requires an investigation by a universal arithmetic, the second group of problems "also evokes *the need for a universal arithmetic*" (282). And, while both groups of problems concern "the indirect determination of number" (283), the second group "has to do with a number determination that is indirect to a far higher degree." Husserl reserves the investigation of the universal arithmetic of the latter for the second volume of his arithmetical studies, which he announced at the time of the publication of the first and, as it turned out, only volume of these studies.[40]

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39. In both his letter to Stumpf and "On the Logic of Signs (Semiotic)," Husserl expressly states that the problem behind this question, namely, the *logic* of the symbolic calculation operative in arithmetic, is something that does not yet exist. In the latter text he writes: "If one takes only the most common and simplest of algorithms, that in which one counts and calculates with numbers, one will search logical works in vain for light on what really makes such mechanical operations, with mere written characters or word signs, capable of vastly expanding our actual knowledge concerning the number concepts—making possible for us accomplishments which were inconceivable to the greatest thinkers of antiquity" (*Semiotic*, 372/50). What is missing according to Husserl is the "*logic* of symbolic representations and judgments" (365/43), which he contends has to set "two goals for itself: It would seek to get to the bottom of the function of symbolic presentations and judgments in the activity of *theoretical* judgment, and above all, to provide *logical* illumination for those algorithmic modes of procedure which have on such a wide scale become the vehicles of progress in the exact sciences" (365/44). Because, on Husserl's view, "the slightest insight into the *logic* of the matter" (370/48) in universal arithmetic is lacking "from the time of a Leibniz, D'Alembert and Carnot up to today," it is reasonable to assume that among the "many attempts" that have "made no essential progress" in this regard he would have to include his own *Philosophy of Arithmetic*.

40. In his "Author's Notice" (*Selbstanzeige*) for the first (and only) volume of *Philosophy of Arithmetic*, Husserl expressed the conviction that "[t]he higher-level symbolic methods . . . that constitute the essence of the universal arithmetic . . . will appear [in the second volume]

§ 52. Summary and Conclusion

The psychological and logical investigations in Husserl's *Philosophy of Arithmetic* contain foundational analyses of arithmetic that are guided by two distinct and incompatible theses. One is the thesis that the basic concepts and calculational operations of universal arithmetic, that is, the numbers and the algorithms of both ordinary arithmetic and the symbolic calculus, have their foundation in the concept of cardinal number. The other is the thesis that both the numbers and algorithms of universal arithmetic have their foundation in a system of signs that are not conceptual but rather formal-logical, in the sense of that part of logic defined as symbolic technique. The juxtaposition of analyses guided by conflicting theses in the single text of *Philosophy of Arithmetic* is explained by a radical shift in their author's view of the logical relation between authentic and inauthentic number "concepts." This shift occurred sometime between the date of the original studies from which this text is drawn and the date of their revision on the occasion of its composition. Husserl's manner of presenting the analyses that exhibit this shift does not call explicit attention to it, but rather suggests a linear development from the view of the foundational status of the concept of cardinal number for universal arithmetic to that of the conceptual independence of the sign system that defines arithmetic. So it is left for the reader to make explicit in Husserl's analyses what he himself does not.

Thus, on the one hand, *Philosophy of Arithmetic* contains "earlier" analyses reflecting the view that the operations of universal arithmetic have their foundation in a symbolic concept of number that is characterized by its idealization of the human mind's limited capacity to present cardinal numbers directly, that is, authentically. Symbolic numbers in this sense are ideal and defined by their logical equivalence to the authentic concept of cardinal number, because, according to Husserl, both number concepts have the same content, namely, a collection of separately collected units or ones. The logical equivalence of these concepts does not entail their identity, however, because authentic cardinal numbers present this object authentically, in the direct intuition of the collected units and the direct presentation of the unity proper to their multitude, while symbolic numbers present it inauthentically, in indirect presentations mediated by signs.

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as one member of a whole class of arithmetics unified in virtue of the homogeneous character of identically the same algorithm" (*PA*, 288). As we shall see, Husserl apparently abandoned the conviction that the *essence* of universal arithmetic is algorithmic, as his logical investigations subsequent to *Philosophy of Arithmetic* sought to provide a logical (and thus a non- or trans-algorithmic) foundation for the logical status of symbolic calculus operative in both formal mathematics and formal logic.

Husserl's logical investigations of symbolic numbers in *Philosophy of Arithmetic* begin with the view that they are surrogates for authentic cardinal numbers, thus for number concepts that refer to a determinate amount of completely arbitrary items belonging to a multiplicity. Moreover, he maintains that the determinate items that make up the content of the authentic number concepts are presented as generically empty anythings or units (ones). As the surrogates or inauthentic stand-ins for the authentic (intuitive) presentation of the content of authentic number concepts, symbolic numbers have the logical status of indirect determinations of cardinal numbers. Because, on this view, the object of symbolic number concepts and authentic number concepts is the same, that is, the object for both is a determinate multiplicity of units, Husserl's analyses of symbolic numbers commence from the supposition that both number concepts—the authentic and the inauthentic—are logically equivalent.

However, as Husserl's analyses of the basic symbolic operations proper to arithmetic unfold, operations he characterizes as 'calculational technique', he gradually comes to realize that both these operations and the symbolic numbers underlying them are *not* direct surrogates for the authentic number concepts and authentic conceptual operations on them. Rather, both the calculational technique operative in arithmetic and its symbolically numerical substratum function as signitive surrogates for the *inauthentic* conceptual system of (systematic) number formations and the conceptual operations on these formations. As signitive surrogates for systematically determined (inauthentic and therefore symbolic) numbers and the systematically determined (again, inauthentic and therefore symbolic) calculational operations upon them, signitively symbolic numbers manifest a complete *conceptual* independence from the authentic *and* inauthentic number concepts together with the concepts involved in the calculation with each.

Signitively symbolic numbers and the calculational technique that operates with them achieve their conceptual independence from their status as the sign equivalencies of the systematic numbers and systematic number operations that form the basis for arithmetical knowledge. The establishment by a universal arithmetic of the rigorous parallelism between the former and the latter insures that the signitive calculative technique has a logical validity equivalent to arithmetical calculation that is conceptual.

Husserl's analyses thematize the shift in calculative technique that occurs with the invention of a sign system in the manner of the "rules of a game." The role of sensible number signs, which provide the stable foundation for their "numerical" interpretation in accordance with conventionally fixed algorithms, is articulated in line with these analyses. In addition, Husserl the-

matizes the fact that it is the intuitive givenness of the former that permits their numerical interpretation in terms of the two principles that govern the expansion of the natural number sequence: addition by ones and the concepts of the more and the less. Moreover, Husserl is aware that at the level of signitively symbolic numbers, neither principle functions in accordance with the authentic conceptual origin of addition and the more and the less; neither functions (respectively) as a principle that relates to determinate quantities of units or to intuitively presented items of a multiplicity. Thus, with respect to the principle of successive number formation, what is at issue at the signitively symbolic level is the sensible *sequence* of number signs—and not the addition by one proper to the units of a multiplicity that comprise the content of the authentic cardinal number concepts. What is at issue at the signitive level with respect to the more and the less is the designation of which of two number signs, as a predecessor or successor in a sequence of number signs, is the greater or the smaller—and not which multitude of units belonging to a multiplicity is more or less.

Yet Husserl does *not* thematize the fact that, in his analyses of the shift that occurs with the invention of the significatively symbolic calculational technique, the concepts to which the signs of this technique refer *already* relate only indirectly to the numbers themselves. As a result, he does not thematize the fact that the rigorous parallelism between 1) the signitively symbolic numbers and 2) the signitively symbolic operations upon them and the "number concepts" is established with respect to number concepts that present the systematic concepts for the formation and designation of the ideally (and therefore *inauthentically*) expanded number sequence. In other words, at the operative level of Husserl's analyses—but not at the level of their reflective thematization—it is the latter, conceptually symbolic numbers and conceptually symbolic calculational operations, that now (in the final analyses of *Philosophy of Arithmetic*) comprise the foundation for the system of arithmetic. Therefore, the contents of the authentic concepts proper to cardinal numbers and the operations upon them that (in the earlier analyses of *Philosophy of Arithmetic*) were characterized as the logically equivalent contents of the symbolic number formations *are no longer so characterized by Husserl*.

Husserl is quite clear, then, that the contents of the authentic cardinal number concepts have become superfluous with respect to the foundation of the signitively symbolic number sequence and the technique of calculation with rule-governed algorithms (i.e., signitive calculation). This clarity, however, does not extend to his final reflections and concluding analyses on precisely which concept of number is the "actual" one underlying the signitive understanding of the calculative technique operative in the universal

arithmetic of known numbers. Thus, these analyses are ambiguous on this point, since they oscillate between the appeal to authentic cardinal number concepts and to signitively symbolic numbers. In other words, the precise sense of the numbers that are "indirectly" determined by universal arithmetic remains unclarified. In the case of the characterization of such numbers as authentic cardinal numbers, the content of the "concepts" of universal arithmetic would be the precisely delimited units of multiplicities—whether or not the amounts (*Anzahlen*) of these units are accessible to thinking. In the case of the signitively symbolic characterization of such numbers, the content of "concepts" of universal arithmetic would be sense-perceptible signs and the algorithmic rules that determine their arithmetical value.

Chapter Fourteen

Klein's Desedimentation of the Origin of Algebra and Husserl's Failure to Ground Symbolic Calculation in Authentic Numbers

§ 53. Implications of Klein's Desedimentation of the Origin of Algebra for Husserl's Analyses of the Concept Proper to Number in *Philosophy of Arithmetic*

Our discussion of the concepts of authentic and symbolic number operative in Husserl's analyses in *Philosophy of Arithmetic* has shown that he was eventually forced to abandon his initial thesis of their logical equivalence. Specifically, we have demonstrated that rather than substantiate the view that the symbolic presentation of number "can [act as a] surrogate, to the furthest extent, for the corresponding authentic presentation" (*PA*, 194) of number, Husserl's analyses conclude by substantiating, in effect, the opposite view. That is, they substantiate the view that the signitively symbolic numbers at issue in arithmetical calculation, which are presented by sensible number signs, *do not refer to the same object as authentic numbers.*

Husserl's analyses clearly show, on the one hand, that the authentic concept of number refers directly to determinate amounts (i.e., the answer to the question 'How many?') of a multitude of determinate objects. Moreover, the latter have the status of generically undetermined—which is to say, physically and "metaphysically" empty and therefore neutral—units or ones (these two concepts being equivalent). On Husserl's view, the authentic number concepts are manifestly not "abstracta," since each one involves the "universal form appertaining to the multitude at hand" (82), that is, one and one; one, one, and one, etc. All of this, on the other hand, is in the sharpest possible contrast with the signitively symbolic number concepts, which refer to neither a determinate multitude of units or ones nor to the universal form of their amount. Rather, signitively symbolic numbers, or, more properly, sig-

nitively symbolic number signs, indirectly determine "number" through the calculational rules—in the manner of the "rules of the game"—for their combination and transformation.

Husserl's concluding analyses in *Philosophy of Arithmetic*, which discover that the "mechanism of the symbolic methodology can break completely free of the conceptual substratum of its employment" (258), do not thematize the implications of the fact that two different concepts of number—authentic and signitively symbolic—are operative in these analyses. Because of this, the logic of calculation with symbolic numbers, in the sense of a theory that would account for their capacity to retain a numerical meaning despite their lack of reference to authentic numbers, remains uninvestigated at the conclusion of *Philosophy of Arithmetic*. Thus, beyond the *assertion* of the rigorous parallelism between 1) the rules for calculating with number (and operational) signs and 2) the *inauthentic* concepts involved in the construction of symbolic number *concepts*,[41] Husserl's analyses in *Philosophy of Arithmetic* fail to shed any light on how the mechanistic technique at issue in such calculation is able to achieve what nobody doubts it is able to achieve: arithmetical knowledge.

In what follows we shall defend the thesis that Klein's desedimentation of the origin of algebra provides an account of its historical context that explains precisely why Husserl's attempt to establish the logical origin of the technique of calculating with symbols (the number and operational signs composing signitively symbolic number formations) on the foundation of authentic numbers and their operational relations not only failed but had to fail. We shall argue that Klein demonstrates that fundamentally at issue in the origin of algebra is a symbolically novel understanding of the concept of number. He shows that this new concept decisively transforms the Greek ἀριθμός-concept, understood as the direct reference to a definite amount of definite objects. In line with this, we shall argue that Husserl's analysis of the authentic concept of cardinal number corresponds to the Greek concept of ἀριθμός and that his analysis of signitively symbolic "numbers" corresponds to the symbolic concept of number that is first made possible by the invention of algebra. In addition, we shall argue that Klein's desedimentation of the origin of algebra shows that it was self-consciously understood by its inventors to be an "art" and not a theory. Finally, we shall argue that this understanding effectively obscures the theoretical presuppositions that make possible the algebraic calculation with symbols, and that this obscurity is mirrored in Husserl's failure to establish the logical foundation of arithmetical calculation with symbols.

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41. See §§ 47–49 above.

On Klein's view the historical context of the origin of algebra discloses that rather than being the realization of the ancient Greek demand (which, as we have seen in our preview of these matters, he traces back to Plato) for a theoretical science of logistic (λογιστική) or calculation, the modern method of calculating with symbols develops as a praxis. This context also discloses that the originators of the "analytical art" (*ars analytice*) characteristic of algebra understood it either as simply *replacing* Greek logistic or as the latter's perfection. In both cases, therefore, the ontological problems inseparable from the ancient demand for a theoretical science of logistic cease to be a factor for the modern understanding of the algebraic method. Consequently, for the most part,[42] the innovators of symbolic calculation were completely unaware of the conceptual presuppositions of their method. Because for Klein these presuppositions involve the transformation of the ἀριθμός-concept of 'number', and because the effect of this transformation is a new conceptuality (*Begrifflichkeit*) that renders the Greek and modern number concepts and therefore their respective *concepts* of 'number' incommensurable, it is our thesis that the desedimentation of this incommensurability provides the historical background for Husserl's failed attempt to establish systematically the origin of the symbolic concept of number[43] on the foundation of the authentic concept of number.[44] At the theoretical level, Husserl's failure signifies his inability to establish the foundation for the logic of the technique proper to signitively symbolic number calculation, which means that the demonstration of our thesis will perforce establish the historical continuity of Husserl's failure to ground such a logic with the unrealized Platonic demand for a theoretical logistic.

§ 54. Klein's Desedimentation of the Two Salient Features of the Foundations of Greek Mathematics

As we have also seen in our preview of these issues (§ 36), Klein's desedimentation of the origin of algebra as Vieta's invention of an "analytical art" begins with a discussion of Neoplatonic mathematics. We suggested that this indirect starting point follows from Klein's thesis that, in order to distinguish

42. Descartes is the notable exception. See the discussion below (§ 114) of the extent to which, according to Klein, he attempted to work out the theoretical basis of the symbolic concept of number.

43. The historical background of the symbolic concept of number/concept of symbolic number is the modern concept that Klein desediments in his *Greek Mathematical Thought and the Origin of Algebra*.

44. The historical background of which is the ἀριθμός-concept desedimented by Klein.

Vieta's innovation from its Greek foundations, the latter need to be reactivated in concert with the reactivation of their own presuppositions. For Klein, what is central to these presuppositions is the meaning of ἀριθμός that is presupposed in Diophantus's *Arithmetic*, as well as in all of Greek mathematics. Thus, because Vieta's innovation represents both an assimilation and transformation of the technique elaborated in Diophantus's *Arithmetic*, Klein begins with a consideration of the latter's Neoplatonic background. We also saw, however, that Klein contends that this background itself is informed by the mathematical presuppositions of Plato's philosophy. Specifically, it is informed by Plato's understanding of arithmetic and logistic and the theoretical ἀριθμός-concept that is presupposed in his philosophical demands for theories of each. Thus, after his initial consideration of Neoplatonic mathematics, Klein investigates its Platonic background, including also the Pythagorean and Eleatic background of this background. Finally, Klein concludes his account of the presuppositions proper to the background of Diophantus's *Arithmetic* with an account of Aristotle's critique of the theoretical mode of being that Plato's philosophy attributes to ἀριθμός.

Klein's desedimentation of the foundations of Greek mathematics uncovers two features that are distinct from the mathematical foundations of Vieta's *Ars analytice*. One is the ἀριθμός-concept of 'number', the definite items of which are posited at the theoretical level as indivisible ones or units accessible only to thought, that is, as "intelligible" monads. Klein's recovery of this understanding permits him to grasp Vieta's transformation of the concept of number as a concept that differs radically from the Greek concept. For Vieta, the concept of number involves a sign that directly refers to "*the general character of being an amount* which belongs to every possible quantity" (*GMTOA*, 182/174), that is, to 'definite amount in general', a concept that therefore "only mediately" refers to "the things or units that may be present in any particular quantity." On Klein's view, at issue here is not a higher level of "abstractness" in Vieta's concept in comparison with the ἀριθμός-concept, as historians of mathematics commonly assume, but rather a novel and therefore unprecedented difference in the conceptuality of the number concept itself. On the basis of Vieta's introduction of the term *symbolum* to designate the "stipulations governing equations" (263–64 n. 226),[45] Klein characterizes the new number concept as "*symbolic in nature*" (183/176).

The other distinguishing feature of Greek mathematics that Klein uncovers concerns the difficulties presented to it by Plato's demand for a the-

45. Klein's note on *symbolum* appears only in the English translation of *Greek Mathematical Thought and the Origin of Algebra*.

oretical logistic. These difficulties are rooted in Plato's ontological presupposition of the indivisible mode of being belonging to monads, which also determines the theoretical understanding of ἀριθμός in Neoplatonic mathematics. These difficulties are rooted in the impossibility—so long as mathematics remains Platonic—of providing a theoretical foundation for "the partitioning of the units themselves, although this is in most cases unavoidable in the course of an exact calculation" (51/44). Klein takes Aristotle's critique of the Platonic χωρισμός (separation) thesis, which postulates the indivisible mode of being of monads, to prepare the way for a "*type* of theoretic logistic which can be built on Peripatetic foundations—the 'arithmetical' textbook of Diophantus" (105/113). However, as this book's title suggests, Diophantus's logistic is not unambiguously logistical. And, indeed, for Klein the relationship, and the distinction itself, between arithmetic and logistic in both Neoplatonic and Greek mathematics generally turns out to be quite a complex affair.

In order to develop our thesis that Klein's desedimentation of the origin of algebra provides the historical background for the systematic failure of Husserl's attempt in *Philosophy of Arithmetic* to establish the foundation of symbolic arithmetic in the authentic concept of number, we shall first consider in detail Klein's account of the relationship between arithmetic and logistic in Neoplatonic mathematics. We shall do so because it is precisely the desedimentation of their unstable relationship that Klein contends points to certain obscurities in Plato's account of their relationship, obscurities he then traces back to Plato's ontological presuppositions regarding the theoretical mode of being of ἀριθμός. Consequently, our consideration of Klein's account of the relationship between arithmetic and logistic will set the stage for our discussion of his reactivation of the ἀριθμός-concept in both Greek mathematics and Greek philosophy. This reactivation, as we have already indicated and now want to stress, is absolutely crucial for his desedimentation of the presuppositions of Diophantus's *Arithmetic* and thus also of Vieta's transformation of the logistical technique operative therein into the modern method of algebraic analysis.

Chapter Fifteen

Logistic and Arithmetic in Neoplatonic Mathematics and in Plato

§ 55. The Opposition between Logistic and Arithmetic in Neoplatonic Thought

For Klein "Neoplatonic mathematics is governed by a fundamental distinction which is, indeed, inherent in Greek science in general, but is here most strongly formulated" (23/10). This distinction is between "that which is in no way subject to change, or to becoming and passing away" and that which is subject to change. Thus, one branch of mathematics "contemplates that which is always such as it is and which alone is capable of being known: for that which is known in the act of knowing, being a communicable and teachable possession, must be something which is once and for all fixed." Klein notes that "whatever pertains to the questions: How large? How many?" belongs to "a certain territory" within the realm of being that has this character of being, and thus is something that can be known. Consequently, "Insofar as the objects of mathematics fulfill the conditions set by the Greeks for objects of knowledge, they are not objects of the senses (*αἰσθητά*)" (23–24/10), which "are subject to change, or to becoming and passing away" (23/10), "but only objects of thought (*νοητά*)." These "mathematical *νοητά* fall into two classes," namely: "(1) the 'continuous' magnitudes—lines, areas, solids; (2) the 'discrete' amounts—two, three, four, etc." As a result, two parts of the noetic branch of mathematics are distinguished, which correspond to these two classes: geometry and arithmetic.

The other branch of mathematics "has for its object the treatment and manipulation of *αἰσθητά* insofar as they are subject to determinations of size or counting" (24/11). Geodetics (the art of measuring land and of measuring in general), logistic (as the art of calculation), and also music (harmonics), optics, and mechanics all belong to this branch. Astronomy, on the other hand, "occupies a special position insofar as it is assigned now to geometry,

now to arithmetic." Klein writes, "Like all of these distinctions, the opposition of a 'pure' science of definite amounts and a 'practical' art of calculation goes back to Plato." However, it is his thesis that "in Plato this opposition is by no means fixed as unambiguously, either in terminology or, more importantly, in content, as it is for the Neoplatonists"—a fact, Klein notes, that "has frequently been overlooked."

Moreover, he points out that the chief Neoplatonic sources[46] on both the nature of and the relationship between arithmetic and logistic refer back to certain passages in Plato, whose "precise interpretation . . . was, in fact, the original concern prompting the Neoplatonic commentators" (29/16). However, because these sources present inconsistent accounts of the exact status of logistic's relation to the objects of sense, Klein maintains that the lack of clarity due to these inconsistencies must point to "certain difficulties inherent in Plato's own conceptions." At the same time, these Neoplatonic sources are in agreement "that logistic teaches how to *proceed* in order to solve problems relating to one or more multitudes of countable objects. It shows how to make definite the amount of such objects, i.e., how to 'calculate.' In calculation the *result* alone, which *varies* as the given multitudes vary, matters" (28/15). They are likewise in agreement that "the possibility of calculation is grounded in certain *unchangeable* characteristics of the definite amounts 'themselves.'" Arithmetic deals with these by studying "their properties and kinds as they are in themselves, not as they may be read off the countable things." Arithmetic is therefore "pure" and "does not 'calculate' with definite amounts of definite objects." Consequently, "Calculation with definite amounts of definite objects is nothing but the 'application' of the states of affairs of 'pure' arithmetic; logistic is nothing but 'applied' arithmetic, which serves, above all, practical ends."

Klein holds, however, that despite the agreement among the Neoplatonic sources regarding the pure or theoretical nature of arithmetic and the applied or practical nature of calculation, they are divided over the exact

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46. Klein's discussion draws on the following sources: Proclus (400 AD) (who cites Geminus [first century BC?] on the difference between arithmetic and logistic), "Proclus Diadochus," *In primum Euclidis elementorum librum commentarii*, ed. Gottfried Friedlein (Leipzig: Teubner, 1873) (henceforth cited as 'Proclus'); an anonymous scholium to *Charmides* 165 E (which is based largely on Geminus), in *Platonis dialogi secundum Thrasylli tetralogias dispositi*, ed. Karl Friedrich Hermann (Leipzig: Teubner, 1884), VI: 290 (henceforth cited as 'Hermann' with volume and page numbers); Olympiodorus's scholia to Plato's *Gorgias*, 450 D and 451 A–C, in "Olympiodori Philosophi Scholia in Platonis Gorgiam," ed. Albert Jahn, *Neue Jahrbücher für Philologie und Pädagogik*, Suppl. 14 (Leipzig: Teubner, 1848), 104–49, here 131–33; and an anonymous scholium to the previous passage in the *Gorgias*, in Hermann, VI: 301.

locus of the distinction between the realm of what is changeable and what is unchangeable. From these accounts it is not clear precisely what the basis is for the distinction between an unchangeable and therefore "pure" constituent and a changeable and therefore applied constituent, respectively, of the objects with which arithmetic and logistic deal (28/16). On the one hand, the distinction is presented as being determined solely by the noetic and sensible nature of the objects with which, again respectively, arithmetic and logistic are concerned, a distinction that therefore provides the basis for their differentiation. On the other hand, this distinction is presented as being determined by a difference in the realm of the definite amounts themselves with which each deals, albeit in different ways.

Thus, in some Neoplatonic accounts (those based on Geminus) the objects of logistic are identified "with that realm of the objects of sense in which the calculations take place," while those of arithmetic are identified "with a realm of pure definite amounts of definite objects." However, in the other accounts (Olympiodorus's and the scholium to the *Gorgias*) the distinction between the changeable and unchangeable constituents that distinguish logistic from arithmetic seems to be transferred to "the distinction between an unchangeable, i.e., noetic, and a changeable constituent" in "the definite amounts themselves." For Klein, this is reflected in what these accounts refer to as the "'material' (ὕλη) of the definite amounts [that] is clearly not attached to the objects of sense but is identical with the 'how many' which each definite amount designates." While this material changes in accord with the definite amount designated as the result of a given calculation, the classifications of definite amounts and their pure relationships remain unchangeable on this view. This preserves the difference between the purity of arithmetic (which investigates these classifications and relationships) and its application in logistic. Thus, while both sets of the Neoplatonic accounts identify—according to whether it is present or absent—*change* as what distinguishes logistic from arithmetic, the second group of Neoplatonic accounts attribute this changing aspect to the definite amounts themselves, which occurs "quite independently of the fact that the objects of calculation might be objects of sense." Klein elaborates this as follows:

> In the definite amount "six," for instance, the multitude "six," with its "how many," its ὕλη, must be distinguished from its εἶδος, namely the "even-times-odd" (ἀρτιοπέριττον – since six is composed of the even factor two and the odd factor three). Arithmetic treats of the *eidos*, logistic of the multitude of "hyletic" monads. (28–29/16)

On Klein's view, however, the accounts of logistic that identify its changing constituent with the ὕλη of ἀριθμοί leave unresolved the following question:

"What, now, is the relation of this conception of the objects of logistic to their inclusion within the realm of αἰσθητά"? (29/16). Unresolved, then—and not just in this account but in all the Neoplatonic accounts—is the question of whether "the multitude which is designated by any definite amount after all always [has] to be referred to the objects of sense." To address these questions, Klein's desedimentation of the presuppositions that form the Neoplatonic background of Diophantus's *Arithmetic* next investigates the problematic status of logistic and arithmetic in Plato, which, as we have seen, Klein identifies as "the deeper reason for this lack of clarity" in the Neoplatonic accounts.

§ 56. Logistic and Arithmetic in Plato

Klein maintains the key Platonic texts[47] that define arithmetic and logistic do not support the usual interpretation, which maintains (on the basis of the Neoplatonic sources discussed above) that they "affirm a direct opposition of arithmetic as a *theoretical* discipline to logistic as the *practical* art of calculation" (30/18). Moreover, he holds that the meaning of these texts' definitions of arithmetic and logistic is "independent of the practical or theoretical character of the two disciplines" (35/24). Finally, although "Both are concerned with 'definite amounts' and 'definite amount' in general, with the ἀριθμός" (33/21), he finds it significant that ἀριθμός does not occur in either definition. Among other things, this rules out arithmetic's being for Plato "first and foremost" (31/19) "'number theory.'"

Thus, "Socrates says that if he were asked 'with what' (περὶ τί) arithmetic deals, he would answer: 'it belongs to that knowledge which deals with the even and the odd, with reference to *how much* either happens to be'" (29/17; *Gorgias* 451 A–B). Regarding logistic, he says that "It deals with the same thing, namely the even and the uneven; but logistic differs {from arithmetic} in so far as it studies the even and the odd with respect to the multitude which they {the single even and odd} make both with themselves and with each other" (451 B–C). On Klein's view, the consistency of the accounts of arithmetic and logistic in the relevant Platonic texts rules out the possibility that their "strangely involved" definitions, in which "the word ἀριθμός is carefully avoided," are "'accidental'" (*GMTOA*, 29/17).

47. The classical definitions of arithmetic are found in *Gorgias* 451 A–B and 453 E and in *Theaetetus* 198 A. The classical definitions of logistic are found in *Gorgias* 451 B–C and in *Charmindes* 165 E–166 B.

Klein's interpretation of Socrates' definitions considers first of all "the context, which determines the manner of posing questions and the mode of conversation in Platonic dialogues, and which Neoplatonic systematizing tends to obscure only too easily" (30/18). Rather than concern "two spheres of knowledge which belong to different levels," that is, the "practical" and the "theoretical," Klein maintains that, for Plato, "the opposition of arithmetic and logistic . . . concerns a knowing which we first acquire in our intercourse with objects of daily life and in which we can thereafter arrive at a certain expertise." For Klein, this knowing is characterized first of all as a "purely 'practical' knowing, which we must acquire and use with a view to the necessities of life" (33/21). This practical knowing, in turn, presents an "art" with two aspects. One involves rendering determinate the exact amount of the definite multitudes of things with which we have to do, that is, the art of correct counting. The ability to count things "presupposes a certain familiarity with definite amounts in general, especially in the case of larger multitudes." The "art of definite amounts" (30/18) or arithmetic therefore involves the capacity to "know and distinguish the single definite amounts," that is, to "'distinguish the one and the two and the three'" (*Republic* VII, 522 C). The other aspect of this art is that of "*multiplying or dividing these multitudes*" (*GMTOA*, 30–31/18), which by approaching "the definite amount enumerated of some given things" (31/18) "with other 'definite amounts,'" signals that "we are not satisfied" with the initial amount in question. This is the case "whether we wish to separate off a 'third' part of the respective multitude or wish to produce a multitude which amounts to 'four' times the given one" (31/19). To "impose on multitudes . . . such multiplications and divisions, or, more generally, in all *calculations* . . . we must *know beforehand* how the different definite amounts *are related to one another* and how they are constituted *in themselves*." The "'art of calculation' – 'logistic'" therefore "concerns the behavior of definite amounts toward one another, i.e., their mutual relations, . . . which first enables us to *relate* definite amounts, i.e., to calculate with them."

Klein finds support for this view of arithmetic and calculation as aspects of a single practical knowing in the fact that "in Plato's language 'counting' and 'calculating' frequently occur together: *Republic* VII, 522 E: λογίζεσθαί τε καὶ ἀριθμεῖν; 525 A: λογιστική τε καὶ αριθμητική; 522 C: ἀριθμόν τε καὶ λογισμόν, which is here intended as a summary of the realm to which the ability to distinguish 'one' and 'two' and 'three' . . . pertains" (*GMTOA*, 32/20–21). Moreover, "In the *Laws* (817 E), 'calculations' (λογισμοί) and 'what concerns definite amounts' (τὰ περὶ ἀριθμούς) prove to be expressly designated as 'one learning matter' (ἓν μάθημα)" (*GMTOA*, 32/21). For Klein, then, "from the kind of knowing which is conveyed in 'arithmetic' and 'lo-

gistic' we may infer the fact of their difference as well as that of their unity" (33/21). On the one hand, both arithmetic and logistic are rooted in the art of correct counting, of which Klein reports "Plato says explicitly in the *Theaetetus* (198 A–B): 'through this art, I think, one is oneself master of the sciences of definite amounts [ἐπιστήμας τῶν ἀριθμῶν] and is able as a teacher to pass them on to another'" (*GMTOA*, 31/19). Thus, their "unity ultimately has its roots in the object of which both treat . . . 'definite amounts of definite objects' and . . . 'definite amounts' in general" (33/21). On the other hand, their difference has its basis in the fact that logistic, as the art of calculation, involves "'operating' with definite amounts, i.e., [it is] an art teaching the procedures to be applied in multiplication, division, taking of roots, and in the solution of verbal problems" (31/19).

According to Klein, however, because both arithmetic and logistic have their source in arts that represent two aspects of one learning matter, and because the latter is rooted in counting, "it follows that 'arithmetical' and 'logistical' knowing are hard to distinguish on this primary level" (32/20). (We shall see shortly that for Klein they are difficult to distinguish at the theoretical level as well.) Thus, "All meaningful operations" (31/19), that is, calculation, including "certain 'mechanical' rules of reckoning" applied to "definite amounts presuppose knowing about the relations which connect the single definite amounts"—that is, they presuppose knowing the art of correct counting. Likewise, "the simplest relation between the parts of a multitude and the multitude itself (as their sum) is given as soon as they are counted or added up, i.e., by a counting process which extends over all members of all partial multitudes" (32/20). Consequently, the art of logistic is inseparable from counting, just as the art of arithmetic is inseparable from the relating of definite amounts. Klein elaborates on this state of affairs as follows:

> Addition and also subtraction are only an extension of counting. Furthermore, all the remaining relations between definite amounts, on which the complicated operations of reckoning are based, may also, in the last analysis, be reduced to that ordering of definite amounts which is ascertained by counting. On the other hand, counting itself already presupposes a continual relating and distinguishing of the counted things as well as of their definite amounts.

Notwithstanding this close connection between arithmetic and logistic, Klein maintains that for Plato

> the fundamental significance of counting necessitates the emphasis on and the relative isolation of ἀριθημητική as such. For Plato, that ability proper to man, *to be able to count*—an ability to which corresponds the *countableness*

> of things in the world—is a fact fundamental beyond all special problems, a fact which determines the systematic aspect of his teaching. (32/21)[48]

Now for Klein "the significance of the fact that in the definitions [of arithmetic and logistic in the *Gorgias* and the *Charmides*] the 'even' and the 'odd' are mentioned, but not the definite amount" (33/21) is explained by the fact that "a 'scientific' definition properly so-called is here applied" (35/24) to each art and thus "to a subject which ordinarily serves only practical purposes." Hence, it is "in response to *this*" (33/21), that is, "the case that an attempt is here being made to bring to bear an alien technique of definition on perfectly familiar facts in order to formulate them more precisely," that explains why, according to Klein, for Plato "the 'odd' and the 'even' are regarded as characteristic of arithmetic and logistic." And this means for Klein that the matter at issue here can only be elucidated "through an analysis of ἀριθμός." However, before pursuing such an analysis, in which the significance of the 'even' and the 'odd'—as the εἴδη of the Greek ἀριθμός-concept—is reactivated by Klein, he first considers "Plato's remarks which bear on the development of 'arithmetical' and 'logistical' knowledge." That is, he considers the distinction Plato draws between practical and theoretical counterparts of each.

According to Klein, "in the *Republic* and the *Philebus* Plato contrasts 'practical' arithmetic *and* 'practical' logistic with their 'theoretical' counterparts" (33/22). Once the "practical aims" (33/21) that characterize the "purely 'practical' knowing" of arithmetic and logistic are "abandoned," Klein writes that, for Plato, "these studies acquire an altogether different standing" (32/21–22), one in which "attention is freed to turn to certain peculiarities of that knowing which is now raised to authentic knowing, to ἐπιστήμη" (32/22). When the practical implications of both arithmetic and logistic are set aside and their "presuppositions are pursued for their own sake" (34/23) by their theoretical counterparts, what distinguishes the former from the latter "is the kind of multitude with which each deals." Practical arithmetic and practical logistic are both "concerned with multitudes of 'unequal' objects—

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48. See Husserl's account of the fundamental importance of 'collective combination' (which, as we have seen, arises in his analyses on the basis of counting) for cognition per se in *Philosophy of Arithmetic*. In this connection he writes: "Collective combination plays a highly significant role in our mental life as a whole. Every complex phenomenon which presupposes parts that are separately and specifically noticed, every higher mental and emotional activity, requires, in order to be able to arise at all, collective combinations of partial phenomena. There could never even be a presentation of one of the more simple relations (e.g., identity, similarity, etc.) if a unitary interest and, simultaneously, an act of noticing did not throw the terms of the relation into relief together and unifiedly grasp them. This psychic relation is, thus, an indispensable psychological precondition of every relation and combination whatsoever" (*PA*, 75).

and obviously *all* objects of sense are such." That is, they are concerned with "'units [μονάδας] which are in some way unequal, such as two armed camps, two head of cattle and two of the smallest or two of the largest of all things'" (32–33/22; *Philebus* 56 D). The theoretical counterparts of these practical disciplines, however, are concerned "with multitudes of wholly homogeneous units, namely precisely those which cannot occur in the realm of objects of sense" (*GMTOA*, 34/22–23). These latter can be pursued only insofar as the bond with the senses that characterizes practical arithmetic and logistic is relinquished, which permits there to be "'posited a unit such that not one single unit of those in a myriad differed from any other'" (34/22; *Philebus* 56 E). Thus, in place of the concern with definite amounts of definite objects that "'have visible or tangible bodies'" (*GMTOA*, 35/23), theoretical arithmetic and logistic treat definite amounts of definite objects that "'admit only of being thought'" (35/24; *Republic* 526 A). This quality "has its basis in the 'purity' of the units, of which 'each and every one is identical and not in the least different and has no parts within itself.'"

On Klein's view, when the definitions of arithmetic and logistic provided in the *Gorgias* and the *Charmides* are considered in conjunction with Plato's remarks on the difference between their practical and theoretical characteristics, it is apparent that "their meaning, although independent of the practical or theoretical character of the two disciplines, can be *understood* only from the *theoretical* side" (*GMTOA*, 35/24). It is this fact that explains for Klein their "strangely elaborate formulation." Theoretical logistic, then, would raise "to an explicit knowledge that knowledge of relations among definite amounts of definite objects which, albeit implicitly, precedes, and indeed must precede, all calculation" (34/23). However, because Plato's remarks calling for a theoretical logistic do not explicitly state exactly what is entailed in the theoretical counterpart to practical logistic, Klein has to "try to imagine what may have been Plato's notion of theoretical logistic" (35/24). It is thus his "conjecture" (44/36) that in the first place "theoretical logistic ought to encompass the knowing about all those *relations*, i.e., ratios (λόγοι) among 'pure' units, on which the success of any calculation depends" (35–36/24). By contrast, theoretical arithmetic "would be reserved for the knowing about these 'pure' *definite amounts themselves*."

In order to corroborate his conjecture about theoretical logistic, especially in its relation and contrast with theoretical arithmetic, Klein turns to "the later mathematical textbooks of the Neoplatonists and the Neophythagoreans" (36/24–25). For "in these textbooks scientific material going back to Platonic and pre-Platonic times was undoubtedly absorbed" (36/25), even if "its arrangement, its nomenclature, and the general man-

ner of presentation may well have undergone great changes." Thus, we will now turn to Klein's account of the role of the theory of proportions in representative texts of Nicomachus, Theon, and Domninus.

§ 57. Tensions and Issues Surrounding the Role of the Theory of Proportions in Nicomachus, Theon, and Domninus

Klein's account of the mathematical textbooks of Nicomachus (first century AD), Theon of Smyrna (also first century AD), and Domninus (fifth century AD) uncovers "a conspicuous relationship between the traditional theory of relations [in their texts] and the logistic of which Plato speaks" (44/35). Not only is "the connection between the 'material' of definite amounts of definite objects and the realm of αἰσθητά . . . still unanswered" in these texts (44/35–36), but likewise unanswered is "the real point of the distinction" (44/36) between their εἶδος and the ὕλη, which "is directly connected with the former" distinction. That is, the distinction at issue in the difference between the εἶδος and ὕλη of ἀριθμοί directly hinges, for Klein, on the answer to the question of the status of the connection between the ὕλη of these ἀριθμοί and the realm of sensible things. He therefore sees the unresolved relationship between the ὕλη of ἀριθμοί and the realm of αἰσθητά as something that "ultimately refer[s] back to the old Platonic oppositions of 'theoretical' to 'practical' studies" and to "the difficulties inherent in the whole conception of theoretical logistic" (45/36).

Klein finds that in Nicomachus[49] and Theon there is "a certain tension between the *factical extension* of arithmetical knowledge and the *conception* of arithmetic as a science which should be confined to the study of definite amounts taken by themselves and not in relation to one another" (39/30). In Nicomachus, this is manifest in the fact that a large portion of his book on arithmetic treats the part of quantity (τὸ ποσόν) conceived of "'as having *some sort of relation to another* [πρὸς ἄλλο] and capable of being thought of only in its relations to another [πρὸς ἕτερον]'" (36/26; Nicomachus, I, 3 – Hoche, 5, 13 ff.), despite the fact that he explicitly assigns arithmetic (and therefore, the ostensible subject of his book) to the study of the part of quantity that is conceived of "'*by itself* [καθ᾽ αὑτό], namely that which has no other relation to another [πρὸς ἕτερον].'" Likewise, in Theon[50] this tension is expressed in

49. Nicomachus of Gerasa, *Introductionis arithmeticae libri II*, ed. Richard Hoche (Leipzig: Teubner, 1866); henceforth cited as 'Nicomachus'.

50. Theon of Smyrna, *Expositio rerum mathematicarum ad legendum Platonem utlium*, ed. Eduard Hiller (Leipzig: Teubner, 1878); henceforth cited as 'Theon'.

the "incoherence" (*GMTOA*, 39/31) of his presentation of the subject matter and relation between proportions in "'arithmetical music [ἡ ἐν ἀριθμοῖς μουσική]'"[51] (38/28) and in "*arithmetic as such*" (38/29), an incoherence that emerges despite the fact that unlike Nicomachus, he "is more consistent insofar as he is careful to keep the realms [i.e., καθ᾽ αὑτό and πρὸς ἄλλο] of the ποσόν strictly separate" (38/28).

According to Klein the realm of the πρὸς ἄλλο in Nicomachus's text is, on the one hand, determined on the basis of "the traditional 'Pythagorean' division of the mathematical sciences into arithmetic, geometry, music, and astronomy" (36/26). As a result, for Nicomachus the study of quantity πρὸς ἄλλο concerns music and natural science, subjects that are related to the realm of αἰσθητά. On the other hand, its study for him also involves the investigation of 1) arithmetical ratios and 2) the theory of proportions[52] and means. While arithmetical ratios have "their most immediate application in the field of music" (37/27), it is also important to note "that they belong to the theoretical foundation of all calculations." Moreover, Nicomachus stresses that proportions and means are useful "for natural science (by which the cosmology of the Platonic *Timaeus* is meant); for music, astronomy, geometry, and last but not least, for understanding the work of the ancients" (37/28). However, despite Nicomachus's separation of arithmetic (as the study of quantity καθ᾽ αὑτο) from the study of the realm of the πρὸς ἄλλο, "the study of proportions and means, which belongs to the realm of the πρός τι [that which is in relation to something], forms the conclusion of" his "'Introduction to *Arithmetic*' (cf. [Nicomachus,] 199, 19–120, 2 and also 64, 21 ff.)" (*GMTOA*, 38/28).

Klein reports that Theon begins by making a distinction between "'the theory of *mere* definite amounts (ἡ περὶ ψίλους ἀριθμοὺς θεωρία) . . . , i.e., arithmetic, and 'arithmetical music' (ἡ ἐν ἀριθμοῖς μουσική) (Theon, 17, 12 f.)" (*GMTOA*, 38/28). Klein maintains that this distinction is "in theory identical" to Nicomachus's distinction between, respectively, καθ᾽ αὑτό and πρὸς ἄλλο. However, in his discussion of the realm of the musical part of arithmetic, its study of ratios (λόγοι), proportions (ἀναλογίαι), and means (μεσότητες) "is closely connected with the presentation of the theory of musical intervals, but at the same time also *clearly contrasted with it*" (38/29). Thus,

51. As we shall soon see below, this corresponds to what Nicomachus designates as the study of πρὸς ἄλλο.

52. Knorr provides the following account of 'ratio' and 'proportion' in Greek mathematics: "A 'ratio' (λόγος) is a comparison of homogeneous quantities (i.e., numbers or magnitudes) in respect of size. A 'proportion' (ἀναλογία) is an equality [*sic*] of two ratios" (Knorr, 15). What Knorr terms 'equality', Klein terms 'sameness'.

the theory of "arithmetical music" is "divided into two parts . . . : the first is concerned with the 'harmony perceptible by sense and made by instruments' (ἡ ἐν ὀργάνοις αἰσθητὴ ἁρμονία) (Theon, 46 f.), the second, which really first provides the theoretical foundation of the first, with 'the harmony apprehended by thought and made by definite amounts' (ἡ ἐν ἀριθμοῖς νοητὴ ἁρμονία)." Finally, "proportions treated by *arithmetic as such* are, in turn, distinguished from the proportions of 'arithmetical' harmonics, because the latter are only special cases of the former." The former concern "the different kinds of definite amounts *taken by themselves*, that is, of the even and the odd together with their subspecies" (38/28–29), the "arithmetical" treatment (in accord with the καθ' αὑτό) of which nevertheless involves "some incidental reference to the relations between the single definite amounts and between their kinds" (38/29). Klein notes that Theon—"'*according to the arithmetical tradition* (κατὰ τὴν ἀριθμητικὴν παράδοσιν)' (Theon, 76, 1 ff.)"—makes this distinction between the proportions whose harmony is apprehended noetically and comprised of definite amounts and those that, apprehended purely arithmetically, provide the basis, as it were, for this arithmetic harmony without themselves presenting a ἁρμονία.

Yet despite the distinctions Theon draws between the theory of proportions treated by, respectively, "arithmetic as such," "the harmony apprehended by thought and made by definite amounts," and the "harmony perceptible by sense and made by instruments," Klein finds that he "'yokes together' (συνέξευκται) 'arithmetical music' (Theon, 16 f.)" (*GMTOA*, 38/30)—the kind "made of definite amounts and grasped by thought"—"and arithmetic." Theon does this "basing himself on Plato, *Republic* VII, 530 D–531 C, but also 'for the sake of the convenience of our study.'" Moreover, "far from neglecting the theory of 'aisthetic' harmony . . . he himself observes . . . 'noetic harmony is more easily understood {starting} from sensible harmony'" (*GMTOA*, 38–39/30; Theon, 47, 5–6), even though "arithmetical harmonics is . . . entirely eclipsed by the purely arithmetical theory of relations" (*GMTOA*, 40/31).

Now for Klein, owing to "the rigor of Greek scientific methodology, we cannot ascribe all these inconsistencies simply to the inadequacies of Theon or Nicomachus." Rather, he explains the inconsistencies as stemming from the influence of two incompatible traditions. On the one hand, there is the Pythagorean tradition whose "theory of proportions owes its first formulation" to the "investigation of musical intervals," which they follow in assigning this theory to musical theory. This explains for Klein the inconsistency manifest in Nicomachus's devoting a large part of his *Introductio Arithmetica* to the realm of quantity (τὸ ποσόν), which is "capable of being thought only

in its relations to another," that is, to the realm of πρὸς ἄλλο, despite his initial systematic separation of the study proper to this part of quantity from the part studied by itself (καθ᾽ αὑτό). On the other hand, "there is the influence of another, classical tradition which goes back to Euclid and Aristoxenus," within which "the theory of proportions, insofar as it is concerned only with the relation between definite amounts, has . . . no separate existence," in the sense of presenting a theory of relations that is able to demarcate arithmetic and therefore pure relations from all relations in harmonic proportions. Consequently, in this tradition the theory of arithmetically *pure* proportions "cannot be naturally isolated from the arithmetical theory of proportions"—that is, from arithmetical harmonics—"whose very foundation it provides," and it is therefore not "passed on" according to any kind of systematic distinction but "according to the arithmetical tradition." Klein's point here is that the classical tradition does not address the distinction between the proportions characteristic of the arithmetical theory of proportions (which, purportedly in accord with pure arithmetic, treats proportions exclusively in terms of the relations between definite amounts) and the proportions involved in the study of arithmetical music (which purportedly treats the relations that compose the noetic harmony of definite amounts considered by themselves). Therefore, although the tradition, and thus Theon, maintain that both kinds of proportion are distinct from the proportions that originate in the harmonies whose source is the realm of αἰσθητά, the nature of the distinction between proportions that are rooted in definite amounts themselves and those composed of the harmonious relations among definite amounts is nowhere explained. That is, what remains unaccounted for is the noetic distinction between proportions that are arithmetically "pure," because they do not have a basis in the noetically apprehended harmonies composed of the relations between definite amounts, and those that are yielded solely by such definite amounts themselves, which, as such, are held to provide the foundation for the proportions that compose noetic harmony.

According to Klein, "Domninus[53] is successful in avoiding all the inconsistencies arising from the Neoplatonic attempts at systemization [considered so far] by assigning the study of both the *kinds* of definite amounts and their *relations* to arithmetic from the very beginning" (43/34). As a result, "we do indeed have here, as in Euclid . . . one single subject matter (cf. the expression 'one learning matter' – ἓν μάθημα – in Plato, *Laws* 817 E),

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53. Domninus of Larissa, "[Introductory Manual of Arithmetic]," in Jean François Boissonade, ed., *Anecdota Graeca* (Paris, 1832), IV: 413–29; henceforth cited as 'Domninus'.

which can be ordered from two main points of view, namely according to the distinction between καθ᾽ αὑτό and πρὸς ἄλλο." Domninus accomplishes this by introducing two sets of distinctions, "in conscious contrast to Nicomachus and Theon" (41/32). One set concerns Theon's distinction between καθ᾽ αὑτό and πρὸς ἄλλο, while the other set concerns the distinction "'according to kind' (κατ᾽ εἶδος) and 'according to multitude' (κατὰ τὸ πλῆθος)." The latter distinction, which is found in Olympiodorus and the *Gorgias* scholium, involves studying, on the one hand, any ἀριθμός "'with a view to the *kind* to which it belongs when taken by itself, either the even or the odd,' the even-times-odd or the odd-times-even, etc. (Domninus, 416, 10 ff.)." On the other hand, it involves the study of any ἀριθμός "'with a view to the underlying *multitude* of units in it' . . . and in this way also it can be 'studied by itself.'" Thus, Domninus understands this multitude to be "'that which underlies and is, as it were, the material [ὕλη] of definite amounts.'"

Once "the definite amounts have been studied *by themselves* according to both their εἶδος and their ὕλη, 'it is necessary to investigate their association with one another' (Domninus, 416, 21 ff.)" (*GMTOA*, 41/33). The latter involves making two types of distinction. One type concerns distinguishing according to kind (κατ᾽ εἶδος) definite amounts that "are prime to one another and definite amounts which have a common measure." The other type involves making distinctions according to material (ὕλη), that is, "'according to the multitude of units in them' (Domninus, 417, 10–12)." At issue here for Klein is determining either that they are "equal, that is to say, they contain the same multitude of units," or that they are related by certain relations involved in counting, which according to "Domninus' own innovation" (*GMTOA*, 44/33) involve ten relations (δέκα σχέσις). Moreover, "it is possible to study definite amounts 'by themselves and in relations to one another at the same time' (Domninus, 417–422)." For example, "Two definite amounts are either prime in themselves and to one another, or composite in themselves but prime to one another, etc." (422). Finally, the "theory of proportions and means represents then a more comprehensive study of their mutual relations according to their material, a study 'concerned with the multitude underlying them'" (423, 11) that is "more comprehensive insofar as in proportions always more than two definite amounts are related to one another" (423–25).

Thus, by assigning to arithmetic from the start the study of both the kinds of definite amounts and their relations, Klein maintains, Domninus is able to avoid the systematic inconsistencies of Nicomachus and Theon. Indeed, "Domninus' second and related set of distinctions affords a better understanding of Olympiodorus and the scholiast of the *Gorgias*" (*GMTOA*,

43/34), both of whom "connect Platonic 'logistic' with the 'material' of definite amounts." For Klein, the same reasons[54] that

> prevent Nicomachus and Theon from identifying the "pure" theory of proportions with theoretical logistic lead both Olympiodorus and the *Georgias* scholiast to take it for granted that logistic is to be understood only as a *practical art of calculation* (i.e., as a set of instructions for the computation of the multitudes in question), yet both understand these quantities, independently of whether they are objects of sense or "pure" units, as the "material" of definite amounts. For only this *material* can in Domninus' sense be subject to those relations which make calculation possible to begin with. (43/34–35)

Klein asserts, then, that Domninus's distinction between κατ᾽ εἶδος and κατὰ τὸ πλῆθος "coincides, on the whole, with that of καθ᾽ αὑτό and πρὸς ἄλλο as far as their area of application is concerned" (43/35). This is the case because in opposition to "the traditional theory of the *kinds* of definite amounts, which corresponds to the study 'καθ᾽ αὑτό *and* κατ᾽ εἶδος,' the introduction of the theory of decadic counting as a study 'καθ' αὑτο and κατὰ τὸ πλῆθος' clearly represents Domninus' own innovation." This innovation treats relationships between the multitudes that emerge in counting as belonging to definite amounts themselves, and thus allows for both 1) the extension of the systematic designation of "definite amounts in themselves" to include their multitudes *and* 2) the differentiation to be made between these multitudes and the systematic designation of definite amounts having a relation to another. That is to say, Domninus considers the relations of definite amounts involved in counting to be distinct from the relations between definite amounts that compose proportions. The innovation responsible for this distinction, on Klein's view, can only be understood as having its source in Domninus's interest in "systematizing," as his reference to "the λογική or λογιστικὴ θεωρία shows" (43/35). By contrast, Klein finds that "the subject matter of the study of πρὸς ἄλλο and κατ' εἶδος, namely the definite amounts which are or are not prime to one another, is very negligible in comparison with the subject matter of the study 'πρὸς ἄλλο and κατὰ τὸ πλῆθος,' which contains all of the traditional theory of relations as well as that of proportions and means." Finally, notwithstanding Domninus's attempt at systemization, for the most part "actually *only* the theory proportions is subject to the approach πρὸς ἄλλο and *at the same time* to κατὰ τὸ πλῆθος."

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54. These reasons involve the impossibility of partitioning the intelligible units that Platonic mathematics posits as the material proper to the theoretical ἀριθμός-concept, even though such partitioning is something that cannot be avoided in exact calculation. Thus, the pure theory of proportions is displaced from logistic because the latter cannot be given a theoretical foundation so long as the units of its ὕλη are posited as indivisible (as demanded by the Platonic thesis). See § 56 above and § 59 below.

Klein compares this last state of affairs with Olympiodorus's and the *Gorgias* scholiast's view that "arithmetic treats of the kinds, while logistic considers the material of definite amounts"; and it is here that he locates "a conspicuous relationship between the traditional theory of relations [represented in the texts of the late Neoplatonists and Pythagoreans] and the logistic of which Plato speaks." This relationship points to the situation that "the question concerning the connection between the 'material of definite amounts and the realm of the αἰσθητά . . . still remains unanswered," as does the related question of "what might be the real point of the distinction κατ᾽ εἶδος – κατὰ τὸ πλῆθος." Indeed, for Klein this distinction "ultimately refers back to the old Platonic opposition of 'theoretical' to 'practical' studies" (44/36), a distinction that has no basis in Plato since the relevant texts, as we have seen, speak of practical *and* theoretical variants of *both* arithmetic and logistic. Notwithstanding Klein's "conjecture" that Plato understood theoretical logistic as "primarily the study of the relations of definite amounts," the fact that "for Plato, too, logistic is originally related to the possibility of calculating" leads Klein to consider whether "in analogy to geometrical usage, it may have contained, besides its own 'theorems,' 'problems' and 'porisms'[55] . . . intended to determine special relations between definite amounts serving as the counterparts in the realm of 'pure' units of the computational problems proper to practical logistic." This leads Klein to a consideration of theoretical logistic and the problem of fractions, a problem that, as we indicated above, has its basis in the ontological presuppositions of Plato's philosophy that inform his theoretical concept of ἀριθμός. It is to these problems that we now turn.

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55. According to Sir Thomas Heath, "the only source of information about the nature and contents of *Porisms* is Pappus" (*A History of Greek Mathematics*, 2 vols. [New York: Dover, 1981], I: 431). He goes on to quote Pappus's *Treasury of Analysis* on the difference between a theorem, problem, and porism: "'For they [the ancients] said that a theorem is that which is proposed with a view to the demonstration of the thing proposed, a problem that which is thrown out with a view to the construction of the very thing proposed, and a porism that which is proposed with a view to the producing of the very thing proposed'" (431). Regarding the distinction between 'constructing' and 'producing', Klein reports that "in Plato's time a debate arose over the question whether 'problems,' since they deal with the 'construction' of definite figures and the 'computation' of specific definite amounts, in short with the *genesis* of mathematical structures, can indeed belong to the realm of those sciences which are turned toward what *always* is (cf. Plato, *Republic* 527 A f.). To be exact, they should be called 'porisms'" (*GMTOA*, 50/231–32 n. 46).

Chapter Sixteen

Theoretical Logistic and the Problem of Fractions

§ 58. The Ambiguous Relationship between Logistic and Arithmetic in Neoplatonic Mathematics and in Plato

Thus far Klein's desedimentation of the Neoplatonic background of Diophantus's *Arithmetic* has disclosed in the Neoplatonic texts an attempt to maintain a systematic distinction between arithmetic, understood as the pure theory of definite amounts in themselves, and the theory of proportions, understood as the theory of the relations that definite amounts have to one another. That is, he has disclosed their attempt to maintain a theoretical distinction between the treatment of ἀριθμοί καθ᾽ αὑτό and that of πρὸς ἄλλο. Moreover, he has shown that this attempt is beset by obscurities and inconsistencies that are rooted in an arithmetical theory that is sedimented in their text, which has its source in Plato and has been passed on according to tradition. This theory aims to investigate the pure relations proper to definite amounts, an investigation that is "intended to stand beside the theory of definite amounts as such, i.e., of their different kinds" (47/39).

Klein also contends that the Neoplatonic texts lack clarity on the precise nature of the underlying material (ὕλη) of ἀριθμοί, insofar as the changing status of this material is held to be responsible for the distinction between 1) the treatment of definite amounts both in themselves *and* according to their kind (κατ᾽ εἶδος) and 2) their treatment in relation to one another. Their accounts vacillate between locating this changing constituent, on the one hand, in the realm of αἰσθητά, and, on the other hand, in the pure ὕλη of the multitude (πλῆθος) proper to the ἀριθμοί themselves. Klein shows that this lack of clarity about the nature of the material of ἀριθμοί is the source of the inconsistency of the Neoplatonic attempts to determine the precise status of λογιστική. Thus, on the one hand, they want to assign λογιστική—in accord with their systematizing inclinations—to the pure the-

ory of proportions, while, on the other hand, as a consequence of their vacillation regarding whether the realm proper to the πρὸς ἄλλο is constituted noetically or aisthetically, they sometimes opt for the latter and characterize it as the *practical* application of pure arithmetic.

The question for Klein, then, is as follows:

> What prevents later writers from interpreting the arithmetical theory of relations, i.e., proportions, as the theoretical logistic? Or, in other words: How did it happen that the Platonic double distinction between theoretical and practical arithmetic, on the one hand, and theoretical and practical logistic, on the other, was reduced to the single distinction between theoretical arithmetic and practical logistic? (45/37)

According to Klein, "Plato's special demand for a theoretical logistic corresponds to the understanding that within the *unified* framework of the purely noetic sciences[56] there should also be a science addressed to the pure relations of definite amounts as such, which would correspond to the common art of calculation and provide its foundation" (46/38). Such a science, which is useful in "'the search for the noble and the good' [*Republic* 531 C], inquires into the *presuppositions* of common calculation and also of harmony, and ignores the manner in which these sciences might be pursued in other contexts." Klein holds, however, that such a division in these subject matters is difficult to maintain, "even if this demand is taken seriously *within* the sciences." Not only, as we have seen, do the Neoplatonic mathematicians encounter difficulties in maintaining the distinction between arithmetic and logistic at both the practical (§ 57) and theoretical (§§ 55, 58) levels, but "A further indication of these difficulties may be seen in the fact that Plato (*Statesman* 259 E) refers to the knowledge of 'the difference among definite amounts' to logistic, although this might as well be said to be the business of arithmetic (cf. 258 D, also *Republic* 587 D)" (*GMTOA*, 47/39).

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56. By 'purely noetic sciences' Klein has in mind not only Plato's demand "for a 'pure' astronomy which ascends from the observed processes of the visible heavens to an understanding of the invisible spheres" (*GMTOA*, 45/37), but also his postulation, "in contrast to the tetrad of 'Pythagorean' disciplines, [of] a science similarly organized but completely freed of sense perception, a science whose ultimate object is the one invisible and inaudible 'cosmic' order on which our world of sense is founded" (46/38). This postulate is evident for Klein in Plato's criticism of "the theory of harmony of the 'Pythagoreans,'" where he "says (*Republic* 531 C) in reproof that though they search for the definite amounts which are responsible for audible consonances, they fail to attempt to determine without reference to anything audible 'which definite amounts are consonant, which are not, and why.'" Evidence that to "this one order corresponds the close inner relationship of all sciences based on measuring and counting" Klein finds in the *Theaetetus* (145 A), where "Theodorous is called γεομετρικός as well as ἀστρονομικός, λογιστικός, and μουσικός."

§ 59. The Obstacle Presented by Fractions to Plato's Demand for a Theoretical Logistic

It is Klein's thesis, however, that fractions are what really stand in the way of establishing a theoretical logistic. If its connection with calculation is attended to, "the crucial obstacle to theoretical logistic . . . [is] the fractionalization of the unit of calculation." Theoretical logistic, as we have seen (§ 56), is "concerned with units which can be grasped only in thought, which are identical to one another and which, above all, *defy partition*." Accordingly, Plato says that "should someone attempt to partition such a unit, . . . all expert mathematicians 'would laugh at him and would not allow it, but whenever you were turning it into small change, they would multiply it, taking care lest the one should ever appear not as one, but as many parts' (*Republic* 525 E; cf. also *Parmenides* 143 A and *Sophist* 245 A)." The Neoplatonists also stress this state of affairs. Theon, for example, articulates the "paradox inherent in the partition of the unit" in terms of the fact that while every definite amount can be divided and thus diminished into smaller definite amounts, "'the one, when it is divided within the realm of sense, is on the one hand diminished *as a body* and is, once the cutting-up has taken place, divided into parts which are smaller than it, while, on the other hand, *as a definite amount*, it is augmented'" (*GMTOA*, 47/40; Theon, 18, 18–21). Thus, "After partitioning, every part assumes the property of being one, i.e., there now exit several 'units'—the one has multiplied itself!"

For Klein, then, because every process of calculation or counting deals with many units, if, in the process of calculation, "we are forced to partition one of these units, then what we do is precisely to substitute something else for the indivisible unit which is subject to partitioning, while the unit itself *is not partitioned but only further multiplied*" (*GMTOA*, 48/40). Again, this holds because "'the one as one is impartible and indivisible' (Theon, 18, 15), 'and even if we multiply without limit it {and therefore every single unit} yet remains—a unit' (Theon, 19, 10 f.)." Thus, for Klein there

> arises the necessity of making a strict distinction between the *one object of sense* which is subject to counting and calculation and the *one as such*, i.e., of keeping each "one" thing strictly separate from all "ones." Each single thing can be endlessly partitioned because of its bodily nature as an object of sense. The unit which can only be grasped in thought is, on the other hand, indivisible simply, precisely in virtue of its purely noetic character: "So that the unit, since it is noetic, is indivisible while the one, inasmuch as it is sensible, can be cut without limit." (*GMTOA*, 48/40–41; Theon, 20, 2–4)

Consequently, "this property of the noetic unit precludes calculation [i.e., its division into fractional parts] with it" (*GMTOA*, 48/41). Thus, unlike the

case in arithmetic where there is "'a least simply,'"[57] in logistic "'the least [is taken] in relation to some class ⟨of objects of sense⟩. For the one *man* becomes . . . the measure of the multitude as the unit is for definite amounts {in arithmetic}.'"[58] The unit of calculation therefore functions as a "'substrate'" and is only indivisible as such. For instance, in calculating with apples, one apple "is that 'least' element which is not further reducible since all computational operations are referred to it as to their ultimate basis. The series of *whole* apples *one* by *one* here furnishes that 'homogeneous' medium in which counting and calculating takes place" (49/41–42). If, in the course of a calculation, it becomes necessary to partition apples,

> the basis of the calculation, namely the one whole apple, still remains untouched. For once an apple has been divided into fractional parts, it loses its significance as that which underlies the counting and is the fundamental element of all calculation, and is instead understood as a *mere thing* such as has the property unlimited divisibility. As the fundamental element of calculation the function of *one* apple is precisely to represent the unit "within the material" (ἐν τῇ ὕλῃ). (49/42)

Therefore, fractions are parts not of pure definite amounts but of the bodily thing that underlies counting, a thing, moreover, that because of its nature can be divided without limit. In the realm of pure definite amounts, "the *unit itself* provides the last limit of all possible partitions: all partitioning 'will stop at one' (καταλήξει εἰς ἕν) (Theon, 18, 11, 13 f.)."

According to Klein, because "in the majority of cases the necessity for introducing fractional parts of the unit of calculation does arise in the course of calculation, there emerges a remarkable maladjustment between the material within which such calculations are performed and that other 'material' of 'pure' definite amounts whose noetic character is expressed precisely in the indivisibility of the units" (*GMTOA*, 50/42–43). As a result, an exact calculation that requires the partitioning of the unit of calculation cannot be carried out within the realm of "pure" ἀριθμοί. Within the Platonic tradition, this state of affairs results in all computational problems being excluded from the domain of the pure sciences. Moreover, "beyond this difficulty arises the question—and this is the crucial question to be asked about theoretic logistic—whether that which distinguishes *exact* calculation, namely operation with fractional parts of the unit of calculation, can really be sufficiently grounded in the science of the possible relations of definite amounts, i.e., in the 'pure' theory of relations, alone" (50/43).

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57. Proclus, 40, 5 ff.
58. Ibid.

Klein's answer is that it cannot. Specifically, while in the arithmetical books of Euclid, "'arithmetic' and 'logistic' matter can hardly be separated, . . . the 'logistic' constituent undoubtedly predominates and is here understood precisely as 'arithmetic'; it is obviously this fact which permits the later 'arithmetical tradition' (ἀριθμητικὴ παράδοσις) *to include the theory of relations as well*" (51/43–44). However, the relations in Euclid's arithmetic concern the theory of incommensurables, "which amounts to a first presentation of the foundations for any calculation which goes beyond simple addition and subtraction, insofar as in such operations it becomes necessary to decompose single definite amounts into their components (factors), to find the greatest common measure (i.e., divisor) of several definite amounts, to express their ratios in the 'least' terms, etc." (51/44). Because calculations in this manner cannot find or provide a foundation—consistent with the Platonic presupposition of the noetic and hence indivisible units employed in pure arithmetic—for the partitioning of the units themselves that are unavoidable in an exact calculation, "the theory of relations in the 'arithmetical' books of Euclid cannot be understood as the noetic analogue of practical logistic." Consequently, Klein maintains that calculation is relegated to the practical arts and sciences and thereby loses the basis for its claim to be an apodeictic discipline. Moreover, the pure theory of relations is displaced from the foundational function Plato assigns to it vis-à-vis practical logistic, "and comes to be assigned now to arithmetic as the theory of the kinds of definite amounts, now to harmonics as the theory of musical intervals based on ratios of definite amounts." Finally, "the discovery of incommensurable 'irrational' magnitudes causes the 'pure' theory of definite amount relations to appear as nothing but a special case of the general theory of relations and proportions of the fifth book of Euclid." With this, the pure theory of definite amount relations "loses all connection with the art of calculation," and the latter (i.e., logistic) comes to comprise "approximately the subject matter of present-day elementary arithmetic" (52/45). Thus, logistic comes to be regarded as technique, especially as one that is "accomplished 'mechanically' with the aid of the fingers and the abacus" (52/44), for rapid addition, subtraction, multiplication, and division, and it "teaches calculation with fractions and probably also extraction of (square) roots with the aid of geometry, develops systems of counting for large definite amounts, especially with a view to astronomical computations, and, finally, solves problems presented in words" (52/45).

Now this practical characterization of logistic, which places it outside theoretical mathematics, is a consequence of "that special conception of the pure definite amounts and their material which governed the Platonic tradition throughout." Thus, in order to understand the full significance of this

conception of logistic in relation to the foundational problematics that generate it, Klein undertakes the desedimentation of the ἀριθμός-concept itself. His goal thereby is to make more comprehensible the Platonic definitions of arithmetic and logistic, and therewith to reactivate fully the presuppositions that informed the horizon of Diophantus's arithmetic, presuppositions that, as we have seen, need to be rendered manifest if the radicality of the conceptual transformation in Vieta's innovation of a symbolic mathematical discipline is to be grasped as such. Once we have succeeded in presenting the results of Klein's desedimentation and reactivation of these matters, our study will be in a position to advance its thesis that these results present, in effect, the historical background of the issues treated in Husserl's analysis of the philosophy of arithmetic, that is, of his—ultimately misguided—attempt to establish the origin of symbolic numbers on the basis of authentic cardinal numbers and arithmetical operations.

Chapter Seventeen

The Concept of Ἀριθμός

§ 60. The Connection between Neoplatonic Mathematics and Plato's Ontology

Klein's desedimentation of the presuppositions of the Neoplatonic mathematical background of Diophantus's *Arithmetic* shows that they are informed by two interrelated strata of presuppositions. Klein reactivates the first of these, which belong to the Neoplatonic stratum proper, by first articulating it and then tracing it to its roots in Plato's philosophy of mathematics. As we have seen, the Neoplatonic stratum is characterized by the peculiarity that theoretical arithmetic does not deal directly with ἀριθμοί but with their kinds (εἴδη). Likewise, logistic is characterized by the peculiarity that, while the nature of the material (ὕλη) proper to the ἀριθμοί with which it deals is rendered inconsistently, a dominant view nevertheless emerges that this material is sensible and that, therefore, logistic is not a science (ἐπιστήμη) but an art (τέχνη). We have also seen that Klein traces the roots of this stratum to the absence in Plato of any reference either to ἀριθμός or to ἀριθμοί in the definitions of arithmetic and logistic. It is Klein's thesis that the definitions proper to each, as having to do with the εἴδη of the odd and the even, point to the fact that "their formulation [in Plato] presupposes a *theoretical* point of view" (63/59). However, "the rigor of these definitions consists precisely in the fact that they articulate only one of the two characteristics of the ἀριθμός," that is, their kinds, while they "*avoid the indefiniteness which attends the term* 'ἀριθμός' insofar as by itself it does not reveal the sort of definite objects it is a definite amount of, i.e., of *what* the definite amount is meant to be a definite amount of." Consequently, even though these definitions presuppose a theoretical interest, they do not presuppose that the ἀριθμοί themselves are theoretical, that is, that they are ἀριθμοί of "pure" units. Thus, the definitions hold irrespective of whether sensible or noetic material (ὕλη) is understood to underlie counting and calculation and therefore arithmetic and logistic. For Klein, however, owing to

the fact that only sensible "units" "are amenable to the partitioning which exactitude of calculation requires" (64/60), the Neoplatonic mathematicians Olympiodorus and the *Gorgias* scholiast "are forced from the very beginning to regard the 'hylic' monads, i.e., the monads which form the ὕλη of the definite amounts," as sensible.

Klein thus holds that the desedimentation of the Neoplatonic stratum belonging to the presuppositions that form the background of Diophantus's *Arithmetic* leads back to the ontological considerations in Plato's philosophy regarding the theoretical mode of being of ἀριθμός. Specifically, they lead back to Plato's account of indivisibility of the pure monads that compose the multitude in an ἀριθμός that is the object of arithmetical and logistical mathematical knowledge. To bring these considerations to the fore, Klein first desediments the concept of ἀριθμός operative in Greek mathematics and then reactivates Plato's account of its theoretical mode of being.

§ 61. Counting as the Fundamental Phenomenon Determining the Meaning of Ἀριθμός

Klein emphasizes that "The fundamental phenomenon which we should never lose sight of in determining the meaning of ἀριθμός is counting, or more exactly, the *counting-off* [Abzählen], of an arbitrary multitude of things" (53/46). In counting, the differences of the things counted are disregarded such that they "are taken as uniform when counted." Thus, in counting apples, their differences (size, color, shape, etc.) are overlooked, and they are taken simply as *apples*; "or apples and pears are counted as *fruit*, or apples, pears, and plates which are counted as '*objects*.'" In counting, the last word spoken is "the *definite amount*, the ἀριθμός of the things involved." Plato expresses this in the *Theaetetus* (198 C): "'Socr.: We will then understand counting as nothing else than examining how large a determinate amount is in a given case. Theatet.: Just so.'" Thus, for Klein, the Greek ἀριθμός "indicates in each case a *definite amount of definite things*. It proclaims that there are precisely so and so many of these things. It means the *things* insofar as they are present in this definite amount and cannot, at least at first, be separated from the things at all" (*GMTOA*, 53/46).

Because ἀριθμός is always a definite amount of definite things, it "is always and indissolubly related to that of which it is the definite amount" (54/48). Thus, "Plato speaks of definite amounts (*Republic* 525 D) which have 'visible and tangible bodies'" (*GMTOA*, 53/47), an expression that Klein claims "must be taken quite literally." According to him, Aristotle illustrates this when he says in connection with the equality of definite amounts, "'It is

also quite rightly said that the *definite amount* of sheep and dogs is the *same* if each is *equal* to the other, but the *'decad'* is *not the same* {in these cases}, nor are the ten {sheep and dogs} the same {ten things}' (*Physics* Δ 14, 224 a 2 ff.)." Klein also reports that Alexander's commentary on the *Metaphysics* holds that "'every definite amount is a *definite amount of something*'"[59] (*GMTOA*, 54/48), which is why "the ἀριθμός can be included in the category of the πρός τι," namely, that which is in relation to something.

This relational character of ἀριθμός "cannot be adequately described by speaking of 'concrete' or 'specified' as opposed to 'abstract' or 'unspecified' numbers" (55/48). That is because "even a 'pure' definite amount, i.e., a definite amount of 'pure' units, is no less 'concrete' or 'specified' than a definite amount of apples." Consequently, Klein holds that the ἀριθμός is distinguished by a "twofold determinateness: it is, first of all, a definite amount of objects *determined in such and such a way*, and it, secondly, indicates that there are *just so and so many* of these objects." It is for this reason that only the general concept of ἀριθμός may be characterized as "'the comprehensive unity of a multitude,'" but not "the definite amounts intended by the *content* of that concept" (55/48–49). Klein uses the scholastic term *actus exercitus*[60] to characterize "the process of counting" (55/49) that yields ἀριθμοί, because in this process "it is only *the multitude of the counted things* which are in view." As a result, "Only that can be 'counted' which is *not one*, which is before us in a certain definite amount: neither an object of sense nor *one* 'pure' unit is a *definite amount* of things or units." Hence, in Greek mathematics, "'the unit' as such is no ἀριθμός, which only seems strange if we presuppose the notion of the 'natural number sequence.'" *Two* things or units are therefore the smallest ἀριθμός of things or units. The unit, being smaller than the smallest ἀριθμός, has on account of this the "character of the 'beginning' or 'source' (ἀρχή)" that makes counting and ἀριθμοί initially possible. Klein appeals to a wide range of Greek authorities to substantiate the non-numerical status of the 'one' or 'unit'. Typical of their view on the matter is Aristotle's: "'More knowable than the definite amount is the unit; for it is prior and the source of every definite amount' (παντὸς ἀριθμοῦ) (*Topics* Z 4, 141 b 5 ff.)" (*GMTOA*, 59/53–54). On Klein's view, it is precisely this state of affairs, that a multitude of objects "can be grasped as *one* definite amount, that the many can be '*one*'" (55/49), that "was treated explicitly as a fundamental *problem* within Pythag-

59. Alexander of Aprodisias, *In Aristotelis metaphysica commentaria*, ed. Michael Hayduck, *Commentaria in Aristotelem Graeca*, I (Berlin: Preußische Akademie der Wissenschaften, 1891), 85, 5.

60. That is, the act as exercised in contrast to the *actus signatus*, or the act that signifies a concept.

orean and Platonic philosophy." Thus, no matter how far this problem leads beyond the consideration of ἀριθμοί, "it always remains tied in with it."

§ 62. "Pure" Ἀριθμοί

Klein maintains that Plato's treatment of arithmetic and logistic provides what "can here serve as our guideposts" regarding how the conception of pure, as opposed to visible or tangible, ἀριθμοί emerges on the basis of counting. We have seen that for Klein the requirements of daily life include the "continual practice of counting and calculation [that] gradually fosters within us that familiarity with definite amounts and their relations which Plato terms 'arithmetic and logistic art.'" Thus, the problem of scientific arithmetic and logistic emerges when the question is posed regarding "those definite amounts which we have at our disposal *before* we begin counting or calculating." These definite amounts "must clearly be independent of the particular things which happen to undergo counting," and it is this consideration that gives rise to the question: "of *what* are these the definite amounts?" To answer this question, Klein claims that our interest has to shift from the daily concern with those objects that "are various and impermanent and yield ever-changing results" (56/49–50) when they are subject to counting and calculation. In raising the scientific question concerned "with understanding the very possibility" (56/50) of these activities, "with understanding the meaning of the fact that *knowing* is involved and there must therefore be a corresponding *being* which possesses that *permanence* of condition which first makes it capable of being 'known'" (see § 54 above), the special nature of the object of theoretical arithmetic and logistic comes to the fore. Specifically, the "soul's turning away from things of daily life, the change in the direction of its sight, the 'conversion' (περιαγωγή) and 'turning about' (μεταστροφή – *Republic* 518 D) which is implicit in this new way of posing questions" leads to another question: Must not that which is *known* in theoretical arithmetic and logistic satisfy the special conditions of exhibiting a *being* that possesses the *permanence* of the *knowable* as such?

Plato answers this question, according to Klein, affirmatively. Specifically, "the object of arithmetic and logistic" is "that which alone of all things is in the strict sense knowable." Moreover, this object "is always already known to a certain extent." The dual requirement of a being that is permanent and always already known in some fashion is "exactly fulfilled by the 'pure' units, which are 'nonsensual,' accessible only to the understanding, indistinguishable from one another, and resistant to all partition." Hence, "the 'scientific' arithmetician and logistician deals with *definite amounts of pure*

monads." Klein maintains that, on Plato's view, "Everyone is able to see—if only it is pointed out to him—that his ability to count and to calculate *presupposes* the existence of 'nonsensual' units" (*GMTOA*, 56/51). That is, "Only a careful consideration of the fact that it is really necessary to suppose that there are definite amounts of definite objects different from the ordinary kind, if the possibility and the successful execution of counts and calculations are to be understood, forces us into the further supposition that there must indeed be a special 'nonsensual' material to which these definite amounts refer" (56/50).

Consequently, "an *unlimited* field of 'pure' units presents itself to the view of the 'scientific' arithmetician and logistician. . . . The single multitudes which may be chosen from this field are precisely those 'pure' definite amounts (of units) with which he deals. This is how the traditional 'classical' definitions of ἀριθμός are to be understood" (56–57/51). Klein substantiates this claim with copious references to both Aristotle and the Greek mathematicians. For example, Aristotle: "'for each definite amount is many because it {consists of many} ones' (Aristotle, *Metaphysics* I 6, 1056 b 23)" (*GMTOA*, 57/51). And Euclid: "'[a definite amount is] the multitude composed *of units*' (τὸ ἐκ μονάδων συγκείμενον πλῆθος) (Euclid, VII, Def. 2)." Thus, Aristotle says, "'Multitude is, as it were, the genus of the definite amount' (*Metaphysics*, I 6, 1057 a 2 f.)." A consequence of the latter position is that "'the definite amount' and 'one' are opposites (Aristotle, *Metaphysics* I 6, 1056 b 19 f.)" (*GMTOA*, 57/52). Nevertheless, Klein reports that Aristotle speaks of the 'one' "metaphorically as being a 'certain, although small, multitude' ([*Met.*], 1056 b 13 f.), namely the multitude 'one.'" Likewise, Chrysippus (third century BC) says that "'the unit is the multitude 'one' (μονάς ἐστι πλῆθος ἕν)."[61] Klein claims that the Greek definitions of the sequence of ἀριθμοί are founded on the possibility of the "successive reproduction of the 'unit'" or the 'one,' even though "The truth is that the unit can be spoken of as a 'multitude' only improperly, 'confusedly.'" In support of this, he quotes Domninus: "The whole realm of definite amounts is an advance from the unit to the unlimited by means of the excess of one unit {of each successive definite amount over the preceding}'" (Domninus, 413, 5 ff.).

For Klein, "the unit is . . . that permanently same and irreducible basic element which is met with in all counting—and thus in every definite amount" (*GMTOA*, 58/52). Consequently, "To determine an amount means to count off in sequence the given single units, be they single objects of sense,

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61. Chrysippus in *Iamblichi in Nicomachi arithmeticam introductionem liber ad fidem codicis Fiorentini*, ed. Ermenegildo Pistelli (Leipzig: Teubner, 1894), 11, 8–9.

single events within the soul, or single 'pure' units." To be countable, these units must be "identical" (58/53) and yet "separated and clearly 'determined' (διωρισμένα)" from one another. Insofar as the units are units of counting, they possess "homogeneity" and "perfect wholeness." As such, they possess the "essential marks of the field of 'pure' units": indivisibility and discreteness. The latter is responsible for the "'discreteness' of definite amounts, namely . . . the fact that the single units which are 'parts' of definite amounts do not, in contrast to the sectional parts of continuous magnitudes, have a 'common terminus' (κοινὸν ὅρον – cf. Aristotle, *Categories* 4, 4 b 25)." According to Klein, such discrete units can "form the 'homogeneous' medium of counting only if each unit, whatever its nature [i.e., sensuous, psychic, or noetic], is viewed as an *indivisible whole*." In support of this he cites Aristotle, for whom "a definite amount is *always* 'a multitude of indivisibles' (πλῆθος ἀδιαιρέτων) (Aristotle, *Metaphysics* M 1085 b 22)." Consequently, not only is the one (μονάς) the source (ἀρχή) of ἀριθμός, in the sense that an ἀριθμός is always "'a definite amount of . . . ,'" but also, insofar as knowledge of the unit is presupposed in "the possibility of recognizing a definite amount of units as such," the unit also has "priority of 'intelligibility.'" Finally, because the units are multitudinous, "The possibility of extending the count over ever more units is unlimited" (*GMTOA*, 59/54). However, because of the indivisible character of the single units, "in the decreasing direction there is at last a barrier; here 'it is necessary to stop on reaching the indivisible' (ἀνάγκη στῆναι επὶ τὸ ἀδιαίρετον – Aristotle, *Physics* Γ 7, 207 b 8), i.e., the indivisible unit."

Klein's desedimentation and corresponding reactivation of the concept of ἀριθμός does not amount to the claim that "the Greeks" possessed a univocal understanding of this concept. Such a claim, to the effect that there is a concept of ἀριθμός that was shared by all Greeks, would, of course, be impossible to substantiate. Rather, Klein's claim is that the various accounts presented by the Greek mathematicians and philosophers of the mode of being belonging to the pure units that comprose ἀριθμοί or of the kind of priority belonging to the unit with respect to an ἀριθμός all "stem from one and the same original intuition, one oriented to the phenomenon of counting." Hence, Klein's desedimentation of these accounts of ἀριθμός traces what they have to say back to a phenomenon—counting—whose reactivation then lends clarity to "all the possible differences of opinion regarding the *mode of being* of the 'pure' definite amounts or the 'pure' units themselves, along with the *kind of priority* of the unit over the definite amount."

§ 63. Why Greek Theoretical Arithmetic and Logistic Did Not Directly Study Ἀριθμοί

On Klein's view the scientific disciplines of Greek arithmetic and logistic—in contrast "with that 'practical' knowledge which is satisfied with 'knowing' . . . definite amounts without understanding what this 'knowing' implies" (*GMTOA*, 59/54)—consist "in finding such arrangements and orders of the assemblages of monads [*Monadverbänden*] as will completely comprehend their variety under well-defined properties, so that their unlimited multitude may at last be brought within bounds." That is to say, because the "unlimitedness and the homogeneity of the field of monads permit us to combine units into assemblages of monads, i.e., into definite amounts of units, *in whatever way and as often as we please*," the first task of theoretical arithmetic consists in understanding how this delimitation takes place. Accordingly, for Klein, "in theoretical arithmetic the definite amounts themselves are *not studied directly*" (61/56). And because, as we shall see, it is the *kinds* (εἴδη) of definite amounts that are responsible for this delimitation—most conspicuously, the odd and the even[62]—and because, moreover, these kinds delimit definite amounts "*completely independent of whatever 'material'* [i.e., sensible or noetic] *may happen to underlie any particular case of counting*" (63/59), the concern of theoretical logistic with this material likewise does not directly concern "the determination of each determinate amount as a 'determinate amount of something'" (61/56). For Klein, then, these two states of affairs explain why the definitions of arithmetic and logistic in Plato's *Gorgias* and *Charmides* "*fail* to name the definite amount as the object of either of these sciences" (59/54).

Klein claims that the "point of view" of the "man who knows definite amounts" discloses that this view is oriented toward finding "within the field of 'pure' units itself those properties which will permit us to collect the different assemblages of monads under some few aspects, so as to obtain a complete synopsis of all possible multitudes" (59/54–55). Accordingly, "Greek theoretical arithmetic does, in fact, deal first and last with different *kinds* of definite amounts" (60/56). This means that "it attempts to comprehend all

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62. Klein also refers, in this context, to the concern of "Greek theoretical arithmetic" (60/56) with "the discovery of *kinds* of figures and definite amounts," such as the distinction Plato's *Theaetetus* makes between 'square' and 'oblong' (60/55), "designations which recur in all later arithmetical presentations," or the use of "the *gnomon*" to show "immediately that the εἶδος for all similar numbers is in each case one and the same." Klein elsewhere describes a 'gnomon' as "a configuration of dots (or of lines) which added to a figure of dots (or lines) produces a *similar* figure" (Klein, "The Concept of Number in Greek Mathematics and Philosophy," 46–47), a figure that presents numbers with "shapes" such as triangular, square, and pentagonal.

possible groupings of monads in general under arrangements which are determinate, i.e., which possess unambiguous characteristics and which may, in turn, be reduced to their own ultimate elements, such as the 'same,' and the 'other,' the 'equal' and the 'unequal,' the 'limit' and the 'limitless' (ταὐτόν – ἕτερον, ἴσον – ἄνισον, πέρας – ἄπειρον, cf. *Philebus* 25 A–B)." Thus, theoretical arithmetic is not concerned with the first aspect of the twofold determinateness of ἀριθμός, with "the determination of each definite amount as a 'definite amount of something'" (*GMTOA*, 60/56) that "is given by the pure units or the given objects." Rather, it is concerned with the second aspect of this determinateness, with "the fact that it is always a *definite* amount (of pure units or of things of some sort)." And it is precisely the latter that Klein maintains "can be understood only as the consequence of the special *kind* to which it belongs, i.e., by means of something *which is in itself one* and thus is capable of unifying, of making wholes—of delimiting." Thus, "Only through membership in an εἶδος 'derivable' from such 'sources' (ἀρχαί) does the being of a definite amount become intelligible as determinate, i.e., as *a delimited amount*, as *one* assemblage of just so and so many monads." In other words, "Precisely because *the ἀριθμός as such is not one but many, its delimitation in particular cases can be understood only by finding the εἶδος which delimits its multitude*," that is, "*by means of ἀριθμητική* as a theoretical discipline."

According to Klein, "the most comprehensive εἴδη, those which come closest to the rank of an αρχή and are therefore termed 'the very first' (πρώτιστα) . . . are the *odd* and the *even*" (61/57). Within the realm of ἀριθμοί, the

> "first cut" (πρώτη τομή – Theon 21, 20; Nicomachus 13, 9) . . . in terms of "even" and "odd" . . . affects all definite amounts in such a way that "one whole half of the realm of definite amounts" (ὁ ἥμισυς τοῦ ἀριθμοῦ ἅπας – Plato, *Phaedo* 104 A) falls under the "odd" and the other under the "even," each of these halves nevertheless comprising an *unlimited* multitude of definite amounts.

Notwithstanding this, however, "by means of certain unambiguous characteristics" the unlimited multitudes of each of these halves is "in turn gathered 'into one' (εἰς ἕν)." Thus, all definite amounts that are divisible into two equal parts, that is, that when divided yield no remainder, are 'even'. And "those whose division yields a remainder of one indivisible unit, are 'odd.'" Klein holds that "This latter property of 'being odd' can obviously occur only in a field of discrete and indivisible units, since it always depends on a single, 'supernumerary' unit, indivisible by 'nature.'" By contrast, "the property of being divisible into two equal parts without 'remainder' is . . . a property common to definite amounts and the continuous, i.e., infinitely divisible, magnitudes

which are always capable of being further bisected (cf. Plato, *Laws* 895 E: 'definite amounts also can, I suppose, like other things be divided into two parts')" (*GMTOA*, 62/57). For Klein, this state of affairs has the significant consequence that "*only* 'oddness' is characteristic of that which is countable as such, while 'evenness' represents something within the realm of definite amounts, and of everything countable, which goes, as it were, beyond it—something 'other,' namely the possibility of unlimited divisibility and thus, in a way, the 'unlimited' itself."

It is Klein's position, then, that for Greek mathematics the odd and the even are "determinations that concern the *being* of the definite amount that belongs to the countable as such, namely they concern the being (οὐσία) of an ἀριθμός as something according to which '*how many*' is determined as a definite amount" (63/58). Thus, "'with every definite amount one or the other belongs necessarily, either the odd or the even' (Aristotle, *Categories* 10, 12 a 7 f.)" (*GMTOA*, 62–63/58). As such, the odd and the even are "'two kinds proper' (δύο ἴδια εἴδη) of ἀριθμός" (62/58). However, because "the odd imposes a limit on unlimited divisibility in the form of an indivisible unit," it has a priority over the even.[63] Nevertheless, "the even still appears alongside the odd as a second essential characteristic of the discrete realm of definite amounts." Moreover, the "general familiarity of the distinction between odd and even definite amounts" (63/59) attests that the εἴδη of each belong to the being of ἀριθμοί.[64]

Klein stresses that the role of the distinction between the odd and the even in limiting ἀριθμοί and thus rendering them determinate "*is completely independent of whatever 'material' may happen to underlie any case of counting*" (63/59). Consequently, for him the absence of any reference to ἀριθμός or ἀριθμοί in the *Gorgias*'s and *Charmides*' definitions of arithmetic and logistic indicates two things: 1) "the multitude of arbitrarily chosen assemblages of monads is accessible to ἐπιστήμη only through the determinate εἴδη which can always be found for these assemblages" and 2) "the characteristics of all possible kinds of definite amounts, beginning with the odd and the even, are to be found *indifferently in all countable things, be they objects of sense or 'pure' units*." Thus, the theoretical rigor of these definitions consists in the fact that,

63. Klein conjectures that the "appearance of the περιττόν, the uneven, on the 'positive' side of the Pythagorean table of opposites marks a characteristic reversal of the 'natural' sense of values (cf. especially the English word 'odd')" (234 n. 64). This note appears only in the English translation.

64. Klein reports in this connection that "this distinction—besides that of male and female, and right and left—provides Plato with the model for every 'natural' *diairesis* [division] (cf. *Statesman* 262 A–E, *Phaedrus* 265 E)" (*GMTOA*, 63/59).

while they articulate "that all-pervasive limitedness which is rooted in an εἶδος, so that they can cover the *whole* realm of what is countable," they do not specify whether what is counted are unequal sensible things or pure (i.e., noetic and therefore uniform) units. This means for Klein that

> the rigor of both definitions does not exclude them from the "common understanding," since (1) *the understanding that the ἀριθμοί with which ἐπιστήμη deals are definite amounts of "pure" units is in no way presupposed* and (2) the two εἴδη of the even and the odd appeal to a thoroughly familiar distinction. (63–64/59–60)

On Klein's view, then, arithmetic—as it is understood in the commentaries of Olympiodorus and the *Gorgias* scholiast—"deals with definite amounts insofar as these present assemblages whose unity is rooted in the unity of a certain εἶδος, although this fact usually remains hidden for the most part from the practical intention of counting" (64/60). Consequently, at the theoretical level, arithmetic "studies the respective '*how many*' of each group of monads that falls under a particular εἶδος only indirectly." By contrast, both practical and theoretical logistic aim "of necessity—insofar as it is concerned with the mutual relations of definite amounts—directly at the '*how many*,' at the multitude of what in each case happen to be related to one another or computed, i.e., at the 'material' which underlies each relation or calculation." Whereas Plato, as we have seen,

> leaves completely indeterminate whether this material belongs to the realm of the sensible or the purely noetic (an indeterminacy which insures the universality of the definition), Olympiodorus and the *Gorgias* scholiast are forced from the beginning to regard the "hylic" monads, i.e., the monads which form the ὕλη of the definite amounts . . . as sensible 'units,' since only these are amenable to that partitioning which exactitude of calculation requires.

According to Klein, both of these Neoplatonic commentators are forced, as it were, by the problem of the indivisibility of pure monads maintained by Platonism to "unwittingly enter an ontological realm which is no longer consonant with the Platonic definition of monad." To make this clearer, Klein next turns his attention to "the ontological problems raised by the ἀριθμοί," that is, to the relation between Greek ontology and Greek mathematics. His reason for doing so is that "any attempt to understand Greek mathematics as a self-sufficient science must fail," because "It is impossible to disregard its ontological difficulties which fundamentally determine its problems, its presentation, its development, especially its beginnings," just as it is impossible "to understand Greek ontology without reference to its specifically 'mathematical' orientation" (64–65/61).

Chapter Eighteen

Plato's Ontological Conception of Ἀριθμοί

§ 64. The Interdependence of Greek Mathematics and Greek Ontology

Klein argues that a major consequence of the interdependence of Greek mathematics and Greek ontology is that "the *μάθημα*-character of 'mathematical' states of affairs, its status as a 'learning matter,' serves as a model for all teachable and learnable knowledge," just as it is "knowledge so understood which determines the sphere of Greek ontological inquiry" (65/61). As we have seen, it is his contention that both the mathematics and ontology of the Greeks presuppose that only that which "possesses that *permanence* of condition which first makes it capable of being 'known'" (56/50) can be learned and therefore taught. We have also seen that the theoretical attempts of both the Neoplatonic mathematicians and Plato to articulate the precise statuses of arithmetic and logistic were driven by this stance on the *being* of knowledge. Notwithstanding their different accounts of the statuses of these mathematical disciplines, they were in accord that only a theoretical study of the εἴδη proper to ἀριθμοί can satisfy the ontological condition proper to the permanence of knowledge.

Klein's desedimentation and reactivation of the suppositions determinate of the Greek mathematical background of Diophantus's *Arithmetic*, however, has "so far avoided coming to terms with the ontological point of view which from the very first determined the form taken by the Greek doctrine of the ἀριθμοί" (64/61). As his desedimentation of the Greek ἀριθμός-concept has shown, this point of view has its origin in Plato's philosophy. So, to come to terms with this point of view, Klein undertakes the task of desedimenting Plato's understanding of the "mode of being" (72/71) that "belongs to definite amounts of 'pure' monads," as well as the εἴδη that delimit them and therefore render them determinate. On Klein's view, "our different [i.e., 'modern'] orientation notwithstanding," "nothing hinders us

from doing justice to the originality of ancient science by allowing ourselves to be guided only by those phenomena to which the Greek texts themselves point and which we are able to exhibit directly" (66/63). It is to his reactivation of such phenomena that we now turn.

§ 65. The Pythagorean Context of Plato's Philosophy

"There can be no doubt," Klein says, "that Plato's philosophy was decisively influenced by Pythagorean science, whatever the exact connection between Plato and the 'Pythagoreans' may have been" (71/69). Thus, "those definitions of arithmetic and logistic which were the basis of the preceding reflections seem to point to a Pythagorean origin." Not only is it the case, as we have seen, that "these definitions in no way presupposed the existence of 'pure' definite amounts; . . . they [also] referred to *everything* countable as such, and thus, above all, to the objects of sense in this world." Consequently, for Klein, "The definitions seem to have preserved the original *cosmological* significance of these two sciences." Indeed, he states that this is the case "especially since we must keep in mind that the 'very first kinds' (πρώτιστα εἴδη) of definite amounts, the 'odd' and the 'even,' represented for the Pythagoreans the 'limit' and the 'unlimited,' i.e., the ἀρχαί of *all* beings, whose union, whose 'mixture,' first brings the world as such into being." Not only is "This view, which is part of the cosmology of the *Philebus* and the *Timeaus*, . . . directly connected with the general theory of opposites of the Pythagoreans," but it also connected with "the symmetry of the pairs of opposites in the *Charmides*" and "in the *Gorgias*" (71/69–70).

Thus, it would seem that the conception in both the *Charmides* and the *Gorgias* of the object of arithmetic and logistic is "conceived from an ontological point of view identical with that of the Pythagoreans" (71/70). This semblance is dispelled, however, by the entirely different mode of being that Plato assigns to mathematical objects in comparison with Pythagorean science. Klein writes that the Pythagoreans

> saw the true grounds of the things in this world in their *countableness*, inasmuch as the condition of being a "world" is primarily determined by the presence of an "*ordered arrangement*" (τάξις)—and this means a *well*-ordered arrangement—while any order, in turn, rests on the fact that the things ordered are delimited with respect to one another and so become countable. (67/64)

Based on this, he concludes that for "the men traditionally called '*Pythagoreans*'" (66/63) "the arithmetical determinations of things concern their *being itself*," and that, as Aristotle reports, "'the authentic being of all things is pre-

cisely their definite amount' (*Metaphysics* A 5, 987 a 19)" (*GMTOA*, 67/64). Consequently, "those affections (πάθη) which are to be found in everything counted *as such*," that is, their "properties, conditions, and ways of behaving would be . . . identical with the elements and sources of their countableness (*Metaphysics* A 5, 985 b 23–26, 29; 986 a 6, 15–17; A 8, 990 a 2 ff., 19)" (*GMTOA*, 67/64–65). Moreover, "They evidently considered a vague 'structural' similarity a sufficient basis for speaking of an 'imitation' of definite amounts by things (*Metaphysics* A 6, 987 b11 f.; Theophratus, *Metaphysics* 11 a 27 f.)," a view that Aristotle criticized due to the "'superficiality' of their definitions" (*GMTOA*, 68/65).

Klein claims that the Pythagorean manner of definition was driven by their search for "the *being* of things by reducing and *assimilating* them to states of affairs 'primarily' exhibited in the realm of counted assemblages as such." This use of definition, which accounted for the being of particular things "in terms of a certain *kind* of definite amount or ratio," was also driven by the search for a primary definite amount or ratio, because "There is always a *first* and 'smallest' definite amount (or ratio) of a particular kind such that a particular property belongs to it *primarily*." In line with this, the first definite amount or ratio "therefore represents the 'root' (πυθμήν) of its kind." Consequently, the ἀριθμός *ten* along with *one* are especially significant, the latter because it is the "*fundamental element* of all counting," the former because "the definite amounts comprised in the decad, together with their mutual relations, are themselves the most important 'roots.'" Klein writes, "This Pythagorean method of identifying the being of things which have analogous characteristics thus extends to the decad and is elaborated in the later Neopythagorean and Neoplatonic tradition so as to become the most important tenet of 'arithmology' (to use a modern term coined for this purpose)" (68/66). Because "the guiding thought of this tradition is always that of the perfect arrangement (διακόσμησις) or order (τάξις) of the Whole," the controlling point of view throughout remains cosmological. Within this tradition, then, "the very countableness of things, and thus the sequence of definite amounts, which makes this τάξις possible, is understood as a definite 'being-in-order' of things, and thus the sequence of definite amounts is understood as the *original order of the being* of these things" (69/66). This order is not understood, however, as "a linear chain whose links are all 'of the same kind' but an 'ordering' in the sense that each definite amount precedes or follows in the order *of its being*, i.e., is related as prior and posterior (πρότερον and ὕστερον – cf. Aristotle, *Metaphysics* Δ 11, 1018 b 26–29)."

Thus for Klein "the science of the Pythagoreans is an *ontology of the cosmos*," which means that their arithmetical and logistical science has as its

"true object the being of the very constituents of the world" (*GMTOA*, 69/66). While this conception doubtless was motivated by, among other things, "an insight into the dependence of musical consonances on 'logistic'—the basis of its possibility is the 'natural' conception of the ἀριθμός as characterized earlier" (69/66–67). That is, "Only when 'definite amount' means a definite amount of *things*, when the *counted things themselves* are meant whenever a definite amount is determined, is it possible to understand their being (precisely insofar as they are things) as 'definite amount'" (69/67). Aristotle reports repeatedly that the Pythagoreans "do *not* make definite amounts *separable* {from the things} (οὐ χωριστὸν ποιοῦσι τὸν ἀριθμόν)," which means that "they do not go so far as to suppose the existence of 'pure' definite amounts of 'pure' units, although *they were the very men* who concerned themselves with definite amounts not for a practical but for a theoretical purpose, who conceived of the ἀριθμός as ἀριθμὸς μαθηματικός, as scientific definite amount (*Metaphysics* M 6, 1080 b 16 ff.)." Klein avers that, despite their being wedded to "the opinion that being extends just as far as sense perception" (*GMTOA*, 69/67), they were nevertheless "on the way toward discovering a mode of being of a higher order." Indeed, he holds that "we may conjecture that they saw the genesis of the world as a progressive *partitioning* of the first 'whole' *one*, about whose origins they themselves, it seems, were not able to say anything conclusive (cf. *Metaphysics* M 6, 1080 b 20 f.; N 3, 1091 a 15 ff.)." Nevertheless, it seems that for them the *one* "already contained as fundamental constituents, as 'elements,' the two fundamental kinds of the realm of definite amounts, the 'odd and the even,' namely 'limit' and 'unlimited' (cf. *Metaphysics* A 5, 986 a 17 ff. and Theon 22, 5 ff.)." Both this first one and all subsequent ones resulting from partition, that is, the definite amounts themselves, "they therefore regarded as having *bodily extension*: 'they understood the monads to have magnitude' (τὰς μονάδας ὑπολαμβάνουσιν ἔχειν μέγεθος – *Metaphysics* M 6, 1080 b 19 f.)" (*GMTOA*, 70/ 67). This "allowed them to reduce the *measurableness* of things to their *countableness*—until the discovery of 'incommensurable' magnitudes proved the impossibility of this reduction" (70/67–68).

Klein maintains, however, that the Pythagoreans went beyond their understanding of "the *bodily monads* as the specific 'stuff' of the *real* [dinglich] being of things (which relates them to the atomists)" (70/68), "to see in the *qualities of the kinds* of definite amounts models which things 'imitate.'" Hence, in accord with their principle "that the being of things with analogous characteristics is identical, they were able to define their being [i.e., the being of the 'kinds'] as *being a definite amount*." Klein cites Aristotle, who reports that they considered "'definite amounts to be the source of the things

that are, both *as their material and as their characteristics, as well as their states*'" (*Metaph*. A 8, 990 a 18–19). This means that both the characteristics of ἀριθμοί, namely their kinds (εἴδη), as well as the ἀριθμοί themselves, are responsible for the being of things, since, as Aristotle again reports, "'Those {different things} [i.e., the kinds of definite amounts and the definite amounts themselves] would be the same with one another since *the same kind of definite amounts belongs to them*'" (N 6, 1093 a 11–12). Regarding these εἴδη, Klein also cites the Pythagorean Philolaos, who in reference to εἴδη of the odd and the even, writes: "'There are many shapes belonging to each of the two forms.'"[65] Klein says that "it is clear . . . that by the μορφαί [shapes] of the εἴδη are to be understood their subspecies, i.e., certain characteristics or qualities of the kinds." He also holds that not "since Boeckh" (*GMTOA*, 70/69), who remarked about these characteristics "'that they were developed by the ancient arithmetician with a special industry which appears to us as petty,'"[66] has the "significance of these πάθη or μορφαί or εἴδη of definite amounts . . . been properly acknowledged" (70/68–69). That is not only because "the εἴδη of definite amounts and of their relations, together with their correlative roots (πυθμένες) . . . form, as we have seen, the true object of Greek arithmetic but also the foundation of all cosmological speculation as originated by the Pythagoreans and developed through the centuries up to Kepler" (70/ 69). Here Klein announces, in anticipation of his analysis of the origin of algebra, that "we shall have to examine the role given to this εἶδος concept in the mathematics of Vieta."

Despite the similarity between the Pythagorean science and Plato's philosophy, Klein holds that "there exists an indissoluble tension" between the role each assigns to mathematical objects. This tension "manifests itself right within the Platonic *opus* in the opposition between the Socratic dialogue and the 'likely tale' (εἰκὼς μῦθος) [in Plato's *Timaeus*] representing the cosmos" (71/70). For Plato, mathematics is the "'bond' (δεσμός) which ties dialectic and cosmology together and affects *both* decisively." As a bond, mathematics holds a "'middle position'" between the objects of dialectical inquiry (the εἴδη) and the objects of praxis (somatic things). Thus, "by giving this 'mediating' role to mathematics Plato assigns to mathematical objects a totally different mode of being than they can possibly have in Pythagorean science." In this connection, Klein emphasizes the fact that in discussing ἀριθμοί "Aristotle never tires of stressing that Plato, in opposition to the Pythagoreans, made

65. Klein's citation refers to Diels, I3, 310, 13–14, Philolaos fragment 5.

66. Here Klein cites August Boeckh, *Philolaos der Pythagoreers Lehren* (Berlin: Voss, 1819), 60.

them 'separable' from objects of sense, so that they appear '*alongside* perceptible things' (παρὰ τὰ αἰσθητά) as a separate realm of being" (71–72/70). As we have seen, this realm is comprised of "the field of 'pure' units, which are indivisible, of the same kind, and accessible only to thought" (72/71). Thus, for Klein, the novelty of Plato's account of ἀριθμοί explains "his general interest in the superiority of the purely noetic over everything 'somatic,' . . . the stress which he lays on the existence of the purely noetic over everything 'somatic,'" and the fact that the "emphasis with which the thesis of 'pure' monads is propounded is indicative that ἀριθμοί were ordinarily, and as a matter of course, understood only as definite amounts of sensible and tangible things."

§ 66. Plato's Departure from Pythagorean Science: The Fundamental Role of Ἀριθμοί of Pure Monads

Klein holds, then, that Plato's "thought of 'pure' definite amounts of definite objects separated from all body is originally so remote that it becomes the philosopher's task precisely to point out emphatically the fact that they are *independent* and *detached*, and to secure this fact against all doubt" (72/70–71). As we have seen, this task is fulfilled by following the "course which leads from our actual counting to the conception of 'pure' monads" (72/71), a course that Plato gives an account of by showing that our ability to count and to calculate is rooted in the fact that our διάνοια (the faculty and activity of thinking) "*authentically* means, when it counts or calculates such things"—that is, "a definite multitude of objects of sense"—"the existence of 'nonsensible' monads which can be joined together to form the definite amount in question." Klein says that "the special mode of being which belongs to definite amounts of 'pure' monads begins to be comprehensible in the light of their foundational role" vis-à-vis that for which it provides a foundation, that is, the ordinary definite amounts. Hence, for Klein, "The being of the foundation takes precedence over that which is so founded, i.e., it is prior (πρότε-ρον), because the second cannot be without the first, but the first can be without the second." Aristotle bears witness to this "primordial independence and 'detachment' which is rooted in the founding function" "when in the explication of the various meanings of prior and posterior (πρότερον – ὕστερον, *Metaphysics* Δ 11) he remarks expressly, in commenting on the meaning of these terms which has just been explicated, that 'Plato made use of this distinction' (ᾗ διαιρέσει ἐχρήσατο Πλάτων – 1019 a 2–4)."

Klein maintains that the distinction in question "is connected with the much-discussed 'hypothesis' (literally: sup-position) theory of Plato" (*GMTOA*,

73/71). Plato's well-known division into two domains[67] (*Republic* 510 B–D) of "that which is accessible to the understanding," of, namely, what "is and can be thought (νοητόν)" characterizes the relation of what is so divided as follows. Certain objects that are perceived by the senses nevertheless refer beyond themselves to something that is imperceptible to the senses, something "which is for thought" (νοητόν) because of this very imperceptibility. Moreover, because the latter is somehow linked to the objects of the senses that refer to it, it provides the foundation for what is perceived by the senses. Thus, Plato maintains that in such cases "the examining soul is *compelled* to *suppose*, i.e., to make to underlie," the objects of sense "the νοητόν in question." Such objects of sense function as "an *image of* something 'other,'" "which underlies them as a foundation and images itself in them" (*GMTOA*, 73/72). Consequently, Plato holds that "'Here the soul must of necessity begin its search {for objects of thought} from *suppositions*' ([*Republic*] 510 B)." Thus, "In such cases we do examine these things by means of the senses; yet we do not *mean* these things but rather that which comprises their 'foundation' and which they *image*" (*GMTOA*, 73/72). In the case of the geometers, what their souls mean are not the "characteristic qualities" exhibited by the figures they draw, but what is "imaged in" them, "e.g., the rectangle which is, in its purity, accessible only to thinking." In that of logisticians, even though they "have before their eyes the 'odd' and the 'even' as the form [*Gestalt*] of whatever objects their deliberations move over," it is nevertheless the case that such "deliberations, being pursued in thought, do not aim at these visible things but at the 'pure' definite amounts or their εἴδη that are imaged by them and taken as their basis by thinking."

While this suppositional procedure characterizes "above all" that of geometers and logisticians, "the efficacy of διάνοια" that is "directly displayed" in their conduct "is by no means confined to these alone" (73/73). Klein claims that for Plato διάνοια "obviously has an essential, perhaps *the* essential, part in all human activity and self-orientation." He maintains this because Plato's procedure by hypothesis "is *not* a specifically 'scientific' method but is that original attitude of human deliberation prior to all science which is revealed directly in speech as it exhibits and judges things" (73–74/73). Klein takes Socrates' characterization of it in the *Phaedo*, in its contrast to the physiological study of nature, as the "'second-best sailing' (δεύτερος πλοῦς)," "which consists of 'taking refuge in reasonable speech' (*Phaedo* 99 E)," to signal "nothing else than a return to the *ordinary* attitude

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67. Namely, one in which the soul makes its inquiries using hypotheses and the other in which it inquires by means of the εἴδη themselves.

of the διάνοια." Hence, he says, "it is for this reason that Plato can characterize the method of 'hypothesis' as 'simple and artless and perhaps naive' (*Phaedo* 100 D)."

On Klein's view, then, reasonable speech guided by the διάνοια always supposes "something 'other' to underlie the objects we perceive, namely νοητά; these, albeit appearing in the mirror of our senses, are the true objects of our study, though we may not even be aware of making such 'suppositions'" (*GMTOA*, 73/74). By rendering these suppositions perspicuous, that is, "transparent and thereby learnable," the "'devices'" of the διάνοια make "completely explicit what the διάνοια has in effect been accomplishing *prior to* any science." The process of making the latter explicit is the work of "a privileged mode of deliberation in which this 'supposing' is raised to the rank of a conscious procedure; this is the origin of every science and every skill (cf. *Philebus* 16 C)" (*GMTOA*, 74/73). This means "one can grasp the authentic content of the former ordinary accomplishment of the διάνοια only through this privileged mode of deliberation." According to Klein, the "*authentic* sense of the διάνοια" (74/74) is grasped in precisely "those τέχναι which are most highly developed, *surveying* and above all, *counting and calculating* (cf. *Euthyphro* 7 B–C), that 'common thing of which all arts as well as all thinking processes and all sciences make use' (*Republic* 522 C) and without which any τέχνη would lose its character as τέχνη (*Philebus* 55 E; cf. *Republic* 602 D)" (*GMTOA*, 74/74). Not only is this why Plato assigns διάνοια to the exemplary realms of geometry and logistic, but it is also why the arguments of the *Republic* "establish the leading role played in education by mathematical subjects, especially counting and calculation" (74/74).

§ 67. Διάνοια as the Soul's Initial Mode of Access to Νοητά

What is at issue for Klein in Plato's account of διάνοια, therefore, is the soul's initial mode of access to the νοητά. Despite their non-sensible mode of being, the νοητά nevertheless remain tied—so long as they are apprehended by the διάνοια—to αἴσθησις (sense perception). They remain so tied because of their suppositional character as the foundation of sensible objects. This means that for Plato "the διάνοια itself is not able to appreciate the full range of significance of its accomplishment" (79/79), because, as we shall see, "its own νοητά, which it 'supposes' to underlie αἰσθητά, appear altogether lucid and without further need of foundation." Thus, "this quarry of the διάνοια, as it is brought in especially by the mathematicians, must first be handed over to the *dialecticians* for proper use (*Euthydemus* 290 C; *Republic* 531 C–534 E)." That is because the διάνοια "at first always attains to noetic structures of oppositional

quality" (*GMTOA*, 78/78), which means that the mode of being of the delimited field of pure units that underlies the sensible objects it counts and that makes possible its calculations cannot be grasped except by means of its opposite, "namely from the unlimited many" (77/77). Hence, according to Klein, "the διάνοια discovers the 'one' *not by itself alone but among many ones*," and it thereby "relates the *one* to the *other* ones, an activity which, in effect, is nothing but—counting."

Klein reactivates Plato's account of the emergence of διάνοια from sense perception on the basis of the argument in the *Republic* (523–525) regarding the "objects of perception which of necessity leave the perceiving soul unclear about their nature and which thus of necessity call up deliberation to aid the soul in discovering it" (*GMTOA*, 75/74). Thus, in contrast to 1) "objects of perception which directly satisfy the perceiving soul because they are in themselves sufficiently clear, so that there is no necessity to appeal to any authority beyond perception—there is no necessity *to deliberate* in any way on such perceptions," "the very structure" of 2) some perceived objects, is such as to necessitate that the soul "go beyond 'bare' perception and 'call up' and 'awaken' 'reckoning and thought' (λογισμόν τε καὶ νόησιν – [*Republic*] 524 B)" (*GMTOA*, 75/75). On the one hand, with respect to the former type of object, for instance, when "I look at the fingers on my hand, I perceive each of them directly *as a finger*. Plain perception is the end of the matter: Within this perception there is nothing 'problematical'—it is complete in itself, it does not leave behind a feeling of a lack of clarity such as might move me to deliberate and to investigate what there authentically is to the finger, *what* a finger might *be* ([*Republic*] 523 D)" (*GMTOA*, 75/74–75). On the other hand:

> The situation changes completely when I perceive the characteristics of each of my fingers in turn, such as its largeness or thickness or softness. Now I perceive in any one finger *at one and the same time also* an opposite: its smallness, its thinness, and its hardness. I therefore perceive opposites in the same perception, and in the same object of perception. Here something which clearly does not belong together is in an obscure way "mixed," *as deliberation immediately informs me*. (75/75)

It is at this point that "reckoning and thought" are called up, becoming "active spontaneously, as it were, in direct succession to perception." They inform the soul that here it is a question of "something *twofold*, namely on the one hand softness and on the other hardness, and similarly largeness on the one hand and smallness on the other (cf. *Theatetus* 186 B)" (*GMTOA*, 76/75). Reckoning and thought therefore permit the soul "to recognize *both together as* '*two*,' as *this one* thing—softness or largeness—and *that other one* thing—hardness or

smallness—no longer 'mixed' with one another, but as distinct: '*Each is one, but both are two*, and the two are separated by thought . . . not fused together but distinct' (ἕν ἑκάτερον, ἀμφότερα δε δύο, τά γε δύο κεχωρισμένα νοήσει . . . οὐ συγκεχυμένα ἀλλὰ διωρισμένα – [*Republic*] 524 B–C)."

The διάνοια is able to effect this separation of the "'mixture' which assaults the senses" (*GMTOA*, 76/76) "by way of *comparison* (cf. *Theatetus* 186 B)" (*GMTOA*, 76/75). Thus, the *same* finger is small or large in comparison with its left or right neighbor. Simmias is large, "not insofar as he is Simmias, but insofar as he can be compared with Socrates." Moreover, "The same Simmias is, on the other hand, small—again, not insofar as he is Simmias, but insofar as he can be compared with Phaedo" (76–77/75–76). Consequently, "The largeness and the smallness of an object, can . . . be recognized by thought as something twofold only when the object is recognized at one time as 'being-larger-than . . .' and another as 'being-smaller-than . . .'" (76/76). Therefore, for Klein, "Only by *relating* the one condition to the other is the διάνοια enabled to suppose two distinct structures" "to underlie each obscure αἰσθητόν."[68] Its objects, "the first kind of νοητά, are therefore attained as a result of the fact that that which is accessible to the senses is, by reason of its 'relational character' ('Bezüglichkeit' – Natorp), recognized as a *manifold*." Consequently, according to Klein,

> the recognition of the many as *many*, and thus also of the other *as other*, means precisely that the single constituents of the many are distinguished from one another and at the same time, in all their distinctiveness, related *to one another*. The διάνοια continually surveys and compares the many aspects which αἴσθησις offers it.

This means that the "διάνοια is never directed toward a single being as such; rather, its view always so encompasses a series of beings that the members of this series are carefully distinguished from, and thus simultaneously related to, each other."

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68. The relating of one condition to another is properly the work of logistic. In the case at hand, the comparison of one finger to the two on either side of it with respect to their size involves two ratios. Specifically, one and the same finger is small when compared at one time to its left neighbor. This is one ratio, i.e., being-smaller-than. At another time, one and the same finger is large when compared to its right neighbor. This is another ratio, i.e., being-larger-than. As we have seen, according to Klein the διάνοια resolves the obstacle of one and the same finger's being both big and small by discerning that the being both big and small of the "one" finger is not itself *one* but *two*. That is to say, 'big' and 'small' are each one but together two. And discerning this requires that the ratios in the being-larger-than and being-smaller-than are recognized as such by logistic, that is, recognized as comparisons between two things with respect to their magnitude.

§ 68. The Limits Inherent in the Dianoetic Mode of Access to Νοητά

It is thus Klein's claim that in Plato "the διάνοια is based essentially on 'account-giving and counting' (λογίζεσθαι τε καὶ ἀριθμεῖν), namely on the ability to recognize many as *so* many, to see many *as many*, i.e., to distinguish the constituents of the many and at the same time to relate them to one another" (76–77/76–77). If it should happen, in the clarification of the opposition in the finger or any other object, that "the perception of an 'incompatibility' (ἐναντίωμα) in the sense that the *same* object appears as *one* and also as the *opposite of one* [i.e., 'many'], 'so that nothing appears any more as one than as the opposite' ([*Republic*] 524 E; cf. *Theaetetus* 186 B), then here too the διάνοια would have to intervene to remove the 'obstacle' and to allow the examining soul to reach 'clarity' (σαφήνεια) concerning the 'one'" (*GMTOA*, 77/77). Therefore, Plato's account of the obscurity involved in the perception of the finger "leads directly to the question concerning 'definite amounts' and the 'one.'" To be sure, the opposition—between the softness and hardness of the same finger, or Simmias's largeness and smallness—is "easily resolved. For the finger [or Simmias] is called 'one' and 'many' in totally different respects" (77 n. 1/236 n. 85). "Nevertheless in this elementary and easily penetrable form . . . the contradiction [i.e., of the one and the many] is palpable enough to spur the διάνοια on to activity—which is here the sole point." Moreover, because perception in these instances is responsible for our seeing "'the same things at the same time as *one* and as *unlimited* in multitude' ([*Republic*] 525 A)" (*GMTOA*, 77/77), and since this holds not just for one object but "also generally for 'every definite amount' of objects (ξύμπας ἀριθμός), because any 'definite amount' represents precisely a limited multitude of unit objects," then the διάνοια is "compelled . . . to separate the unit as such, 'the one itself' (αὐτὸ τὸ ἕν), from its opposite, namely the unlimited many."

Thus, as we indicated above, for Klein the διάνοια—because of the oppositional character of its clarifying activity—"is unable to come face to face with the one *as it is in itself*. For the διάνοια always deals with a *multitude* of ones; it cannot grasp the one except through the totality of ones, just as it cannot grasp one element of language, i.e., a single sound rendered by a letter, without the remaining sounds (*Philebus* 18 C), nor a single tone without the other tones" (*GMTOA*, 78/78). The discovery by the διάνοια of the "one" not by itself but in relation to *other* ones, "an activity which is, in effect, nothing but—counting," is, according to Klein, "a proceeding of a perfectly ordinary kind. In all the daily routines of life we are dependent on just such interventions." Moreover, the soul, led by the διάνοια,

> turns its attention to this, its very own way, "arousing thinking within herself" (κινοῦσα ἐν ἑαυτῇ τὴν ἔννοιαν) and discovers, within a field of unlimitedly many and homogeneous "pure" units, the "pure" definite amounts of these units; it raises its own relating activity to full explicitness by examining the relations of these definite amounts to one another, thus incidentally laying a foundation for the possibility of making calculations; it causes the εἴδη of these relations of definite amounts, as well as of the definite amounts themselves, to "underlie" the objects of sense as "suppositions," "positing the odd and the even" ([*Republic*] 510 C) as the first of these—yet in spite of all this, it is unable to come to face with the one *as it is in itself*. (*GMTOA*, 78/77–78)

As we have indicated, this inability is rooted in the oppositional character of the noetic structures that the διάνοια recognizes, a character that has its source in the fact that the διάνοια, despite its being directed toward νοητά, "always remains related to that αἴσθησις which first 'called upon it' to clarify an obscure state of affairs" (78/78). Thus, "The διάνοια effects its clarification by recognizing the opposition which underlies the obscurity of the αἰσθητόν *as* an opposition," a recognition that is "the proper function of the διάνοια (cf. *Theaetetus* 186 B)." And again, as we have already seen, the consequence of this is that "it at first always attains to noetic structures of oppositional character." Klein finds these illustrated in the *Theaetetus* in terms of "'being' and 'nonbeing,' of 'likeness' and 'unlikeness,' of the 'same' and the 'other,' and furthermore of the 'one' and the 'definite amount' which deals with these {i.e., with objects of sense}' (*Theaetetus* 185 C–D)." Moving in the realm of opposition, the διάνοια "in conducting its 'comparisons' (ἀναλογίσματα – *Theaetetus* 186 C) . . . discovers as the true 'foundation' of this domain the realm of the 'pure' relations of definite amounts, i.e., the ratios (λόγοι) and proportions (ἀναλογίαι) of the 'pure' definite amounts, because every possible comparison is ultimately founded on these (cf. *Timaeus* 36 E–37 A)" (*GMTOA*, 78/78–79). However, since "the work of the διάνοια is everywhere 'hypothetical' in character" (78/79), it nevertheless remains dependent on sense perception, both in the case of its discovery of the foundation proper to the domain of oppositions in ratios and proportions and in the case of "the *general* theory of proportions." Thus, even though "it can 'replace' the 'more and less' (τὸ μᾶλλον καὶ τὸ ἧττον) which always attaches to the realm of αἰσθητά by the exact relations of definite amounts, thereby accomplishing the most important step toward gaining that true ἐπιστήμη which no longer has any use for αἴσθησις and whose object is the realm of those other νοητά that ascend to something 'unsupposed' (ἀνυπόθετον – *Republic* 510 B, 511 B)," it cannot, as we have mentioned, appreciate the full significance of this accomplishment. And it cannot do so because the νοητά it supposes do not appear to it to need—due to their seeming clarity—any further foundation. Consequently, Klein maintains that, for

Plato, "Only dialectic can open up the realm of true being, can give the ground for the powers of the διάνοια and can reveal Being and the One and the Good as they are—beyond all time and all opposition—in themselves and in truth" (*GMTOA*, 79/79).

According to Klein, then, in Plato's thought the soul's *dianoetic* access to *νοητά* is limited, because the comparison necessary for their discovery yields a structure that is fundamentally oppositional. The pure being of these structures supposed by the soul's διάνοια to underlie the activities of counting and making calculations (as well as of all the oppositions that it discovers in the realm of sense) and, as such, something that is available only to thought, is limited due to its inseparability from its opposite. Thus, the διάνοια cannot think the one apart from the many, the more from the less, the odd from the even, etc. On Klein's view, overcoming this limitation, that is, overcoming the oppositional character of the *νοητά* supposed by διάνοια, is for Plato the work of dialectic, the proper method of which is "'division' (διαίρεσις)" (93/97). Klein holds that the key to this method is to be found in "that ever-same *aporia*" (84/86) manifested by the mode of being of ἀριθμός, its peculiar "one over many" structure. Plato directs "attention to varying aspects" of this aporia, and in the process "indicates the *mode of being of the νοητόν as such*" (88/91). Because of the intrinsic "'weakness of speech' (τὸ τῶν λόγων ἀσθενές) (*Seventh Letter* 342 E)" (*GMTOA*, 91 n. 2/238 n. 103), however, this mode of being cannot be grasped "'*with complete clarity*' (*Sophist* 254 C)" (*GMTOA*, 90/94). That is to say, when it comes to the problem of being, "the λόγος fails!" (91/94), and it fails precisely because "the problem of 'being' has an internal connection with the *aporia* of 'non-being'" (84/86), and therewith, with its opposite. Klein will show that the λόγος cannot fully account for this aporia because it is inseparably tied to the διάνοια, a tie that brings with it the fundamental involvement with the *counting* of that which has being.

Chapter Nineteen

Klein's Reactivation of Plato's Theory of ᾿Αριθμοὶ Εἰδητικοί

§ 69. The Inability of Mathematical Thought to Account for the Mode of Being of Its Object

Klein's interpretation of Plato's dialectical method is guided by the question he has shown was raised, but not answered, by Greek mathematical thought. It is guided therefore by the question of the mode of being proper to the mathematical objects that such thought, in its theoretical guise, cannot help but encounter, a question that it also cannot help but be unable to answer—"for all time" (83/85)—so long as it remains strictly mathematical. The question here, which concerns both the mode of being proper to the "pure" ἀριθμοί as well as to their εἴδη, is what guides Klein's desedimentation and reactivation of Plato's thought of "[t]he Platonic theory of the ἀριθμοὶ εἰδητικοί [eidetic definite amounts]" (88/91), a theory that "is known to us . . . only from the Aristotelian polemic against it (cf., above all, *Metaphysics* M 6–8)." Klein writes:

> Only the ἀριθμοὶ εἰδητικοί make something at all like "definite amount" possible in this our world. *They* provide the foundation for all counting and reckoning, first in virtue of their *particular* nature which is responsible for *the differences of genus and species* in things so that they may be comprehended under *a definite amount*, and, beyond this, by being responsible for the *unlimited* variety of things, which comes about through a "distorted" imitation of ontological *methexis* [participation].[69] (*GMTOA*, 89/92–93)

Thus, Klein's desedimentation of what Aristotle referred to as Plato's "so-called unwritten teachings (τὰ λεγόμενα ἄγραφα δόγματα)"[70] reactivates the role of the ἀριθμοὶ εἰδητικοί that Aristotle attributes to Plato, and it does so not only in terms of their function of making possible the γένη involved in the dialectical method of "dividing according to genera" but also in terms of their role

69. See n. 78 below.
70. Aristotle, *Metaphysics* A 6.

in providing a solution to what Klein terms "the ontological *methexis* [participation] problem" (81/82).[71] Klein's account of this problem represents a major discovery, insofar as he identifies, in Plato's discussion (in certain dialogs) of the relationship between εἴδη, an account of their "ontological" participation that is more fundamental—indeed, literally more radical—than the usual account of the participation of non-eidetic things in an εἶδος. The key to the solution of the problem of participation in its ontological dimension lies for Klein in the realization that "the understanding of the special κοινόν [common thing] character of the definite amount is of crucial importance for the solution of the fundamental Platonic problem proper to the 'community of the kinds' (κοινωνία τῶν εἰδῶν)," a problem whose solution also provides the foundations for the mode of being of ἀριθμοί, their kinds, and, finally, for "not only the inner articulation of the realm of ideas but every possible articulation, every possible division and conjunction—in short, all *counting*" (88–89/92).

§ 70. The Curious Kind of Κοινωνία Manifest in Ἀριθμοί

Klein's reactivation of the unwritten Platonic theory of ἀριθμοὶ εἰδητικοί is of necessity guided by Aristotle's reports about it, which provide the basis for Klein's desedimentation of the "veiled way" (81/82) in which various dialogs, but especially the *Sophist*, are engaged "with the curious kind of κοινωνία [community] which shows itself in definite amounts." According to Klein, the κοινωνία at issue in ἀριθμοί concerns the following: that "*every definite amount of things belongs to these things only in respect to their community, while each single thing taken by itself is one*" (80/81). The kind of community here is distinguished from that of "a property which belongs to several things in common," because, in this case, the property must also be attributed to each single one of them, so that a κοινόν [common thing] is something 'which belongs to *both* {in this case to hearing – ἀκοή and sight – ὄψις} in common, as well as to *either* in its own right' ([*Hippias Major* 300 A–302 B] – cf. *Theaetetus* 185 A)" (*GMTOA*, 79/79–80).

Klein locates in Plato's dialog *Hippias Major* a discussion of both of these kinds of κοινόν. Regarding the property kind of common thing, Hippias "refers to the fact (300 E–301 A) that when something is said of him, Hippias, and of Socrates as holding for *both*, for instance, that 'we are both' (ἀμφότεροί ἐσμεν) just, healthy, wounded, golden, silver, etc., then it is 'en-

71. Klein writes to his friend Gerhard Krüger in 1932, "I believe—however hyperbolic this may sound—that I have 'solved' the μέθεξις problem, I mean in Plato's sense" ("Selected Letters from Jacob Klein to Gerhard Krüger, 1929–1933," 321).

tirely necessary' (μεγάλη ἀνάγκη) that each of these properties should also belong to *each one*" (79/80). By contrast, Socrates points out with respect to the first kind of κοινόν that "'Each of us is one, but that very thing which *each* of us is, *both* of us are *not*; for we are not one but *two*' (301 D)" (80/81). Consequently, "Socrates and Hippias are *both* together *two*, yet *each* of them is not two, but only *one*; and conversely, what *each* of them is, namely *one*, that both together they are *not*" (80/80–81). Moreover, "It follows directly from this that *both* together make an 'even' definite amount, while *each* of them taken separately is 'odd' (cf. *Phaedo* 103 ff.)" (*GMTOA*, 80/81). In addition to "'each' (ἑκάτερον) and 'both' (ἀμφότερον), certain 'irrational' magnitudes which when added make one 'rational' magnitude 'and numerous other such things' (καί ἄλλα μυρία τοιαῦτα) fall under the domain of κοινά which Socrates has in mind." Klein concludes, "We can easily see that this domain can be defined only within the mathematical realm and that definite amounts, above all, have this curious κοινόν character."

§ 71. The Κοινωνία Exemplified by Ἀριθμοί as the Key to Solving the Problem of Μέθεξις

On Klein's view, then, "There are two different kinds of κοινόν, of which one is represented by the 'beautiful' (καλόν), the 'just' (δίκαιον), etc., while the other can be shown to exist within the realm of quantity." The latter kind of κοινόν "is signaled by Plato with the formulaic phrase: 'each {is} one but both {are} two' (*Hippias Major* 300 ff.; *Republic* 524 B; *Theaetetus* 146 E, 185 A, B, 203 D ff. . . . cf. also *Republic* 476 A, 479 C, 583 E and *Phaedo* 96 E–97 A; 101 B, C; furthermore *Parmenides* 143 C, D and *Sophist* 243 D, E; 250 A–D)" (*GMTOA*, 81/81–82). Klein holds that "Plato himself relates this state of affairs to the problem of μέθεξις," to the problem of "how it is possible that one idea in its unity and wholeness is 'distributed' over many things which 'partake' in it" (79/80). Klein identifies two dimensions to this problem: 1) the "dianoetic methexis problem" (80/80), which is exemplified in the distribution of the unity of ἀριθμός over Socrates and Hippias in the example above, and 2) the "ontological *methexis* problem" (81/82), which concerns "the relation of an idea of a higher order to the ideas under it, of a 'genus' to its 'species'" (80/80). Indeed, he holds that "as the elementary form of the problem of the one and the many is to the dianoetic *methexis* problem, so is the latter to the ontological problem of the 'community' of ideas." And, as we have indicated, the second kind of κοινόν that Klein identifies, the kind exemplified by ἀριθμοί, "is of crucial importance for the solution of the fundamental Platonic problem of the 'community of the kinds' (κοινωνία τῶν εἰδῶν)" (81/82).

Klein holds that the *Sophist* "brings this problem [i.e., the community of the kinds] to the fore," and thereby places "under examination . . . nothing but the special constitution of the κοινόν of which the *Hippias Major* speaks" (85/87). Guided by Aristotle's reports about Plato's theory of ἀριθμοὶ εἰδητικοί, Klein desediments "[t]he notion of an 'arithmetic' structure of the realm of ideas" (87/89), which "permits a solution of the ontological *methexis* problem." On the basis of this solution, an account is yielded of the mode of being proper to ἀριθμοί and their kinds that, given the fact that "the *aporia* of 'being' is here [in the *Sophist*] left unresolved" (85/88), is still nevertheless able to point out that "the διάνοια . . . at the end of its 'dialectical' activity, [must] come to see that the 'conjunction' of *opposites* is in truth the 'co-existence' of *elements other in kind*" (92/96).

§ 72. The Κοινωνία Exemplified by Ἀριθμοί Contains the Clue to the "Mixing" of Being and Non-being in the Image

Klein argues that the *Sophist* deals with the fundamental fact that "the difficulty of the problem of 'being' has an internal connection with the *aporia* of 'non-being'" (84/86). Thus, the question about being "is in itself twofold," insofar as "At bottom we are dealing with *one* difficulty: 'being and non-being have equal parts in this quandary' ([*Sophist*] 250 E)." Moreover, for Klein, the quandary in question concerns the "mode of being of the 'image'" (*GMTOA*, 81/83), which "can only 'be' if 'non-being' and 'being' can '*mix*' with one another" (84/86). Hence, "The main task of the *Sophist* is that of exhibiting the foundations of the 'possibility of being' of a sophist as identical with the ultimate foundation of every possible articulation of being itself" (81/82). This possibility is rooted in "The fact that everything which is can be 'duplicated' by an image (cf. *Timaeus* 52 C), an image which is, in some enigmatic way, precisely *not* that which it presents [*darstellt*], so that it is at once *this* being and '*another*,' is ultimately founded on the 'mirror-like quality' of being itself." Consequently, Klein writes that "This primal character of being is . . . the effect of the 'twofold as such,' the 'indeterminate dyad' (ἀόριστος δυάς)." To anticipate the conclusion Klein draws concerning the role of ἀριθμοὶ εἰδητικοί in the clarification of being—so far, that is, as this is possible given the limits of the λόγος—this twofold structure of being is exemplified by "the first 'eidetic definite amount,'" which is "the eidetic 'two': it presents the γένος of '*being*' *as such*" (90/93).

On Klein's view, the sophist presents, "[w]ithin human existence" (81/82–83), the "perfect 'imaginal' embodiment" (81/83) of the ἀόριστος δυάς (indeterminate dyad). Thus, "Whether or not the sophist's being is a pos-

sible one must then, in the final analysis, be considered to depend on showing that the ἀόριστος δυάς is the ἀρχή of all duality and thus of all multiplicity." This means that to understand the sophist, "the primal phenomenon of 'imageability' . . . must occupy the center of inquiry." This phenomenon initially manifests itself in the sophist's "claim 'to know everything' (πάντα ἐπίστασθαι)" (81–82/83), which must be taken seriously if the enigma of the sophist and therefore the image is to be unraveled, "even if such a claim can only be made playfully—as Theaetetus says [to the Stranger]: 'You are jesting somehow' (Παιδιὰν λέγεις τινά – [*Sophist*] 234 A)" (82/83). Indeed,

> to do justice to this claim, we must understand it as the highest form of "play," namely as "*imitative play*" (234 B; cf. 231 A), for the whole activity of the sophist is to be understood as an "imitation of reasonable speech" (μίμησις . . . περί τοὺς λόγους), made "through the ears" (234 C), analogous to the activity of the painter (cf. *Protagoras* 312 C) who can, with the aid of color, *produce* "images and namesakes of being" (μιμήματα καὶ ὁμώνυμα τῶν ὄντων) for the eyes, and who can thus simulate a whole world (234 B; cf. *Statesman* 277 C and also 288 C).

The question must be asked whether the sophist's "imitation is a 'true' or only an 'apparent' one, i.e., whether it corresponds to the relations of similarity in geometry where the "proportions" of the originals are always preserved, or to those of the 'theory of perspective' (ὀπτική) or 'scene painting' (σκηνογραφία) where they are distorted according to the laws of perspective" (82/83–84). Thus, here, "within the realm of μίμησις, of 'image making' (εἰδωλοποιική), the opposition of being (εἶναι) and seeming (δοκεῖν – [*Sophist*] 236 A–C)" emerges, and the question of whether the sophist's imitation has to do with one or the other kind of μίμησις "is endlessly hard to decide because it begins by taking two things for granted: a 'yes' and a 'no,' a 'being' (ὄν) and a 'non-being' (μὴ ὄν)" (*GMTOA*, 82/84). The difficulty here is rooted in the equivalence of this question with the question "about the 'imageability' of being as such," which means that "this is the ultimate question to which that concerning the possibility of the sophist's being leads." Hence, Klein claims that when the sophist asks about "the enigmatic being quality of the 'image' (εἴδωλον) which spans ([*Sophist*] 239 D–240 A) 'likeness' (εἰκών) and 'mere appearance' (φάντασμα), [he] mirrors himself within himself, as it were, in an unlimited image." Indeed, "That he demands an answer such as comes from 'reasonable speech' (ἐκ τῶν λόγων) marks him, to crown it all, as the highest 'imitator'—for is not this just the demand of the 'philosopher,' of Socrates, who once entered on the 'second-best sailing' (δεύτερος πλοῦς) by taking refuge in the λόγοι (*Phaedo* 99 D–E; cf. *Statesman* 285 E–286 A) and who now plays the silent listener?" (*GMTOA*, 82–83/84).

According to Klein, "the ability to determine the being quality of an 'image' depends on the solution of the problem concerning the 'non-,' for in the image as such 'being' and '*non*-being' are inextricably intertwined ([*Sophist*] 240 A–C)" (*GMTOA*, 83/84). Moreover, "characteristically the presentation of the 'first and greatest quandary' (μεγίστη καὶ πρώτη ἀπορία – [*Sophist*] 238 A 2; D 1), which concerns non-being, already indicates the close connection between 'speaking' (λέγειν) or 'thinking' (διανοεῖσθια) the μὴ ὄν [non-being] on the one hand and the possibility of counting, i.e., the existence of ἀριθμοί (238 A–239 B) on the other" (*GMTOA*, 83/84–85). For Klein this is apparent in the following exchange between the Stranger and Theaetetus: "'Then we posit every definite amount whatsoever as belonging to being?'" (83/85), to which Theatetus replies: "'At least if there is anything to be posited as being at all.'" So Klein claims that, for Plato, "what is 'countable' is always understood as 'being,' and 'being' is always understood as 'countable.'" What is is always spoken about "either in the 'singular' or in the 'plural' (leaving apart the 'dual'): even *one* thing is only 'one' among *many* things." Moreover,

> the direct connection between . . . "thinking" (διανοεῖσθαι) and "accounting for and counting" (λογίζεσθαι καὶ ἀριθμεῖν) becomes especially visible when we turn toward non-being or, going further, make "non-being" itself the object of study, for even when we speak of the μὴ ὄν and μὴ ὄντα, of non-being and non-beings, we articulate even that which defies all articulation, namely—nothing!

Klein holds that this is the reason why the "Eleatic Stranger—'as ever, so also now' (καὶ γὰρ πάλαι καὶ τὰ νῦν)—declares himself no match for 'non-being.'" Klein therefore finds it of the utmost significance that the Stranger "pretends to expect of his interlocutor, the young Theaetetus, 'straight speaking about non-being,'" as the latter, not only because of his youth but also because he is a mathematician, is "incompetent . . . for all time to deal with the problem of the μὴ ὄν within his own realm." Nevertheless, Klein avers that because Theaetetus is a mathematician he "yet holds in his hand, as it were, the keys to its solution." For "In truth, neither of the two *by themselves* ('forget about you and me' – [*Sophist*] 239 C) but only *both together* ('at the same time from the mathematical side and from universal reasonings' – Aristotle, *Metaphysics* M 8, 1084 b 24 f.) can approach a solution" (*GMTOA*, 83/85–86).

§ 73. Partial Clarification of the Aporia of Being and Non-being Holds the Key to the "Arithmetical" Structure of the Νοητόν's Mode of Being

As we have indicated, the key to the approach to a solution to the problem of non-being is rooted for Klein in the attention Plato pays to the "ever-same *apo-*

ria" (84/86) that is inseparable from the *κοινόν* manifested by ἀριθμοί. When this aporia is formulated by both the Stranger (i.e., the philosopher, who approaches it from the side of "universal reasoning") and Theaetetus (i.e., the mathematician, who approaches it from the "mathematical" side), a partial clarification of the image's mode of being is grasped. And, with this, the ἀριθμός structure of the *γένη* (kinds) that allows being and non-being to mix is indicated, as is the ἀριθμός structure of the εἴδη proper to the odd and the even that underlie "each single definite amount of pure units" (89/92), that is, that underlie each noetic ἀριθμός. In other words, Klein maintains that the partial clarification of the aporia of non-being holds the key to the notion that an "arithmetical" structure (i.e., the ἀριθμοὶ εἰδητικοί) belongs to the mode of being of the *νοητόν* as such, including the mode of being of both the εἴδη and the unity of the pure monads in ἀριθμοί, each of which eludes the mathematician's theoretical thinking. The mode of being of the εἴδη of ἀριθμοί eludes such thinking because it can get beyond neither the aporia of its supposition that the *unitary* qualities of oddness and evenness are distributed among an unlimited *multitude* of ἀριθμοί nor the aporia of its supposition that the *one* is always a one among *many*. Both of these presuppositions thus prevent mathematical thinking—"for all time"—from coming face to face with the mode of being proper to either the εἴδη of ἀριθμοί or the "one itself" presupposed by its thinking.

If an image—which, as we have seen, *is* enigmatically *not* what it presents—is to "be" at all, then 'being' and 'non-being' must somehow "mix." Klein argues that this "holds just as much for the being of 'seeming,' of mere 'appearance,' of 'lie,' of the 'false,' and of 'error'" (84/86). The problem of non-being is therefore inseparable from that of being, in the sense that

> in asking about "non-being" at all, *we are already directed by the question "about being"* (περὶ οὐσίας – [*Sophist*] 251 C–D; "about that first and greatest founder" – περὶ τοῦ μεγίστου τε καὶ ἀρχηγοῦ πρώτου – 243 D; cf. also the traditional subtitle of the dialogue: "About Being" – Περὶ τοῦ ὄντος), just as we must of necessity come upon the "philosopher" in our search after the "sophist" (cf. 231 A–B; 253 C).

The converse also holds, because of the internal connection previously mentioned between being and the aporia of non-being. This is the reason why Klein maintains that for Plato there is only "*one* difficulty," one question, "which is in itself twofold." Indeed, he holds that Plato avers that "This is what the 'ancients' as well as the 'moderns' have failed to recognize." In missing this, they "are unable even so much as to see a difficulty which arises in all their solutions of the problem of being." Of "those who allow the 'whole' to be more than only one, that is, those who reduce everything to *two* basic constituents, as for instance the 'warm' and the 'cold,'" it must be asked: "'But what then are

you addressing in both *when you say that both and also each is*?' ([*Sophist*] 243 D–E)" (84/86–87). If "this 'being' [is] 'a third thing besides those two,'" then "there would be—in contradiction to the thesis—*three* basic constituents" (84/87). Moreover, "neither can 'being' coincide with *one of both*, for then only *this one* could be said to be, and consequently there would 'be' only this *one*." Finally, "if they *are* only 'together' then they 'are' precisely only *together*," which is how the stranger puts it in a question posed to Theaetetus: "'But do you then want to call *both together being*?'" Klein stresses, however, that "in the present case this would mean: the 'warm' and the 'cold' would no longer *be* by themselves separately, but there would be clearly only *one*, something 'tepid,' or more generally, a 'middle thing.'"

What is at issue here in Plato's question regarding the difficulty of those who allow the whole to be more than one, on Klein's view, is the "special constitution of the κοινόν" (85/87) identified in the *Hippias Major*. However, at this, "the lowest level of ontological deliberation," the state of affairs "That 'both together' (ἄμφω) are indeed 'one' and yet remain 'two' (cf. *Parmenides* 143 C–D)" cannot be exhibited, "because the two 'substrates,' the 'warm' and the 'cold' themselves can be mixed." Nevertheless, when the discussion in question shifts such that the problem of the κοινόν comes to concern "the relation of 'rest' (στάσις) and 'change' (κίνησις)" as the two aspects of a whole, the state of affairs identified in the *Hippias Major* is indeed exhibited. For "It turns out ([*Sophist*] 249 D) that 'change' *as well as* 'rest' must 'both together' (συναμφό - τερα) be assigned to 'being.'" Thus, even though "κίνησις and στάσις are 'most opposite to one another' (ἐναντιώτατα ἀλλήλοις – 250 A 8 f.) and therefore completely uncombinable; yet *both* and *each* of them '*is*': 'Stranger: And do you say that *both* and *each of these* alike *are*? Theaetetus: Yes, indeed I do.'" Just as in the case of the problems unrecognized by the ancients when they allow the whole to be twofold and therefore more than one, for instance, in the example above of the hot and the cold, in the case at hand "only this much can be ascertained (250 B 2–D 4): to say 'change' and 'rest' 'both and each' (ἀμφό - τερα καὶ ἑκάτερον) 'are' cannot mean that 'being' coincides with one of them." Moreover, "it is just as impossible, if they are 'both {together said} to be' (εἶναι ἀμφότερα), to posit their 'being' as a 'third' thing beside them' (250 B 7) or 'outside both of them' (250 D 2), 'taking both together and then disregarding them to look at the community of their being'" (*GMTOA*, 86/ 88). This is impossible, for if 'being' is understood to "be 'according to its own nature' (κατὰ τὴν αὑτοῦ φύσιν) and therefore *neither* rest *nor* change," then 'being' "would be precisely not change and rest 'together' (συναμφότερον)," and this "appears to be the 'most impossible of all things' (πάντων ἀδυνατώτατον)—for what is not at rest is surely changing, and what is not changing rests!"

§ 74. The Κοινωνία among Ὄν, Κίνησις, and Στάσις Composes the Relationship of Being and Non-being

For Klein, then, the *Sophist* does not resolve the aporia of being but rather formulates it in a way that allows "the problem concerning 'both together—each of both—neither of the two' to come to the fore." He maintains that it is this formulation that permits the realization that "The strange κοινωνία among ὄν, κίνησις, and στάσις is none other than that between 'being' and 'non-being'" (85/88). To show this, Klein first traces the course of the conversation in the *Sophist* as it is initially broadened beyond the topic of the relationship between change and rest to include the problem of the one and the many. The formulation of the latter problem in connection with the question of the "κοινωνία τῶν εἰδῶν (or τῶν γενῶν)" proves to be crucial for his account of the "Platonic theory of the ἀριθμοὶ εἰδητικοί" (88/91) as the solution to the ontological *methexis* problem. Primarily on the basis of Aristotle's reports in the *Metaphysics* about the ἀριθμοὶ εἰδητικοί in Plato's unwritten teachings, Klein shows not only that they are intended to make intelligible the inner articulation of the community of εἴδη and γένη, but also—as has already been suggested—all articulation, including every possible division and conjunction, that is, counting. Finally, Klein shows that the *Sophist*'s investigation of the incompatibility of κίνησις and στάσις "permits the διάνοια to understand the 'duplicity' of 'being,' namely, that it means not only ever self-identical 'rest,' but also conjointly 'change,' and that in this manner alone, the 'imaging' [*Ab-bildung*] of being in re-cognition [*Erkennen*], that is, in knowing [γιγνώσκειν] and being known [γιγνώσκεσθαι] – 248 B ff.), and, beyond this, all image making (cf. *Cratylus* 439 E–340 A)" (*GMTOA*, 91–92/95–96) is possible.

For Klein, then, "the relation of στάσις to κίνησις forms the nucleus of all the subsequent discussion [in the *Sophist*]" (85/88). The "universal extension of the original problem of the 'two' aspects, i.e., of the meaning of 'at one and the same time' (ἅμα) or of 'both' (ἄμφω, cf. *Phaedo* 96 E–97 A)" (86/88) leads to the question of "how the 'many' are conjoined to form the 'unity' of any being." At issue in this question "is that 'gift of the gods to human beings' (*Philebus* 16 C), namely the 'astounding' (ibid., 14 C) assertion that each thing is 'one and many' (ἓν καὶ πολλά) 'at once' (ἅμα)" (85–86/88). Initially raised "in the most universal terms, namely in reference to any being," it is next "transformed into the narrower *ontological methexis* problem, into the question of the κοινωνία τῶν εἰδῶν (or τῶν γενῶν)" (86/88). Three possibilities, "and no more" (86/89), emerge regarding this κοινωνία, only one of which is realizable. These possibilities are: "(1) There is no κοινωνία at all. (2) All the εἴδη are mutually related. (3) There is partial κοινωνία, in the sense

that some εἴδη can 'mix' with each other but others not." The first two possibilities "are not in fact realizable," which means that "the third alone, of necessity, remains (most explicitly: [*Sophist*] 252 E; cf. also 256 C)." Klein holds then that "*the very formulation of this possibility indicates the ἀριθμός structure of the γένη*; for what is it but the division of the whole realm of εἴδη into single groups or assemblages such that each εἶδος, which presents a unique eidetic 'unit' (ἑνάς), i.e., a μονάς (*Philebus* 15 A–B), can be 'thrown together' with the other ideas of the same assemblage, but not with the ideas of other assemblages?"

§ 75. The Contrast between Ἀριθμοὶ Μαθηματικοί and Ἀριθμοὶ Εἰδητικοί

Klein concludes from this that "The εἴδη . . . form assemblages of monads, i.e., ἀριθμοί of a peculiar kind." Unlike the mathematical assemblages of monads, which are, "as we have seen, completely similar and can therefore all be 'thrown together' (Aristotle, *Metaphysics* M 7, 1081 a 5 f.: 'capable of being thrown together and indifferent' – συμβληταὶ καὶ ἀδιάφοροι)," "the assemblages of εἴδη, the ἀριθμοὶ εἰδητικοί, cannot enter into [just] any 'community' with one another." This is the case for Klein because "Their 'monads' are *all* of a different kind and can be brought 'together' only 'partially,' namely only insofar as they happen to belong to one and the same assemblage, whereas insofar as they are 'entirely bounded off' from one another (πάντῃ διωρισμέναι – *Sophist* 253 D 9) they are 'incapable of being thrown together,' incomparable (ἀσύμβλητοι)." Consequently, "The monads which constitute an 'eidetic definite amount,' i.e., an assemblage of ideas, are nothing but the *conjunction* of εἴδη which belong together" (*GMTOA*, 87/89–90). Moreover,

> They belong together because they belong to one and the same εἶδος of a higher order, namely to a "genus," a γένος (as, for instance, "human being," "horse," "dog," etc., partake in "animal") *without "partioning" it among the* (finitely) *many* εἴδη *and without losing their indivisible unity only if the* γένος itself exhibits the mode of being of an ἀριθμός. Only the ἀριθμός structure with its special κοινόν character is able to guarantee the essential traits of the community of εἴδη demanded by dialectic; the indivisibility of the single "monads" which form the ἀριθμός assemblage, the limitedness of this assemblage of monads as expressed in the joining of many monads into one assemblage, i.e., into one idea, and *the untouchable integrity of this higher idea as well*. (87/90)[72]

72. Klein writes in his 1932 letter to Krüger (ibid.): "There is in Plato a kind of 'formula,' which runs: ἀμφοτέρα δύο, ἑκάτερον δὲ ἕν (e.g., *Republic* 524b, *Theaet.* 185b). That is: determinate numbers—*and only these*—have the peculiarity that they apply to *several* things *together*, but to no individual thing of this group, of this ἀριθμός—and vice versa. If we have five apples,

Klein therefore concludes that for Plato "What the single εἴδη have 'in common' is theirs only *in their community* and is not something which is to be found '*beside*' and '*outside*' (παρά and ἐκτός) them (cf. also *Philebus* 18 C–D)."

Thus, unlike the unity and determinacy of the noetic ἀριθμοί that the διάνοια supposes to underlie its praxis of counting, which have their basis in the delimitation of the field of unlimitedly many monads as a consequence of the special kinds to which each *definite amount* belongs (see § 57 above), "the unity and determinacy of the ἀριθμός assemblage is here rooted in the content of the idea (ἰδέα)." On Klein's view, the content of the idea is something that "the λόγος reaches in its characteristic activity of uncovering foundations 'analytically.'"[73] This means that a "special kind of definite amount [e.g., 'odd' or 'even'] with a particular quality [e.g., 'oddness' or 'evenness'] is not needed in this realm, as it was among the dianoetic definite amounts . . . , to provide a foundation for this unity." Each whole in question is determined by the ἰδέα whose content provides the unity of the parts comprising the eidetic ἀριθμός assemblage, and therefore—in contrast with the delimitation provided by the kinds proper to mathematical ἀριθμοί—is what is responsible for the delimitation of the unity of an ἀριθμὸς εἰδητικός. Moreover, "each *eidetic definite amount is*, by virtue of its eidetic character, *unique in kind*, just as each of its 'monads' has not only unity, but also uniqueness." These unitary but unique monads stand in "contrast to the unlimitedly many homogeneous monads of the mathematical region of definite amounts, which can be assembled arbitrarily into specific definite amounts as often as one likes," which means that "it is impossible that the *kinds* of definite amounts corresponding to those of the dianoetic realm" should exist in the realm of eidetic definite amounts.

'five' *only* applies for them *all together*, but not for *each* of these apples. Conversely, each individual apple is certainly 'one,' but all together they are not 'one,' but rather 'five.' This is elaborated at length in *Hippias Major* (300a–302b) and there too linked intimately with the μέθεξις problem. What, then, does this state of affairs mean for the great problem of the κοινωνία of εἴδη, which alone is the real problem of μέθεξις? Well, simply this: if, e.g., 'dog,' 'horse,' 'stag,' 'lion,' etc. (a *finite* amount of such εἴδη) relate to the γένος *in the same manner* as the monads of an ἀριθμός to this ἀριθμός as their integrated totality, then the γένος does *not* 'divide' itself into the εἴδη, just as 'five' does not divide itself into the five monads! Thus: the ἀριθμὸς εἰδητικός is the ontological possibility of the ἰδέα as a whole that stands in many relations in *kind* to other εἴδη and yet is indivisible. The ἀριθμὸς εἰδητικός is nothing other than the Platonic solution of the μέθεξις question. Hence the obstinacy and intensity of Aristotle's attack! Of course, such a solution is possible only on the basis of the 'natural' concept of ἀριθμός, as you well know . . ."

73. See § 102, where Klein's articulation of the "analytic" method in ancient mathematics is discussed, and esp. n. 134, where this method is discussed in connection with the dialectical method manifest in Plato's dialogs.

Klein makes use of Aristotle's report that "there are three kinds of ἀριθμοί" (88/91) to clarify the similarities and differences between sensible, mathematical, and eidetic definite amounts in Plato. Ἀριθμὸς αἰσθητός (a sensible definite amount) and ἀριθμὸς μαθηματικός (a mathematical definite amount) are alike, in "that they occur in *multitudes* and are of the *same kind* (Aristotle, *Metaphysics* B 6, 1002 b 15 f.: '{Mathematical objects} differ not at all in being many and of the same kind')." They are different, of course, in that the former is not everlasting and unchanging, while the latter is precisely both. Ἀριθμὸς εἰδητικός (an eidetic definite amount) and ἀριθμὸς μαθηματικός are alike in that they are both everlasting and unchanging, but unlike in that "each εἶδος" that belongs to the multitude unified by an ἀριθμὸς εἰδητικός "is, by contrast [with each monad that belongs to the multitude unified by an ἀριθμὸς μαθηματικός], irreproducible and truly *one* (*Metaphysics* A 6, 987 b 15 ff.: 'Mathematical objects differ from objects of sense in being everlasting and unchanged, from εἴδη, on the other hand, in being many and alike, while an εἶδος is each by itself one only')." This clarification allows Klein to articulate the three kinds of definite amounts as follows. Sensible or "aisthetic definite amounts present nothing other than precisely the sensibly perceivable[74] *things themselves* present to αἴσθησις in this definite amount." By contrast, "mathematical definite amounts form an independent domain of objects which the διάνοια reaches by seeing the exemplary fulfillment of its own activity in 'reckoning ⟨i.e., account-giving⟩ and counting' (λογίζεσθαι καὶ ἀριθμεῖν)." And the "eidetic definite amount indicates the *mode of being of the νοητόν as such—it defines the εἶδος ontologically as a being which has multiple relations to other εἴδη in accordance with their particular nature and which is nevertheless altogether indivisible*."

§ 76. The Foundational Function of Ἀριθμοὶ Εἰδητικοί

Besides Aristotle's polemic against what Klein terms "the Platonic theory of the ἀριθμοὶ εἰδητικοί," Klein's desedimentation and reactivation of the peculiar ἀριθμός structure of their κοινωνία takes its clue from the various reports about Plato's "lecture *On the Good* ('Περὶ τ᾽ ἀγαθοῦ')." Both the latter and repeated references in Aristotle (Klein cites *Metaphysics* Λ 8, 1073 a 20, M 8, 1084 a 12 ff., 25 ff., and *Physics* Γ 6, 206 b 32–33 at *GMTOA*, 88/92) indicate that Plato "limited the realm of eidetic definite amounts to *ten*" (88/91). However, Klein thinks it "is questionable whether Plato sketched out more

74. The phrase *sinnlich wahrnehmbaren* (sensuously perceivable) is not translated in the English translation.

than the general framework of the theory." Nevertheless, Klein's desedimentation of the Greek ἀριθμός concept, together with his reactivation of the basic problematic of Greek theoretical arithmetic as dealing not with ἀριθμοί but with their delimitation by their εἴδη (see § 57 above), permits him to reactivate in turn 1) the "separate, independent and 'absolute' being" (89/93) of ἀριθμοὶ εἰδητικοί "in relation not only to *αἰσθητά* but also to the 'pure' mathematical definite amounts," 2) the grounding of the "'sequence' of monadic definite amounts" in the "original *order* of *eidetic* definite amounts," and, finally, 3) the ground of the ordering from "higher" to "lower" of the *γένη* themselves that comprise the ἀριθμοὶ εἰδητικοί.

Regarding (1), Klein holds that "the eidetic definite amounts might, in their foundational function, be most easily compared to the Pythagorean 'roots' (*πυθμένες*) of the mathematical realm of definite amounts" (88/92; see § 62 above). That is because "the ἀριθμοὶ εἰδητικοί are intended to make intelligible not only the inner articulation of the realm of ideas but every possible articulation, every possible division and conjunction—in short, all *counting*" (88–89/92). On his view, Aristotle's polemic against Plato's theory "exhibits the many contradictions which must arise from the transfer of the *universal* character of the countable as such to the distinct nature of each εἴδη" (88/92). However, "*it is precisely the unmathematical employment of the ἀριθμός-structure which is essential*" for Plato on Klein's view. With regard to the εἴδη of ἀριθμοί, theoretical arithmetic and logistic are unable to provide "the true grounds for the existence of such εἴδη of definite amounts and of each single definite amount of pure units"[75] (89/92). This is the case, because "While the arithmetician and the logistician 'suppose' certain εἴδη to 'underlie' the unlimitedly many monads of his domain in order to have 'hypothetical' grounds on which they may be comprehended into single monadic assemblages," only the dialectician is able actually *to give* these grounds by pointing out how "the ἀριθμοὶ εἰδητικοί make something of the nature of definite amounts possible in this our world." Thus, on Klein's view, there can "be arbitrarily many definite amounts, such as hexads or decads, in the realm of 'pure' units as well as in the realm of sensibles," "[o]nly because there are εἴδη which belong together, *whose community in each case forms a 'kinship'* which

75. Stanley Rosen's critique of Klein's interpretation of the *μέγιστα γένη* in the *Sophist* "on the basis of Aristotle's contention that these forms are eidetic numbers" (Stanley Rosen, *Plato's Sophist* [New Haven, Conn.: Yale University Press, 1983], 48–57, here 48) overlooks Klein's reactivation of this aspect of the mode of being proper to mathematical ἀριθμοί, an aspect, as we have seen, that Klein maintains Greek theoretical arithmetic attempts to address with its hypothetical method. Because the presentation of this aspect, or, more precisely, of this problem, forms the indispensable context for Klein's account of the function of ἀριθμοὶ

εἰδητικοί to provide the foundation for mathematical ἀριθμοί, as well as for his account of them as a solution to the ontological *methexis* problem, Rosen's claim that Klein's "attempt to understand Platonic dialectic by means of an arithmetical paradigm" (49) "is unsuccessful on internal technical grounds" needs to be reevaluated.

Rosen argues that Klein adopts the paradigm "of the combination of numbers" (50) in order to resolve "the problem of the combination of the elements of intelligible structure," e.g., the problem of how the "Formal elements like *man* and *horse* must be able to combine with an element like *animal*, without dissolving the integrity of each element in the combination." He goes on to articulate this paradigm as follows: "Thus, for example, the numbers *two* and *three* (italics designate 'the numbers themselves') combine into the number *five* with no sacrifice of their independent identities, but with the addition of new properties (such as odd and even) which they may lack independently." In Klein's discussion of mathematical ἀριθμοί, however, no such paradigm can be found. Specifically, as we have seen, for him the Greek theoretical arithmetic that formed the context for Plato's philosophy is concerned with the problem of accounting for the delimitation of the unlimited field of noetic monads by investigating the εἴδη of ἀριθμοί. Moreover, for Klein, as we shall see below (§ 88), prior to Aristotle these εἴδη were *not* understood as the "properties" of ἀριθμοί, but, on the contrary, as their "kinds." As such, they were *supposed to* account for the delimitation of each ἀριθμός as a determinate amount of monads, e.g., two, three, etc. Hence, one errs in calling them "properties" of ἀριθμοί, since, strictly speaking, the specific ἀριθμοί were supposed to come into being only as a consequence of the delimiting characteristics accomplished by their εἴδη. As we have also seen, the reasons for the unsatisfactoriness of this mathematical *supposition* are, according to Klein, alluded to in Plato's dialogs, and amount to the claim that the method of hypothesis—as employed by theoretical arithmetic—cannot account for the "one and many" mode of being of both the εἴδη and the monads appealed to (see § 69 above) in its investigation of the even and the odd "with reference to *how much* either happens to be" (see § 56 above).

When Rosen's appeal to the numbers themselves putatively at issue in Klein's arithmetical paradigm is considered with respect to Klein's understanding of theoretical arithmetic presented above, it is necessary to challenge the appositeness of his critique. Rosen's claim, that what is at issue here is the ability of "any arithmetical number" (52) to "combine with any other arithmetical number to form a community in which none of the participants loses its identity (something that is not true for empirical or practical numbers)," does not get at what Klein reactivates as the distinctive feature of the κοινωνία in the one and many structure of ἀριθμοί. Namely, that the peculiar unity of an ἀριθμός, as an assemblage of just so many units (e.g., 'two' or 'three'), is something that only applies to the aggregate of units as a whole. That is to say, e.g., that the ἀριθμός of "two" chickens or the ἀριθμός of "two" monads—contra Rosen, it makes no difference on Klein's account of this aspect of the problematic whether the units are aisthetic or noetic—involves a one and many structure, the unity of which, e.g., the 'two', manifestly does not belong to the individual units; thus, in the example at hand, it does not belong to the chickens and monads. Hence, for Klein, the combination of the ἀριθμοί 'two' and 'three' involves the addition of counted units, and not, as Rosen would have it, the combination of "the numbers themselves" such that "their independent identities" are preserved in the number *five* itself. For Klein, Rosen's notion of 'numbers in themselves' would be out of place in the consideration of the theoretical *arithmetic* that formed the context of Plato's dialogs.

Thus, when Rosen maintains that, "As Klein observes, the units of eidetic numbers, as distinguished from those of theoretical numbers like *eight* and *ten*, cannot form such a community" (52), it is evident that he is conflating two issues that must be kept distinct in order to achieve access to Klein's view of ἀριθμοὶ εἰδητικοί. Rosen thinks that because the units of ἀριθμοὶ εἰδητικοί "are not free to combine with one another indiscriminately," they cannot "form a community in which none of the participants loses its identity." However, for Klein the indiscriminate combination of the units that make up a mathematical ἀριθμός is distinct from the issue

must, due to the 'arithmetical' tie of its 'members' as an eidetic 'definite amount,'[76] be designated as *the six* or *the ten*." Moreover, only because of this "unmathematical" ἀριθμός structure "can definite amounts exhibit such definite, unifying kinds . . . as the 'even-times-even' or the 'triangular.'"[77]

§ 77. The Order of Ἀριθμοὶ Εἰδητικοί Provides the Foundation for Both the Sequence of Mathematical Ἀριθμοί and the Relation of Family Descent between Higher and Lower Γένη

Beyond providing the foundation for both the many sensible and pure ἀριθμοί and the εἴδη that delimit them, the ἀριθμοὶ εἰδητικοί, "in virtue of their distinctive nature," are "responsible for the *differences of genus and species* in things that makes them graspable in a *definite 'amount'*" (89/92–93). In addition, it is "through a 'distorted' imitation of ontological *methexis* (. . . cf. *Philebus* 16 C–

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of whether it is possible for the units in an ἀριθμὸς εἰδηκός to combine without losing their identity. On the one hand, there is the ἀριθμός structure of the "one and many" community that allows the discrete items that comprise the many to maintain their integrity, in the sense that, notwithstanding their unity as parts of the whole ἀριθμός, the whole proper to this ἀριθμός is something that manifestly *does not* characterize each of its parts taken singly. On the other hand, there is the difference between the homogeneity of the many monads that comprise a pure ἀριθμὸς μαθηματικός and the heterogeneity of those that characterize an ἀριθμὸς εἰδηκός. As we have seen from our discussion of Klein's account of the ἀριθμός structure, the first issue is distinct from the second, because *both* the many mathematical and the many eidetic monads *are* capable of "commingling" "without loss of identity ensuing for the constituent" elements. The second issue, which addresses the *difference* in the character of the communities brought about by the commingling made possible by the ἀριθμός structure of each, that is, the issue of the combinability of monads belonging to different ἀριθμοί that characterizes mathematical but not ἀριθμοὶ εἰδητικοί, is thus a difference that shows up for Klein *within* a common context. For, contrary to Rosen's claim, *both* mathematical *and* eidetic ἀριθμοί "form a community in which none of the participants loses its identity."

Rosen's judgment, then, that the question of whether "the version of a doctrine of eidetic numbers attributed to him [Plato] by Klein" (53) is "a genuine solution to the ontological *methexis* problem," is a question that "can be answered in the negative without any hesitation," must be rejected. The failure of the critique of Klein upon which it is based to keep distinct the issues detailed above, has as one consequence keeping open the possibility that Klein's version of a Platonic doctrine of ἀριθμοὶ εἰδητικοί may indeed be successful "on internal technical grounds." Because of the shortcoming in his critique, Rosen's claim that the authenticity of the thesis Klein attributes to Plato cannot be established, on the grounds that a "careful scrutiny of the passages cited by Klein from the dialogs shows no explicit statement at all" of it, is also in need of reassessment. For Rosen's failure to grasp the basics of Klein's thesis rules out his being able to recognize whether or not parts of Plato's dialogs do indeed present, as Klein maintains, evidence—albeit "in a veiled way"—of an engagement with the strange kind of κοινωνία that shows itself in ἀριθμοί.

76. The German *als eidetische "Anzahl"* (as an eidetic "definite amount") is not translated in the English translation.

77. See n. 62 above.

E; *Timaeus* 43)" (89/93)[78] that they are responsible "for the *unlimited* variety of things." It is this "foundational function" (89/93) that "guarantees" the separate, independent, and absolute being of the ἀριθμοὶ εἰδητικοί in relation to both sensible and pure mathematical ἀριθμοί. For Klein, then, "[w]hat the Pythagoreans undertook with respect to the world of sense in which they believed that all beings were comprised . . . Plato now undertakes to do with respect to the world opened up by the λόγος, the world of νοητά which has true being." This means that for Plato "the 'definite amount'-being of the νοητά signifies their ordered being, their τάξις." This τάξις, however, cannot be explained or otherwise accounted for by theoretical arithmetic or logistic, because for these sciences the mode of being of νοητά is hypothesized as "a completely homogeneous field" formed by mathematical monads (see § 59 above). The sequence of monadic ἀριθμοί, which by its very nature as a sequence is inseparable from the difference that composes each ἀριθμός qua its sequential ordering, transcends the hypothesized homogeneity of the monad field, and therefore cannot be accounted for by theoretical mathematics. But it can be "grounded in the original *order* of the eidetic definite amounts." Thus, Klein maintains that "Every eidetic definite amount is either superior or inferior in this order with respect to its 'neighbor.'" Indeed, because of this eidetic ordering, "a subsumption of all these definite amounts under one idea common to all, namely 'definite amount in general,' is quite impossible." This is because the order at issue here is determined precisely by the *difference* of each ἀριθμὸς εἰδητικός from its "neighbors," which *ipso facto* rules out a quality in common that is necessary for them to fall under a common idea. Klein quotes Aristotle in this context: "'In respect to things in which there is a prior and a posterior, what is in these cannot be something apart from them' (Aristotle, *Metaphysics* B 3, 999 a 6 f.)" (*GMTOA*, 89 n. 1/237 n. 100).

Klein's account of the function of ἀριθμοὶ εἰδητικοί to ground the sequence of monadic ἀριθμοί shows that their grounding function extends to an ordering of the γένη yielded by the partial κοινωνία τῶν εἰδῶν. What is ordered in this case are the relations among the incomparable assemblages of εἴδη whose unity and determinacy is rooted in the content proper to the εἶδος or ἰδέα of a higher order that unites them. Thus, he writes that "the relation

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78. Klein, at this point, does not elaborate precisely what is at issue in the "'distorted' imitation of ontological *methexis*," but instead refers to his text's initial and final discussions of the relationship between dianoetic and ontological *methexis*. The earlier discussion articulates the relationship between dianoetic and ontological *methexis* in terms of a "proportion" (see § 71 above), while the latter (see § 72 above) articulates the difference between the preservation of the proportions of the originals in the relations of similarity in geometry and the distortion of such proportions according to the laws of perspective. Regarding this final discussion, see § 83 below.

of 'family descent' between the higher and lower ideas corresponds to the *'genetic' order of the eidetic definite amounts*" (89–90/93). That the genetic order of ἀριθμοὶ εἰδητικοί "is genuinely Platonic can be seen, if not 'with complete clarity' yet clearly enough, from the *Sophist*" (90/93). Manifest for Klein in that dialogue's discussion of the "'greatest genera' (*μέγιστα γένη* – 254 C)," which occurs in the context of the goal "to grasp 'being' as well as 'non-being' in a manner suited to the present mode of examination" (90/94), is the beginning of this order. The ordered mode of being of ἀριθμοὶ εἰδητικοί responsible for the sequence of monadic ἀριθμοί is also shown to be, in turn, responsible for ordering—from higher to lower—the *γένη* proper to the distinct assemblages characteristic of the *κοινωνία τῶν εἰδῶν* in terms of their "familial descent." Klein shows how the *Sophist*'s discussion of the partial *κοινωνία τῶν εἰδῶν* points to "the eidetic 'two'" (90/93) as "the 'first' eidetic definite amount" in the genetic order of ἀριθμοὶ εἰδητικοί. As the 'first' ἀριθμὸς εἰδητικός, its *γένος* is the most "original and *comprehensive*," because the "'higher' the *γένος* . . . the less articulated the eidetic definite amount." As such, it "presents the *γένος* of *being as such*, which encompasses the two εἴδη 'rest' and 'change.'"

§ 78. The Inability of the *Λόγος* to "Count" the *Μέγιστα Γένη* Points to Its Limits and Simultaneously Presents the First Ἀριθμὸς Εἰδητικός

Klein shows all of this by bringing into relief how the *Sophist*'s investigation of the possibility of a partial *κοινωνία τῶν εἰδῶν* (or *τῶν γενῶν*) with respect to the *μέγιστα γένη* points to the failure of the "dianoetic-dialogic method" to "count" the *γένη* involved in the kinds of community that emerge from its investigation. After disclosing this failure, he is able to reactivate the "eidetic two," "which consists of *στάσις* and *κίνησις*," as the "paradigm" (92/96) for "'an arithmetic community' among eidetic monads which are not capable of 'being mixed' although they 'belong together.'"

Klein begins by showing how in its attempt to count the *γένη* of *στάσις*, *κίνησις*, and *ὄν*, "the *λόγος* cannot conclude the count with 'two' because it says that *στάσις* and *κίνησις* 'are' not only together but also 'singly'" (91/95). Because the mixing of *στάσις* and *κίνησις* has been ruled out and indeed stressed (see § 71 above), "both '*are*,' and from this follows the 'triad' of *στάσις*, *κίνησις*, and *ὄν* ([*Sophist* 254] D 12)" (91/94). However, Klein points out that earlier in the dialog "it has already been shown that the *ὄν* is *not* to be understood as a 'third thing beside' or 'outside these' (*τρίτον παρὰ ταῦτα* or *ἐκτὸς τούτων*), since this would lead to the 'most impossible thing

of all' (250 B–D)," namely, ὄν according to its nature excluding both *στάσις* and *κίνησις*. From this, Klein concludes that "In respect to ὄν, *κίνησις* and *στάσις*, the λόγος fails!"—"because it must count 'three' when *in truth* there are only 'two,' namely *στάσις* and *κίνησις*, which are 'each one' and 'both two'! (ἑκάτερον ἕν and ἀμφότερα δύο)" (91/94–95). Thus, unlike "the case of 'two mathematical monads,'" which the λόγος counts "each . . . by itself as *only one* and precisely *not as 'two*,'" such that the *διάνοια* understands their arithmetical *κοινωνία*, expressed by the ἀριθμός 'two', to pertain only to "both together" (i.e., the λόγος does not regard each of the monads encompassed by the ἀριθμός 'two' to be, *when taken singly*, 'two'), it is otherwise in the case of the "two" *γένη* of *στάσις* and *κίνησις*: for here the *διάνοια* understands their "arithmetical" *κοινωνία*, expressed by the *γένος* ὄν, to pertain to both together *as well as to each of them singly*. That is, the λόγος regards each of the *γένη* encompassed by ὄν—*στάσις* and *κίνησις*—to have ὄν, as well as both together to have it. Consequently, when the λόγος attempts to give an account of 'being' (ὄν), it "counts" the ὄν of *στάσις* and *κίνησις*, "both together," as "one" *γένος*, and takes *στάσις* singly as another "one," and, likewise, *κίνησις* is taken singly as still another "one"; that is, it counts "three" *γένη* where there are only "two."[79] Because this 'two', however, emphatically does not emerge from counting and therefore does not present a mathematical ἀριθμός, Klein takes its emergence to present an ἀριθμὸς εἰδητικός: specifically, to present the eidetic 'two', which—as we have noted—for him presents the "*γένος* of '*being*' *as such*" (90/93).

§ 79. Θάτερον as the "Twofold in General" Allows for the Articulation of Being and Non-being

Klein thus concludes that, for Plato, "[ὄ]ν, *κίνησις*, and *στάσις*, *in spite of their* '*arithmetical*' *κοινωνία*, *cannot be* '*counted*' *at all*" (91/95), and that it is "this that defines the 'failure' of the λόγος." Regarding the *κοινωνία* of the "three" *γένη* presupposed by the "dianoetic understanding," it is able to get "clear only

79. Klein writes that "This state of affairs is obscured for us by the 'symbolic' concept of number . . . , but it poses a nearly insurmountable difficulty for Eleatic, Pythagorean, and Platonic philosophy" (*GMTOA*, 91 n. 1/238 n. 102). This is because, as we saw in Chapter 12 above and shall see again below, the "symbolic" concept of number directly designates a concept and therefore only indirectly designates a determinate amount of items. Thus, the problem here is nearly unintelligible when it is approached from the conceptual level of the symbolic numbers that we take for granted. The problem of how to understand the "unity" of the items designated by their "definite amount" (i.e., ἀριθμός) (be it mathematical or eidetic) can only appear obscure on the basis of an understanding of 'number' that rules out, from the start, direct reference to such items that are inseparable from the Greek concept of ἀριθμός.

about this much, that each" of them, "insofar as it is 'itself' precisely what it is, and is grasped in its 'self-sameness,' is an 'other' than the 'two' others: 'Then each of them is other than the two, but the same with itself' (254 D 14 f.)." This consideration leads, according to Klein, to "the introduction of yet a further 'pair' [of γένη]: 'self-sameness' and 'otherness' (ταὐτόν and θάτερον)," which both "compounds" this "crucial quandary of the διάνοια" and brings with it "the solution which must suffice within the dianoetic realm." It compounds it insofar as this pair of γένη range through and therefore pervade "*all* the γένη." That this is the case is established when it is seen that "the possibility of 'otherness' is dependent on the 'self-sameness' of the participants in the relation of 'otherness.'" This means that the kind of κοινωνία they exhibit is unlike the kind exhibited by the κίνησις and στάσις pair, which does not pervade all γένη—for, most obviously, they do not pervade each other. According to Klein, "the κοινωνία between ταὐτόν and θάτερον ... permits the διάνοια to understand the 'duplicity' of 'being,'" which is manifest in its meaning both στάσις and κίνησις, because in order to be understood at all the significance of each has to be grasped, in their self-sameness, as other than the other. In addition, their κοινωνία also permits the διάνοια to understand "that by this means alone" (92/96) (i.e., the duplicity of being), "the 'imaging' of beings in re-cognition, that is, in knowing [γιγνώσκειν] and being known [γιγνώσκεσθαι], and all image making, is possible." Finally, just as it is the opposition encountered in the sensible realm that presents the obstacle that first awakens διάνοια, so, too, "at the end of its 'dialectic' activity," it is the duplicity of being that allows it to "come to see that the 'conjunction' of *opposites* is in truth the 'co-existence' of *elements other in kind*" (92/96). Thus, for Klein, the duplicity of being emerges as "the ontological foundation and justification of the method of 'division' (διαίρεσις)" (93/97), by which, in "a continual duplication of the εἴδη—that the λόγος grasps in the 'division' of the γένη—it makes the 'genetic' order of the eidetic definite amounts possible" (94/98).

Klein claims that, on closer inspection, the κοινωνία between ταὐτόν and θάτερον "is nothing but another expression for the internal 'twofoldness' of θάτερον itself" (91/95), because, for Plato, "'the 'other' is always in relation to an 'other' (255 D 1; cf. D 6 f.)." Hence, θάτερον, by pervading all "γένη whatsoever" (255 E 3 f.), "accordingly is both a connecting bond (δεσμός) and the cause of their division (αἴτιον τῆς διαιρέσεως – 253 C 3)," and, as such, presents "*the 'ultimate source' of all articulation whatsoever*." This means that "'Otherness' makes possible an 'arithmetic community' among eidetic monads which are not capable of 'being mixed' although they 'belong together'" (92/96). Klein claims further that the eidetic 'two,' as "the paradigm for such a κοινωνία" and as containing στάσις and κίνησις, can be

articulated only when "'being' itself" is understood to be "accompanied of necessity by a 'not'; just as *στάσις* is *not* *κίνησις*, so *κίνησις* is *not* *στάσις*." It follows that "since precisely only 'both together' amount to 'being,'" *κίνησις* is that which, in confrontation with *στάσις*, is the 'other' *without which even στάσις itself cannot be*"—and vice versa. According to Klein, the significance of this for Plato is that "Otherness turns out to be the ontological meaning of 'non-being,' which can never be separated from 'being.'" Just as θάτερον, "analogously to the vowels among the letters (253 A 4–6)" (91/95) and, therefore, by its "very 'nature' (φύσις)" "'being broken up into parts'" (93/97), pervades all the *γένη*, so too "the shadow of 'non-being' necessarily attends all the 'being' of that which is" (92/97). 'Non-being,' therefore, "is only 'being *other*,' 'not something the *contrary of*, but *only other than*, being' (οὐκ ἐναντίον τι . . . τοῦ ὄντος ἀλλ' ἕτερον μόνον – 257 B; cf. 258 B)."

§ 80. Recognizing "the Other" as the "Indeterminate Dyad"

By allowing for the articulation of being *and non-being*, Klein maintains that, for Plato, θάτερον is "akin to the discerning ἐπιστήμη" (93/97), and that it is so "certainly not accidentally," since it makes possible "everywhere" (92/97) "an 'opposition' of one being to an other being," and thus "'a confrontation of being with being' (ὄντος πρὸς ὂ ἀντίθεσις – 257 E 6)" (92–93/97). For Klein, then, the κοινωνία of ταὐτόν and θάτερον allows the duplicity of being, as both *στάσις* and *κίνησις*, to be grasped and, in addition, makes possible "the 'imaging' of beings in 're-cognition,' that is, in knowing and being known, and, beyond this, all image making" (92/96). Likewise, on Klein's view, θάτερον, as the source of this duplicity, "is the reason for the possibility of a 'mistake' or 'interchange' of the 'one' and the 'other' or of 'being' and 'non-being,' a possibility on which all 'contradiction' (cf. 232 B), all 'illusion,' all 'error,' and every 'lie' depend (260 B–264 B, also 266 D–E)" (93/97). Klein says of the peculiar status of θάτερον, that "It is always, as it were, only a 'part' (μόριον) of itself, namely an '*other of another*' (ἕτερον ἑτέρου)—a 'counter-part'" (93/98). As a result, he maintains that "as the ἀρχή of all doubleness it [i.e., θάτερον] must be recognized as the 'twofold in general,' and in the context of Plato's search for foundations, what has been referred to as the ἀόριστος δυάς."[80] As the source of the duplicity of both being and non-being, the δυάς is ἀόριστος because "it does not present 'two' beings of some particular kind such as are mutually delimited and uni-

80. Among the many references Klein provides to Aristotle's mention of the ἀόριστος δυάς (in connection with the question of whether the εἴδη are ἀριθμοί), the following is typical: "For the indeterminate dyad makes things double (ἡ γὰρ ἀόριστος δυὰς δυοποιὸς ἦν)" (Aristotle, *Metaph*. M 8, 1083 b 35–36).

vocally determined.[81] Rather, in endowing the being [*Sein*] of each being [*Seienden*] with 'imageability,' it 'doubles' everything, and so first allows each to become a 'being' at all—it is 'two-making' (δυοποιός – Aristotle, *Metaphysics* M 8, 1083 b 35 f.; M 7, 1082 a 15)" (93–94/98).

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81. David Lachterman maintains that Klein's account of the relationship between Being and the Other, "while deeply provocative, leaves a number of questions unsettled" (David Lachterman, Rev. of Klein, Jacob, *Plato's Trilogy: Theaetetus, The Sophist, and The Statesman*, in *Nous* 13 [1979)], 106–12, here 111). Lachterman holds that "Klein's thesis that Being is the first in the series of eidetic numbers," when combined with his consideration that "if the Other is taken to be the Indeterminate Dyad, then it is one of the two principles of the forms and the eidetic numbers," renders it "unclear how a principle can participate in one of the items ontologically subordinate to it, or under its command" (112). This lack of clarity appears to be the case for Lachterman because, on the one hand, Being and the Indeterminate Dyad are among the μέγιστα γένη and therefore must "participate in, or mingle with" (111) one another. On the other hand, "The Other, into which not-being is analyzed" (110) must, as a result of its connection with not-being, be *subordinate* to Being. Hence his question: how can Being participate in something that is ontologically inferior to it? The unstated basis of this lack of clarity seems to be the idea that the participation relationship between Being and the Indeterminate Dyad cannot be symmetrical, because the latter, as a result of its connection to non- or not-being, has a lower ontological status than the former.

If the issue Lachterman raises is unpacked in terms of our consideration of Klein's account of the different modes of being belonging to the duplicities that characterize Being respectively as the eidetic two and the ἀόριστος δυάς, his concern would appear to be the following: how can the duplicity of Being, which consists of στάσις and κίνησις, participate in a "principle" that it not only is not, non- or not-being, but that is also its opposite, the unlimitedness of the twofold in general? If, however, the referent for what Lachterman articulates as "the two *principles* of the forms and eidetic numbers" (my emphasis) is considered as its proper identity for Klein, i.e., as the strange kind of arithmetical κοινωνία that is responsible for the unity of eidetic elements different in kind, then Klein's thesis about the relationship between Being as the eidetic two and the Other is indeed able to settle the question posed by Lachterman. This is the case, on the one hand, because according to this thesis the kind of κοινωνία characteristic of Being is different from that of the Other: even though each are what Lachterman refers to as "principles," for Klein such principles are neither homogeneous nor symmetrical, but, on the contrary, indexes for two different kinds of κοινωνία. Specifically, the unity of elements different in kind characteristic of Being, as the eidetic ἀριθμός 'two', encompasses precisely what is many but *not* unlimited, while the unity of elements different in kind characteristic of the ἀόριστος δυάς encompasses precisely what is unlimited. That is, rest and motion do not pervade all γένη (most obviously, they do not pervade each other) and, therefore, all beings, while the Other pervades all γένη and, therefore, all beings. On the other hand, what, as their ἀρχή, is responsible for each different kind of κοινωνία is something that is beyond the opposition of Being and non-being, namely, the One Itself. As the Whole responsible for delimiting Being as something that, notwithstanding its duplicity, is itself one, and also for generating non- or not-being as something that, despite its negativity, is not the contrary of Being but "only *other than* Being," i.e., unlimited in multitude, the necessity belonging to the priority of the One Itself is beyond Being and non-being alike. It is beyond Being, because it is that which is referred to but unaccountable for by its duplicity. It is beyond non-being, because it is the source—albeit one that is distorted by what it generates—of ταὐτόν, which, as we have seen, is "nothing but another expression for the internal 'twofoldedness' of θάτερον itself."

§ 81. The "One Itself" as the Source of the Generation of Ἀριθμοὶ Εἰδητικοί

Klein holds that, beyond the generation of beings, the ἀόριστος δυάς also makes possible the "'genetic' order of the eidetic definite amounts" (94/98). It does so "by a continual 'duplication' of the εἴδη," although, more precisely, "only the *even* eidetic definite amounts can be generated in this way . . . , while the corresponding *odd* eidetic definite amounts" do not arise from duplication but "through a 'delimitation' on the part of the ἕν" (94 n. 1/239 n. 110). Thus, in contrast to mathematical ἀριθμοί, all of which—namely, both the odd and even—are generated by the successive reproduction of the unit, that is, of the one that is a one among many ones (see § 59 above), "even" ἀριθμοὶ εἰδητικοί are generated by duplication and the "odd" by delimitation. Moreover, Klein maintains that the λόγος can only grasp εἴδη in διαίρεσις "because at the 'head' of the τάξις of eidetic definite amounts, at once completing and instituting them, is the *One Itself* in its 'absolute' priority" (94/98). He writes:

> Since it is beyond all articulation, beyond the "two," and thus "beyond being itself" (ἐπέκεινα τῆς οὐσίας – *Republic* 509 B), it is not, like the mathematical unit, a one among many ones, but rather the original, perfect, *all-comprehensive Whole* (cf. *Sophist* 244 D–245 D; also *Parmenides* 137 C, 142 D). As the "Whole" it is that which needs no "other" at all, that which is altogether "complete." In this sense it is "*the perfect itself*," namely, the model of every possible "relative" wholeness which is "delimited" in respect to an "other": it is the "Idea of the Good" (ἰδέα τοῦ ἀγαθοῦ).

The absolute priority of the "One Itself" initially comes into relief with the consideration that neither of the "two" sources responsible for the delimitation of one being by another—namely, ταὐτόν and θάτερον—can account for the unity of the delimited beings *except in terms of their opposition to one another*. On the one hand, θάτερον cannot do so for the obvious reason that, as the twofold in general, it is not responsible for unity but duality. On the other hand, ταὐτόν cannot do so either, because, as we have seen, its κοινωνία with θάτερον makes it simply another expression of the "twofoldness" of θάτερον. This means that the unity, the wholeness, or, in other words, the self-sameness *itself* of that which arises in its opposition to another is something that cannot be accounted for in terms of that which is responsible for this opposition. That there is such self-sameness "itself," that is to say, a unity or wholeness that is beyond all opposition and thus beyond all duplicity, is something that the very articulation of duplicity (i.e., division) makes manifest when each of what is divided is said precisely to be "one"—and thus not said to be what "both together" are, namely "two." However, so long as the

unity of what is articulated in division is grasped in terms of the mathematical one, namely, as a one that can arise only in relation—and therefore in opposition—to the other ones that comprise the many of which it is one, the "one itself" appealed to by such articulation will remain presupposed and therefore unaccounted for. Hence, Klein's reactivation of the necessity recognized by Plato, that the absolute priority of the "One Itself" as a "Whole" possessing a perfection and completeness that precludes its self-sameness from having any relation to what is other than itself. As such, its unity is beyond both the eidetic 'two' of *ὄν* and the 'twofold in general' of the *ἀόριστος δυάς*. It is precisely the status of the "One Itself" as beyond all articulation that allows it to serve as the model for the relative wholeness of both sensible beings and the *γένη* or *εἴδη* whose self-sameness and therefore unity arises in the delimitation composed of their contrast to an other.

That the "One Itself" is at the head of the order of *ἀριθμοὶ εἰδητικοί*, "at once completing and instituting them," follows from its status as both 1) the model for the self-sameness proper to the unity of everything that is articulated—in the case at hand, the *γένη* or *εἴδη* that comprise each *ἀριθμὸς εἰδητικός*—and 2) the "Whole" that, needing no "other" at all, is "the perfect itself," with a unity beyond all articulation. In the first case, the "One Itself" is responsible for instituting the *ἀριθμοὶ εἰδητικοί*, because as the absolute priority of its unity, as a unity beyond all opposition, it makes possible the relative wholeness proper to each of the *γένη* or *εἴδη* that compose them. In the second case, it is responsible for completing the *ἀριθμοὶ εἰδητικοί*, insofar as the "incomparable" character of its unity, as a unity that needs no "other," makes possible the uniqueness of the *κοινωνία* of each *ἀριθμὸς εἰδητικός* as either superior or inferior in relation to its "neighbor." More precisely, the incomparable character of the unity of the "One Itself" confers limits upon the *ἀόριστος δυάς* and thereby generates both the "even" and the "odd" *ἀριθμοὶ εἰδητικοί*. On the one hand, it generates the former by initially rendering determinate the "twofoldness in general" of the *ἀόριστος δυάς*, the result of which is the eidetic 'two', and thereafter rendering determinate each successive "even" *ἀριθμὸς εἰδητικός* generated by the original "duplication" of the eidetic 'two'. On the other hand, it generates the latter (i.e., the "odd" *ἀριθμοὶ εἰδητικοί*) through the *ἕν* by making *ἀριθμοὶ εἰδητικοί* that are indivisible.[82]

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82. The quotes around the terms 'odd', 'even', and 'duplication' are intended to indicate that, in the context of *ἀριθμοὶ εἰδητικοί*, each of the monads that comprises them are "incomparable" and that they are therefore, in this important respect, disanalogous with mathematical *ἀριθμοί*. For, as we have seen, in contrast to the monads that comprise each noetic mathematical *ἀριθμός*, which can be combined indiscriminately with the monads of other such *ἀριθμοί*, the monads that comprise *ἀριθμοὶ εἰδητικοί* cannot be so combined. In the case of the generation of

§ 82. Γένη as Ἀριθμοὶ Εἰδητικοί Provide the Foundation of an Eidetic Logistic

On Klein's view "'arithmetic' has priority over 'logistic'" (93 n. 1/238 n. 106) in Plato's thought. As a consequence, Klein maintains that "however great a role the latter may, especially in the theory of proportions, play for Plato," the "doctrine of the γένη as eidetic definite amounts must, finally, also furnish the foundation of an *eidetic logistic*" (94/98). The priority of arithmetic over logistic in the mathematical realm comes about due to the fundamentality of that with which arithmetic deals, the ordering of ἀριθμοί ascertained by counting, over that with which logistic deals, calculating the *relations* between ἀριθμοί. For, as we have seen (§ 53), Klein holds that despite the close and, indeed, inseparable relationship between arithmetic and logistic for Plato, addition, subtraction, and all the other logistical relations between definite amounts are, in the final analysis, reducible to those obtained by counting. It is this state of affairs that leads to the "relative isolation" of arithmetic as such, as the significance of the ability to count, namely, the countable order of the things in the world to which counting corresponds, is the fundamental fact that, according to Klein, determines the systematic aspect of Plato's teaching. Hence, Klein also holds that in the eidetic realm, the logistically established ἀναλογία by means of which certain things can only be understood has its ultimate ground in the basic compositions and kinds that characterize the ἀριθμοὶ εἰδητικοί themselves. He writes that, "For instance, sound-mindedness (σωφροσύνη) and justice (δικαιοσύνη) in Book IV of the *Republic* (cf.

"even" and "odd" ἀριθμοὶ εἰδητικοί, then, both the delimitation through the ἕν of the "monads" in the former and their duplication through the ἀόριστος δυάς in the latter cannot be understood in strict analogy with, respectively, the divisibility into two equal ἀριθμοί, or the opposite of this effected by the supernumerary ἕν, which are characteristics proper to assemblages of mathematical monads. Of course, achieving clarity regarding exactly how the generation of ἀριθμοὶ εἰδητικοί is to be understood is something that the most basic premise of Klein's reactivation of Plato's thought on this matter rules out, and it does so on the grounds of its claim that for Plato the εἴδη or γένη that the διάνοια has to presuppose in order to investigate the *distinction* between being and non-being *cannot be counted*. Thus, only this much appears to be clear: that non-being, as the image of being, can be what it is not only *if* what it is not, namely being, is granted a priority over its "duplication" through its image. That this priority cannot be established, however, as long as the investigation of problem of 'being' remains at the level of its duplicity, that the solution to its establishment must be found beyond being and therefore beyond the source of this duplicity in the eidetic 'two', and that it is therefore in the direction of the "One Itself," as the ἰδέα τοῦ ἀγαθοῦ, to which Plato points as the source of this priority, all of this appears to be—if not clear, then, at least, clear enough. Nevertheless, as we will see below (§ 83), establishing the priority of the "original" over the "image" in this way "is brought . . . at the price of the transgression of the limits which are set for the λόγος" (*GMTOA*, 98/99), with the result that the *is* becomes, from the "point of view" of "the ordinary mode of predication," "no longer understandable."

I 337 A–C!; Aristotle, *Nicomachean Ethics*, E 6, 7), or the τάξις of elemental materials in the *Timaeus*," which "can be only understood by means of ἀναλογία, proportion," are nevertheless just like "the relation of ontological to the dianoetic *methexis* problem," only resolvable with their reduction "to the 'community' relations in the realm of εἴδη." That is, Klein argues that while "the relation of the ontological to the dianoetic *methexis* problem" (see § 68 above), "as well as the original-image relation in general" (94/98–99), is (like those just mentioned in the *Republic* and *Timaeus*) "first of all graspable precisely in 'logistical' considerations" (94/99), this relation is nevertheless something that "we can glimpse in its *original form*" only with the reduction of the logistical relations to the relations of εἴδη that compose the ἀριθμοὶ εἰδητικοί. In other words, the non-mathematical, eidetic proportions through which certain relations are initially only graspable, are, in their original form, precisely the community relations of εἴδη with which the "conception of ἀριθμὸς εἰδητικός" is concerned.

Klein's discussion of the relationship of the ontological to the dianoetic *methexis* problem elaborates the fundamental dependency of the proportional or logistical relations in eidetic logistic on the relations that compose the κοινωνία (community) of the εἴδη. On his view, the "*imagistic character of the whole* methexis *relation*," namely, of the relation of ontological to dianoetic *methexis*, is "usually overlooked in the discussion of the *methexis* question." This relation is overlooked, he says, "insofar as this discussion is concerned with the dianoetic realm, i.e., the relation of an εἶδος to a series of αἰσθητά." Earlier, Klein had articulated the relationship between dianoetic and ontological *methexis* in terms of the following proportion: "as the elementary form of the problem of the 'one and many' [i.e., precisely 'the relation of an εἶδος to a series of αἰσθητά' just mentioned] . . . is to the dianoetic *methexis* problem, so is the latter to the ontological problem of the 'community of ideas'" (see § 68 above). Thus, when the discussion of the *methexis* question is limited to the elementary form of the problem of the one and the many, the *methexis* of many αἰσθητά in one εἶδος, what is missed is the problem proper to the relationship of *this methexis* to the *methexis* of the relations of community within the realm of the εἴδη. As we just saw, Klein characterizes this relationship "as the original-image relation in general" and avers that it is "first of all graspable precisely from 'logistical' considerations." However, he also maintains that "Only the reduction of this relationship [i.e., the relationship of dianoetic to ontological *methexis* as the original-image relation in general] to the community relations in the realm of the εἴδη allows us to glimpse the *methexis* question in its *original* form," a reduction that presents "the conception of the ἀριθμὸς εἰδητικός" "precisely as one of the possible solutions to this question."

§ 83. Plato's Postulate of the Separation of All Noetic Formations Renders Incomprehensible the Ordinary Mode of Predication

In addition to providing the foundation for the differences between the genus and species of things, which allows them to be counted, Klein also attributes (see § 74 above) to the ἀριθμοὶ εἰδητικοί the responsibility for bringing about their unlimited variety, through what he refers to as "a 'distorted' imitation of ontological *methexis*" (89/93) that occurs in the dianoetic realm. The *methexis* question in the dianoetic realm concerns the mimetic and therefore *imagistic* relation of the many to the one, a relation that is responsible for the unlimited variety of things. According to Klein, this *methexis* in Plato's thought is itself an *image*—albeit a "distorted" one—of the *methexis* in the ontological realm, which concerns the *eidetic* relation of the one and the many.[83] As "one of the possible solutions" to the ontological *methexis* question, "the ἀριθμὸς εἰδητικός indicates in itself the possibility of an immediate unification of the many" (95/99). Because the non-mathematical nature of such an ἀριθμός prevents the λόγος from giving a completely clear account of Plato's "final answer to the problem of the 'one and the many,'" Klein holds that "this solution is brought, as we have seen, at the price of the transgression of the limits which are set for the λόγος, for, from this point of view, the ordinary mode of predication, such as: 'the horse *is* an animal,' 'the dog *is* an animal,' etc., is no longer understandable." When it is said that 'the dog is an animal' and 'the horse is an animal', the γένος 'animal' is united with the εἴδη 'dog' and 'horse' in the following manner: both together as well as each by itself *are* inseparable from being an animal; animal is common to dogs and horses and each dog and horse *is* an animal. This manifestly does not hold, according to Klein, in the case of the "arithmetical" unity belonging to the structure of an ἀριθμὸς εἰδητικός, because the γένη united by the γένος responsible for their κοινωνία *are* not this γένος. While it makes sense to say that a dog *is* an animal and a horse *is* (likewise) an an-

83. As a "distorted" image of ontological *methexis*, the dianoetic proportions do not accurately duplicate the proportions of the original to which they are related imagistically (see § 72). In the case at hand, the original is composed of the communal relations of the εἴδη that make up an ἀριθμὸς εἰδητικός. Of course, what is at issue in the reference to proportions here are not equivalent numerical ratios (see n. 52 above) but, as it were, the adjustment of the proportions in the image to reflect accurately how the original appears according to its aesthetic perspective, an adjustment that paradoxically results in the image seeming to be more like the original itself than would be the case if the proper mathematical proportions of the original were duplicated in the image. This paradoxical state of affairs, moreover, is—as will be seen directly below—precisely what is reflected when the eidetic solution to the one and the many problem is compared with its natural solution and shown to fall short of the perfectly comprehensible logical sense that is inseparable from the natural one.

imal, it does not make sense to say that *στάσις is* ὄν and *κίνησις is* (likewise) ὄν. Moreover, the incomparable nature of the monads that comprise an ἀριθμὸς εἰδητικός means that, "Above all, the 'natural' meaning intended when a multitude of things is called an ἀριθμός is lost." That is, what is lost is the sense that the many things that compose an ἀριθμός are countable and therefore exactly delimitable.

Thus on Klein's view, when the doctrine of γένη as ἀριθμοὶ εἰδητικοί is employed to provide the foundation of an eidetic logistic, the natural meaning of that which is at first comprehensible only in terms of the proportions established by logistical considerations is no longer comprehensible. In the case at hand, the reduction of the eidetic proportion (ἀναλογία)—which is characteristic of the dianoetic *methexis*'s distorted imitation of ontological *methexis*—to its *foundation* in the community relations belonging to the multitude of γένη or εἴδη in an ἀριθμὸς εἰδητικός presents what Klein calls attention to as "difficulties." They include not only those pertaining to the intelligibility of these relations to the ordinary mode of predication noted above, but also those that pertain to the questions connected with the mode of being of an image. The problem of discerning something as *not* being what it presents itself *to be*, that is, of *recognizing* something *as an image*, remains unresolved by the doctrine of γένη as ἀριθμοὶ εἰδητικοί. That is because this doctrine establishes the *ontological* conditions that have to prevail for an image to be at all, namely, the "mixing" in some sense of being and non-being. What remains unresolved in this account, however, are the conditions that make it possible to grasp an image *as* an image in the first place. In other words, establishing the possibility of the mode of being of an image is not tantamount to establishing how it is that the initial distinction between image and original is made at all. To establish this, the priority of the original over its image must be secured in a manner that rules out the very duplicity that makes possible the recognition of the original *as* original. In the visible realm, the comparison by the διάνοια of the perception of visible things with their images is sufficient to—seemingly—accomplish this. This holds insofar as *perception* appears able to grasp the one visible thing that is the source of the image as something that is beyond all opposition and therefore beyond the duplicity of that which is grasped in relation to something else. Of course, as soon as this "one" thing is perceived in relation to other things, this semblance is exposed as such and its priority called into question. Reestablishing the priority of the one, of the original, over all relations and thus over the duplicity that makes relations possible, is, as we have seen, the task Plato attributes to dialectic according to Klein. We have also seen, however, that dialectic's own proper method, διαίρεσις, leads not only to "the postu-

late of the 'separation' (χωρισμός) of all noetic formations, and in particular the χωρισμός of ἀριθμοὶ μοναδικοί, the definite amounts of 'pure' monads," but also to the χωρισμός of the absolute priority of the One Itself. For Klein, only Aristotle's critique of the postulate of χωρισμός "exposes the root of" the difficulties presented by the ἀριθμὸς εἰδητικός solution to the problem of the one and the many, when this solution is considered from the perspective of the natural meaning of things. It is Klein's desedimentation and reactivation of this critique that we now take up.

Chapter Twenty

Aristotle's Critique of the Platonic *Chorismos* Thesis and the Possibility of a Theoretical Logistic

§ 84. Point of Departure and Overview of Aristotle's Critique

According to Klein, "the obvious point of departure for the Aristotelian criticism of the Platonic school is the ontological standing attributed by them to the mathematical realm, in particular, to the definite amounts of pure units" (95/100). As we have seen, this standing is based in the "exemplary μάθημα character of mathematical objects, their undeniably pure noetic quality, their 'indifference' with regard to sensuously perceivable things," all of which the Platonic *chorismos* thesis takes "to indicate directly the possibility of the existence of noetic structures which are independent and 'detached,' i.e., separated from, all that is somatic." On Klein's view, however, Aristotle's critique questions neither "mathematical science itself" nor the fact that "mathematical inquiry has a special field of objects, as, for example, arithmetic has the field of pure monads (cf. *Posterior Analytics* A 10, 76 b 4 f.)." Rather, he contends that Aristotle is "concerned with proving the Platonic conception of the *mode of being* of mathematical objects false, 'so that our controversy will be not about their being but its mode' (. . . *Metaphysics* M 1, 1076 a 36 f.)."

Klein articulates two interrelated aspects of Aristotle's critique. On the one hand, it contains an argument against the Platonic view of the mode of being proper to ἀριθμός as independent of the objects of which it is the ἀριθμός, as having a generic unity, and as being something that is a κοινόν, something that is a whole above and alongside of the parts—namely, the definite amounts of units—of which it is the whole. On the other hand, it determines the mode of being of mathematical objects, especially of pure ἀριθμοί, by abstraction (ἐξ ἀφαιρέσεως) from the objects of sense. From this it follows that the former objects are separable but not detached from the lat-

ter, that the unity of ἀριθμός derives from the unity possessed by each object insofar as it is the measure of the count in question, and that the demarcation of one ἀριθμός from another is comprehended only with respect to objects that are actually counted. Klein maintains that Aristotle's critique eliminates the ontological obstacles—posed by the Platonic notion of the monad's indivisibility—that stand in the way of the realization of a *theoretical logistic* and thus prepares the way for its realization in the arithmetical textbook of Diophantus.

§ 85. Aristotle's Problematic: Harmonizing the Ontological Dependence of Ἀριθμοί with Their Pure Noetic Quality

For Klein the point of departure for Aristotle's criticism emerges "in the course of an analysis of what is meant in ordinary speech" (*GMTOA*, 95/101). Klein cites Aristotle at length:

> The fact that it becomes possible to call something a "whole" by indicating the "parts" of which the whole "consists," and that, therefore, "for declarative speech" (τῷ λόγῳ) the parts precede the whole does not mean that in respect to their "being" (τῇ οὐσίᾳ) these parts can lay claim to priority over the being of the whole. (cf. *Metaphysics* M 2, 1077 b 1 ff. and Δ 11, 1018 b 34 ff.) (95–96/101)

Thus, "to call an object that appears before us, for example, a human being, a 'white' human being, presupposes the partial assertion 'white,' and yet no other being is meant than precisely this white human being (cf. on this *Metaphysics* Z 4, 1029 b 13 ff.)" (96/101). For Aristotle, then, it is clear that the "'white' meant has no existence 'outside' of this human being and 'separate' from him." As a consequence, in its being it is "bound to the given 'whole,' namely to 'this white human being,'" and, as such, "It cannot exist if this human being of which it is predicated does not exist." This is to say, "Its *being* is therefore *dependent* on the being of the *human*." On Klein's view, for Aristotle it is "[*e*]*xactly* in this same way" that "the assertion 'three trees' presupposes the assertion 'three'"—in other words, "what the assertion 'three' means has no existence 'outside' of the trees of which there are said to be three." And this is because the "definite amount of trees, i.e., 'three,' has no proper, no independent 'nature' (φύσις, cf. *Metaphysics* M 6, 1080 a 15; 7, 1082 a 16; 8, 1083 b 22, and elsewhere)." Hence, both the "being 'so many' of the trees" and "their being, for instance, 'green,' is dependent on their being *trees*."

Klein traces Aristotle's understanding of the ontological status of ἀριθμός to its "'natural' meaning," in which "the assertion that certain things

are present 'in a certain definite amount' means only that *such a thing* is present in just this definite multitude: 'To be present in a definite amount is to be some definite amount of a ⟨given⟩ object' (. . . *Physics* Δ 12, 221 b 14 f.)." This significance of ἀριθμός, combined with its dependence, rules out for Aristotle the sufficiency of determining its mode of being "as independent, and separate or 'absolute,'" even in the case of the pure units that must be presupposed "in order to understand the prior knowledge of definite amounts which is revealed in our daily calculating and counting" (96/102). On Klein's view, this holds because Plato's determination of the mode of being of pure ἀριθμοί on the basis of the "attempt to 'account' for . . . the possibility of counting and calculating" misses for Aristotle precisely the "dependence and bondage" that is "indicative of the being of a definite amount." The Aristotelian problematic, then, is "precisely to bring this being character of all possible definite amounts, and thus also of the 'pure,' 'mathematical' definite amount, into harmony with the purely noetic quality of the latter."

§ 86. Aristotle on the Abstractive Mode of Being of Mathematical Objects

Aristotle attempts to accomplish this with his "so-called 'theory of abstraction'" (98/104), which Klein maintains "is, after all, not so much a 'psychological' explication of certain cognitive processes as an attempt—fraught with heavy consequences for all later science—to give an adequate ontological determination of noetic objects like the μαθηματικά." According to Klein, mathematical objects for Aristotle "have their being ἐξ ἀφαιρέσεως ⟨by abstraction⟩, that is, . . . they are 'lifted off,' 'drawn off' ('abstracted') from sensuously perceivable things" (98/104). Such objects, "which in respect to their being are not 'detached,'" are studied by science "*as if* they were detached or separated from sensuously perceivable things." Thus, one "'thinks the mathematical objects which are not separate as separate whenever one thinks them.' (. . . *On the Soul* Γ 7, 431 b 15 f.)." Klein traces the reason for this back to Aristotle's answer to the question of how we are able "to extract all the single parts (μέρη), the single 'constituents' of a thing which we get hold of in the λόγος one after another, e.g., 'this' 'round' 'white' 'column,' from the concrete [*dinglichen*] context into which they are fitted in accordance with the possibility of their *being* and to examine each of them separately" (*GMTOA*, 96–97/102). Klein characterizes Aristotle's answer as follows:

> in each case we *disregard* certain attributes of the thing in question, ignoring the nexus of being which links them all to one another. This "disregarding of . . ." is able to produce a new mode of seeing which permits something to

> come into view *in* αἰσθητά which, for all their variety and transitoriness, suffers no change but remains always in the same condition, thus fulfilling the demand that it can be an object of some science, of an ἐπιστήμη. (97/102)

Thus, "'Each thing may be viewed best in this way—if one posits that which is *not* separate ⟨i.e., which has no separate existence⟩ *as separate*, just as the arithmetician and the geometer do.' (. . . [*Metaph.* M] 1078 a 21 ff.)" (97–98/103).

On Klein's view the "'lifting off'" (98/104) characteristic of abstraction "is only a different expression for precisely that aforementioned 'disregarding' of all the other content of things." For "In this 'disregarding of . . .' the sensuously perceivable things wither away, as it were, and become mere 'pieces' [*Stücken*] or mere 'bodies.'" They are "Thus deprived of their aisthetic character and, to a great extent, robbed of their individual differences." The possibility of "an apodeictic discipline" is based on this "'disregarding of' every particular content," which allows both the ἀριθμός aspect and geometrical dimensions of αἰσθητά to be, "as it were, read off" (97/103) them, "'though not insofar as these are sensuously perceivable bodies, but insofar as they are just such {namely so extensive or so many}' (. . . *Metaphysics* M 3, 1077 b 20 ff.)." These aspects and dimensions of αἰσθητά "are no longer subject to the senses, though they do not achieve any independent being, i.e., a being alongside of the αἰσθητά." Even though, however, "their being remains dependent on the being of sensuously perceivable things," "insofar as the mathematical aspects become visible in their 'purity,' detached from all other content, they may be isolated within the 'whole' and may be, without detriment to their dependence, 'lifted off,' as it were, from the 'whole.'" Abstracting them in this way poses no danger "For their more exact investigation," because "'if someone, positing things as separated from that which {otherwise} goes with them, examines them for that about them by which they are such {namely 'separable'}, there will be no more falsification because of this than when someone draws something on the ground (for the purpose of demonstrating geometric theorems) and says that it has the length of a foot when it has not; for the falsity is not in the premises {as such}.' (. . . *Metaphysics* M 3, 1078 a 21 ff.)" (98/103).

According to Klein, when the things that serve as the basis of abstractive disregarding "are no longer regarded even as 'bodies' but only as 'pieces,' these things have been transformed into 'neutral' monads" (98/105). He quotes Aristotle to this effect: "'The mathematician makes those things which arise from abstraction his study, for he views them after having *drawn off* all that is sensible . . . , and he *leaves* only the ⟨object of the question⟩ 'how many?' and 'continuous magnitude' (. . . *Metaphysics* K 3, 1061 a 28 ff.)"

(98/104–5). Hence, in the case of pure ἀριθμοί, it is precisely "this 'neutrality' of things which have withered away into mere countable 'pieces'" (98/105) that "constitutes the 'purity' of the[ir] 'arithmetic' monads." Therefore, contrary to the Platonic *chorismos* thesis, it is "[n]ot *original* 'detachment' but *subsequent* 'indifference'" (99/105) that "characterizes the mode of being of pure definite amounts (*Metaphysics* M 2, 1077 a 15–18)." As the way in which "mathematical formations"—here ἀριθμοί—"first become objects of *science* . . . this mode [of being] of their content, the 'what' of all these formations, is *given in advance*" (98/103–4). Klein maintains that the task, for Aristotle, of determining "how this 'being' itself is to be understood" (98/104) is "no longer the task of mathematics but of 'first philosophy' (πρώτη φιλοσοφία) alone (cf. *Metaphysics* K 4, 1061 b 25–27)." Science, then, "simply has to 'accept' (λαμβάνειν) the 'being' of the various *original* formations, namely of the 'one,' the 'line,' the 'plane,' etc., and to 'derive' from it the 'being' of the rest, i.e., to display the noncontradictory connection of *all* the *pregiven* contents of arithmetic and geometry (cf. *Posterior Analytics* A 10, 76 a 31–36)."

§ 87. Aristotle's Ontological Determination of the Non-generic Unity of Ἀριθμός

For Klein, Aristotle's "attack on the *chorismos* thesis" (99/105) takes aim primarily at Plato's account of "generic identity" as "the ultimate foundation of all possible unity." The Platonic "problem of the 'unity of an ἀριθμός-assemblage—how the 'many' can be understood as 'one' at all," which is solved according to the "Platonic doctrine of the ἀριθμοὶ εἰδητικοί"—is in truth for Aristotle, on Klein's view, neither solved in this way nor able to account for the unity in question. Aristotle therefore rejects the Platonic claim "that the unity of definite amounts ultimately has its roots in the indivisible qualitative wholeness of a γένος." In the first place, "if generic identity entails unity, then 'unity' is attributed to formations which, *strictly speaking, cannot be* '*one*' *at all*" (99/106), especially "in speaking of a 'definite amount,'" since in so doing "we mean precisely *more* than 'one thing.'" Klein quotes Aristotle at length on this important point:

> Some things are one by {immediate mutual} contact {of the parts}, others by mingling, yet others by the disposition {of the parts}; none of this can possibly occur in the monads of which the dyad and the triad {consist}, but just as two men are not one thing over and above *both* of them {being each by himself one}, so is it necessary also with {pure} monads (. . . [*Metaph.*] M 7, 1082 a 20 ff.).

In the second place, "'. . . by reason *of what* the definite amounts are one, or the soul and body, or generally, the εἶδος, no one tells us at all . . .' (. . . cf. *Metaphysics* Λ 10, 1075 b 34 ff.; cf. also K 2, 1060 b 10–12)" (99/105–6).

Klein's reactivation of Aristotle's critique maintains that for Aristotle it "is precisely the supposition of the 'detachment' of the pure monads" that "seduces us to this view," namely, that of the generic "unity" of ἀριθμοί (99/106). For once the supposition is made that "these monads, whose noetic character manifests itself in their 'absolute' *indivisibility* . . . , in their unlimited multiplicity . . . , and in their complete similarity . . . ," are detached and therefore independent of sensuously perceivable things, the basis is removed for such monads "to offer any sort of 'natural' articulation (such as is found in the ever-different and ever-divisible sensuously perceivable things) which might serve as the *original source* of delimitation and unification productive of singular assemblages, i.e., of precisely limited amounts" (99–100/106). Thus, "the possibility of collecting *two* monads in *one* ἀριθμός-combination cannot but be understood as the effect of an original and therefore *independent* εἶδος, be it the 'even'[84] (ἄρτιον), be it the eidetic 'two'" (99/106). However, because "in truth [for Aristotle] the pure monads are nothing other than the sensuously perceivable things reduced to mere countable 'pieces,' 'they do not differ {from αἰσθητά} because they are indivisible {while αἰσθητά are divisible}, for points {which, although they are also purely noetic formations, can nevertheless be immediately 'represented' by sensuously perceivable *signs*} are also indivisible, and yet there is *nothing other aside from their* '*being two*' {i.e., no 'one' thing different from themselves} which might be termed their 'twoness.' (. . . *Metaphysics* M 8, 1082 a 24–26; cf. also M 9, 1085 a 23–31)" (100/106–7). For Aristotle, then, a "definite amount is precisely not *one* thing but a 'heap' (σωρός) of things or monads (cf. *Metaphysics* H 3, 1044 a 4; M 8, 1084 b 21 f.)" (100/107). This means that, unlike for Plato, the "being of a definite amount is not a κοινόν, to be taken as a 'whole' *above* and *alongside*, as it were, the parts of the 'heap' (cf. *Metaphysics* H 6, 1045 a 8–10)." Rather, an ἀριθμός is precisely these parts and *nothing more*. "'For a definite amount is {only} that which has been counted or can be counted.' (. . . *Physics* Δ 14, 223 a 24 f.; cf. Γ 5, 204 b 8.)."

Klein holds that Aristotle recognizes, as does Plato, a "preknowledge of all possible definite amounts" that makes "available indifferent formations, each of which holds the place of 'a definite amount by which we count' (ἀριθμὸς ᾧ ἀριθμοῦμεν – *Physics* Δ 14, 223 a 24 f.; cf. Γ 5, 204 b 8). Moreover, this availability, on Klein's view, should be called—following Plato—"a

84. The English translation renders ἄρτιον as 'odd', which is clearly a misprint.

'stored possession' [κτῆσις] and not a 'possession in use' [ἕξις] (*Theaetetus,* 197 B ff.; cf. Aristotle, *Posterior Analytics* A 1)." However, while Plato takes all of this to indicate a field of "pure" monads and hence ἀριθμοί that are originally "detached" from sensuously perceivable things, Aristotle, on Klein's view, maintains that each of these ἀριθμοί, as a "stored possession," is precisely such as to "*not* coincide with what is counted" and thus something that "must not be spoken of as *one thing*." The "availability" of "all possible definite amounts" in this "inexplicit preknowledge" (100–1/107) does not point, then, to each of them "being one" (101/107), as it does for Plato. That is to say, it does not point to their being "a κοινόν, to be taken as a 'whole' *above* and *alongside*" the multitude of counted or countable objects that compose (for Aristotle) an ἀριθμός properly so-called. Rather, this "availability" for Aristotle, according to Klein, "first comes to be known in the execution of counting, and so it is also rooted in the experience of counting *multitudes* and of culling from them those indifferent formations ἐξ ἀφαιρέσεως." Indeed, "we see that here too we are dealing with 'heaps,' namely with heaps of 'pure' monads, which may be understood as 'separable' but not as originally 'detached'" (100/107). So ἀριθμοί of pure monads are one "only as much as is anything whatever which extends 'over the whole' (καθόλου, cf. *Posterior Analytics* B 19)," which means that they "are as little 'one thing' as any definite amount of sensuously perceivable things (cf. also *Metaphysics* M 4, 1079 a 34–36)" (101/107).

§ 88. Aristotle's Ontological Determination of the Unity of Ἀριθμός as Common Measure

Having criticized the Platonic *chorismos* thesis, as well as the view of ἀριθμός as a κοινόν that follows from it, what remains is "the question *by what* those definite amounts which are 'heaps' are demarcated from one another, how it is possible to call one definite amount just that, namely *one*" (101/107–8). Klein maintains that, for Aristotle, "this question should be posed only concerning that which is *actually* counted" (101/108). That is because "the 'unity' of a definite amount is not of a generic sort," which, as we have seen, is how the Platonic *chorismos* thesis understands it. Rather,

> When we consider that all counting presupposes the *homogeneity* of that which is counted insofar as it is counted . . . , we see not only that every definite amount consists of many units, i.e., is many "ones" (ἕνα . . .), but also that it is *kept together by a common measure*, namely by the unit in question that has become the basis of the count, and, therefore, the means by which "many ones" "as many" initially become—as such—possible. "For each definite amount is 'many' because each is [made up of] 'ones' and because each

> is measured by {its own}'one.'" (πολλὰ γὰρ ἕκαστος ὁ ἀριθμὸς ὅτι ἕνα καὶ ὅτι μετρητὸς ἑνὶ ἕκαστος – *Metaphysics* I 6, 1056 b 23 f.; cf. *Physics* Δ 12, 220 b 20–22.)

Thus, "We comprehend a definite amount as *one* because we do our counting over *one and the same thing*, because our eyes remain fixed on *one and the same thing*." It is therefore

> In *this* sense [that] the "one" (or the *one* thing subjected to counting) makes counting and thus the "definite amount" as such possible: In *this* sense it takes precedence over the definite amount and may be called its ἀρχή (. . . cf. *Metaphysics* Δ 6, 1016 b 17–20; 15, 1021 a 12 f.; N I, 1088 a 6–8). The priority of the one over the definite amount does not follow from a relationship of superiority of genus over species, but rather from the character of the one as "measure."

And it is precisely this state of affairs, namely, that "the 'unity' of a definite amount" is something that "has its roots in the unity which each counted thing has as the 'measure' of the count in question," which rules out this unity being of "a generic sort."

This measure and root of the being one proper to things is also something that "in reverse marks their possibility as countable, and it follows from this and only this that 'to be one is to be indivisible.' (. . . *Metaphysics* I 1, 1052 b 16; cf. I 3, 1054 a 23; furthermore, *Physics* Γ 7, 207 b 6 f.)" (101–2/108). Indivisibility, then, "belongs to things only insofar as they supply the *measure of a possible count*" (102/108). "'Whatever is not subject to division is called one in respect of that by reason of which it is not divisible.' (. . . [*Metaph.*] Δ 6, 1016 b 4–6.)" (102/108–9). The "unity of a definite amount of things" is therefore "only the unity of its 'measure,' namely of the very thing which is subjected to counting and *in that capacity* indivisible" (102/109). Consequently, "the definite amount can be determined in general as 'a multitude measured by the one.' (. . . [*Metaph.*] I 6, 1057 a 3 f.)" (102/109). And it is precisely this that provides the basis, according to Klein, for both Aristotle's rejection of the Platonic view of the "one" as a "κοινόν [common thing] ([*Metaph.*] I 1, 1053 a 14)" and Aristotle's own view of it as "a *measure* (μέτρον – 1053 b 4 f.; 1052 b 18 ff.; Λ 7, 1072 a 33; N 1, 1087 b 33 ff.)."

§ 89. Aristotle's Ontological Determination of the Indivisibility and Exactness of "Pure" Ἀριθμοί

Klein's reactivation of Aristotle's determination of the mode of being of ἀριθμοί concludes with a consideration of how "by confining counting and calculating to 'pure' monads, we turn the *being one* and *being indivisible* of things *as such* into the object of study." Klein traces the genesis of the ob-

jects of this study to Aristotle's claim that "we habitually reduce every count to a neutral expression. We do not say: one apple, two apples, three apples, but rather: one, two, three . . . (cf. M 7, 1082 b 35)." When we do this, "we are already seeing the things in question in their 'reduced' structure, i.e., as indifferent, simply *countable* material." And because "raising this procedure to the rank of a science" is what yields the "*being one* and *being indivisible*" of "the mathematical μονάς," for Aristotle the status of the latter is "nothing other than the *character of being a measure* as such, which has been 'lifted off' things." Indeed, "This is why the arithmetician understands the μονάς as that which is '*totally* indivisible' (*πάντῃ ἀδιαίρετον* – I 1, 1053 a 1 f.; Δ 6 1016 b 25), and therewith also as something that is given as a 'completely exact' (*ἀκριβέστατον* – 1053 a 1) measure." Moreover, "the universal 'applicability' of the 'pure' definite amounts is grounded in this indivisibility and exactness." Thus, "for example, the human being *as human being* is one and indivisible, the {arithmetician} has, however, already posited the {absolutely} indivisible one {namely the μονάς, and indeed, has posited it as 'detached'}, and then sees what might belong to the human being insofar as he is 'indivisible' i.e., what makes him subject to being counted or reckoned with as a 'unit'. (. . . M 3, 1078 a 23–25.)" (102–3/109–10). Finally, what Aristotle says here about the applicability of the μονάς to the human being "may be said of any arbitrarily countable being whatsoever" (103/110), which is the reason why its applicability is "universal."

§ 90. The Influence of Aristotle's View of Μαθηματικά on Theoretical Arithmetic

Klein maintains that "Aristotle's ontological view of μαθηματικά, especially of the 'pure' definite amounts, soon comes to influence the development of mathematical science itself." In contrast to the prominent role played by the concept of the εἶδος within "the Pythagorean and Platonic framework" of theoretical arithmetic, subsequent to Aristotle it "will now have a considerably smaller significance." Not only will εἴδη of ἀριθμοί "no longer be understood as structures which give unity and unambiguous articulation to the realm of definite amounts," but, "if they are to have a place at all, the singular definite amounts must henceforth be considered as εἴδη, and indeed be considered such in the 'derivative' sense (cf. *Metaphysics* Z 4, 1030 a 18–27)." In other words, Klein here is maintaining that one consequence of Aristotle's account of the "abstract" mode of being of definite amounts is that any εἴδη attributed to them must also assume this mode of being, which is "derivative" in the precise sense of its non-independence from the sensible things

that function as its abstractive basis. Moreover, "The 'even,' the 'odd,' the 'even-times-odd,' etc., . . . are now no more than the 'peculiar characteristics' (ἴδια πάθη) of definite amounts (. . . *Physics* B 2, 194 a 3–5)," which means that they are no longer understood as the εἴδη *of* ἀριθμοί. As such, these characteristics "present merely a *quality* (ποιότης) of definite amounts, for instance, the character of being a 'composite' definite amount" (*GMTOA*, 103/110). Finally, the "*being* (οὐσία) of a definite amount is the multitude of units as such, for instance 'six'—where οὐσία is again to be understood in the 'derivative' sense." Thus, in pointed contrast to the Platonic account of the "being" of ἀριθμοί which posits a mode of being proper to the noetic character of each ἀριθμός (e.g., the hexad) as something that has its source in "εἴδη which belong together, *whose community in each case forms a* '*kinship*' which must, due to the 'arithmetical' tie of its 'members' as an eidetic 'definite amount,' be designated as *the six*" (89/92), Aristotle, according to Klein, determines it as follows. The "'*what*' of each definite amount insofar as it is a definite amount is precisely the 'how many' it indicates; thus 'six' units are not in themselves 'two times three' units or 'three times two' units, for this indicates only their 'composite quality,' but 'once six': 'For the being of each definite amount is what it is once, for instance that of six is not what it is twice or thrice but what it is once; for six is once six' (. . . *Metaphysics* Δ 14, 1020 b 7 f.; also b 3–7)" (103/110–11).[85]

Klein reports that this "transformation of the ontological foundation" (103/111) of definite amounts is "directly mirrored" in "the 'arithmetical' books of Euclid (VII, VIII, IX)." Hence, the "'pure' units of which the definite amounts to be studied are composed are here understood precisely only as '*units of measurement*'" (103–4/111). Moreover, the "same approach is indicated by Definitions 8, 9, 11, 12, 14 of the seventh book (namely of even-times-even, even-times-odd, odd-times-odd, prime and composite definite amounts . . .), which define the nature of each definite amount with respect to the *measuring character* of its factors" (104/111). Klein finds this last fact especially significant, for it presents "the very approach which Nicomachus and Domninus, at least, make every effort to avoid," since, like all Neoplatonists, their concern with these classifications is "to provide a (more or less unambiguous) classification of all definite amounts," and not merely to "determine certain of their properties."

85. In his 1939 talk, "The Concept of Number in Greek Mathematics and Philosophy," Klein suggests that "it seems as if Aristotle didn't see the real problem of numbers," because he denied "that there is any unity in a number of things." Klein concludes this lecture with the observation, "This problem still awaits a solution" (52).

§ 91. Aristotle's Ontological Conception of Ἀριθμοί Makes Possible Theoretical Logistic

However, Klein draws "another consequence" (105/112) "[f]rom Aristotle's ontological conception" of ἀριθμοί "insofar as it affects the problem of *theoretical logistic*," which, he maintains, is "far more central in our context." Specifically, Klein holds that Aristotle's criticism of the Platonic view of the monad as "an independent and, as such, simply indivisible object," which shows that "this 'indivisibility' does not accrue to the μονάς as a self-subsisting ἕν, but by virtue of the *measuring character* of any such unit, be it of an aisthetic or a noetic nature," removes the difficulty that stands in the way of realizing on Platonic grounds a theoretical logistic. That is, it removes the obstacle of the indivisibility of the unit of calculation that prevented logistic, "as the theory of those mutual relations of definite amounts that provide the basis of all calculation," from being able to account for calculation with fractions. In other words, Klein maintains "that only Aristotle's conception of μαθηματικά makes possible that 'theoretical logistic' which suffered from the dilemma of being at the same time postulated by the *chorismos* thesis and yet precluded from realization by that very thesis" (105/112–13).

For Klein, then, "Only when the Aristotelian critique has taken effect can a whole series of 'applied' sciences, such as were cultivated in the Alexandrian school, be justified as 'sciences'" (105/112). In support, Klein cites the *Metrics* of Heron of Alexandria, which "starts out from the following observation: 'In order, then, not to have to name feet or ells or their parts in each measurement, we will exhibit our numerical results as {reduced to indifferent} *monads*, for it is open to anyone to substitute for them whatever measure he wishes.'" Commenting on the implications of this, Klein holds that "Nothing now stands in the way of changing the unit of measurement in the course of the calculation and of transforming all the fractional parts of the original unit into 'whole' definite amounts consisting of the new units of measure." Consequently, what was impossible for Platonically conceived theoretical logistic now becomes possible, since "even fractions can now be treated 'scientifically.'" On Klein's view, the "'arithmetical' textbook of *Diophantus*" (105/113) provides "a significant document which is able to give us a concrete notion of the *type* of theoretical logistic which can be built on Peripatetic foundations." Klein's reactivation of the peculiar nature of Diophantus's arithmetic as a theoretical logistic is the subject of the next chapter.

Chapter Twenty-one

Klein's Interpretation of Diophantus's *Arithmetic*

§ 92. Access to Diophantus's Work Requires Reinterpreting It outside the Context of Mathematics' Self-interpretation since Vieta, Stevin, and Descartes

On Klein's view the "self-conception" (122/117) of modern algebra is inseparable from Vieta's foundational appropriation and transformation of the logistical technique employed by Diophantus in his *Arithmetic*.[86] As a result, Klein concludes that "this new self-definition of the mathematical enterprise rests essentially upon a certain reinterpretation of ancient mathematics (as of the whole of science)" (122–23/117). Achieving the goal of clarifying this reinterpretation, in Klein's words, "means nothing less than to gain renewed access to Greek numerical doctrine" (123/117), and it is for this reason that Klein's historical reflection has focused on the Neoplatonic mathematical horizon of Diophantus's work along with its Platonic and Aristotelian contexts. From the vantage of our foregoing exposition of Klein's investigation of these horizons, we are in a position to confirm both the "zigzag" movement[87] of his method and its effective reactivation of "the most general conceptual presuppositions of Greek arithmetic and logistic" (122/117). That is to say, Klein's desedimentation of the Greek ἀριθμός-concept, Plato's articulation of the involved distinction between arithmetic and logistic in general and their practical and theoretical modes in particular, and the different modes of being that Plato and Aristotle attribute to the εἴδη of ἀριθμοί can now be confirmed.

Klein maintains that his "approach" to the significance of "the work of Diophantus," which interprets it "in the light of" the "most general presuppositions of Greek arithmetic and logistic," "cannot but bring us in conflict

86. See Part I, § 23 above.
87. See Part II, § 38 above.

with the standard presentation of Diophantus' *Arithmetic* as a rudimentary stage of modern algebra and hence of our symbolic mathematics in general." This conflict arises because that standard presentation is "intimately connected with the self-conception of modern mathematics since Vieta, Stevin, and Descartes." The problem with the standard view, then, is that it does not recognize "[t]he necessity of abstaining as far as possible from the use of modern concepts in the interpretation of ancient texts" (123/118). Klein observes, and indeed stresses, that an important consequence of the failure to recognize this necessity is that the generality of the Greek mathematical method is mistaken for the *generality of its object*. When this occurs, the use of "[l]etters for indicating magnitudes" (128/124) by Diophantus (and before him, by Aristotle, Euclid, Archytas, and others) is understood from the vantage of "our *own* conceptuality [or conceptual framework]," which, as a matter of course, identifies "the object represented with the means of its representation" (126/122)—and so misunderstands such letters as symbolic designations. For Klein, however, "such a letter is never a 'symbol' in the sense that that which is signified by the symbol is in itself a 'general' object" (128/124). On the contrary, the symbolic understanding of letters presupposes precisely "the conceptual transformation which permits the ancient ἀριθμός to appear as 'numeral' ('*Zahl*')—as opposed to a 'definite amount of definite objects' ('*Anzahl*')—*and concomitantly, as 'general magnitude*.'" Klein's clarification of the modern reinterpretation of Diophantus is therefore tantamount to an *attempt* to trace the transformation of number into the modern symbolic designation from a perspective whose point of departure is the conceptuality of Greek mathematics.

Klein is well aware that "the feasibility of an interpretation not based on modern presuppositions must always be limited" (123/118). He notes that "even if we succeed in ridding ourselves completely of present-day scientific terminology, it remains immensely difficult to leave that medium of ordinary conceptuality which corresponds to our mode of thinking, a mode essentially established in the last four centuries." He nevertheless holds that "the ancient mode of thinking is not totally 'strange' or closed to us," since "the relation of our concepts to those of the ancients is oddly '*ruptured*'—our approach to an understanding of the world is rooted in the achievements of Greek science, but it has *broken loose* from the presuppositions which determined the Greek development." To clarify "our own conceptual presuppositions," then, Klein says that "we must always keep in mind the difference in the circumstances surrounding our own science and that of the Greeks."

Klein locates the difference in circumstances that concern the contrast between the "'natural' basis" (123/119) of Greek science, in which "science

stands in original and immediate opposition to a nonscientific attitude" (123/118), and the replacement of this natural basis "by a *science already in existence*" (124/119), which characterizes the "'new' science" (124/119). For Klein, "Greek ἐπιστήμη" (123/118) involved, "for the first time," the recognition that "the life of 'cognition' and 'knowledge'" is "an ultimate human possibility, one which enables men to disregard all the ends they might otherwise pursue, to devote themselves to contemplation in complete freedom and leisure, and to find their happiness in this very activity." Thus, not only is this science "contrasted with the affairs of the day," but, "In attempting to raise itself above this *nonscientific* attitude, science preserves intact these given foundations" (123/118–19). The situation with "[o]ur science" (124/119), "whose foundations were laid in the sixteenth and seventeenth centuries," is completely different, insofar as its "edifice" is "erected *in deliberate opposition to the concepts and methods*" of the former science, even though "the fundamental claim to validity of this science is recognized." That is to say, for the "new" science, "*Scientia* herself appears as an inalienable human good," even if, as in the case of "the learned science of the schools," it has "become debased and distorted." Thus, although scholastic science's "principles are denied," its "methods rejected," and its "knowledge mocked," science's "place within human life as a whole is placed beyond all doubt" by the "founders of the 'new' science."

These considerations lead Klein to conclude that it is "both possible and necessary to learn to see Greek science from the point of view of . . . its 'natural' basis" (123/119). Instead of approaching this science through "the *conceptual formulation*" (124/119) of a science whose "new insights" are "derived from the traditional concepts," Klein approaches it in terms of "the whole complex of those '*natural*' *cognitions* which are implied in a prescientific activity moving within the realm of opinion and supported by a preconceptual understanding of the world" (123–24/119). Doing so not only enables him—as we have seen—to reactivate the most general conceptual presuppositions of Greek arithmetic and logistic, but it will also enable him to see that the new science of "men like Galileo, Steven, Kepler, and Descartes" (124/119), while no doubt "carried by an original impulse which is quite foreign to the learned science of the schools," nevertheless "shares with scholastic science the most general presuppositions of that 'scientific attitude' which was developed by Greek science in opposition to a 'natural' existence" (124/119–20). Furthermore, seeing Greek science from this angle will enable Klein to realize that even though this new science "returns to the sources of Greek science, which had been neglected by scholastic science" (124/120), it nevertheless "interprets both the presuppositions and the sources from a basis

which is utterly foreign to ancient science." This last point cannot be overemphasized, because its realization not only is *not* self-evident, but it is also discernible only by a cognitive regard that has appropriated something of the conceptuality belonging to the original horizon that informs Greek ἐπιστήμη, an appropriation that therefore places it in a position to see both the discontinuity and the continuity between the conceptualities of ancient Greek and modern science. In other words, to grasp modern science as a "reinterpretation of the ancient body of doctrine, which brings with it a characteristic transformation of all ancient concepts," and to see also that this reinterpretation "lies at the foundations not only of all concept formation in our science, but also of our ordinary conceptuality, which is derived from the former," the radically different circumstances surrounding these two conceptualities must be established and then investigated.

Our articulation of Klein's desedimentation of the most general presuppositions belonging to Greek arithmetic and logistic has considered in detail Klein's first step in establishing these different circumstances, which now places us in a position to consider the next step, involving the establishment of the radical discontinuity between modern and Greek mathematics. Klein carries out this next step by, first, interpreting the logistical technique operative in Diophantus's *Arithmetic* from the perspective of Greek mathematical doctrine. Only on the basis of such an interpretation is it then possible, secondly, to establish the fact that Vieta's interpretation of this technique involves a complete transformation of "the structure of the objects with which mathematics deals" (129/125), a transformation that has its basis in "[a] new kind of generalization, which may be termed 'symbolic abstraction.'" It is possible to recognize that Vieta's reinterpretation of Diophantus occurs within a conceptual framework completely unknown to the latter only when the basis for comparing the conceptualities operative for each has been established, that is, when the conceptuality that informs Diophantus's work is approached from the horizon of ancient rather than modern mathematical science. Moreover, only on the basis of the contrast that emerges from this comparison is it possible to see that "the nature of the modification which the mathematical science of the sixteenth and seventeenth century brings about in the conceptions of ancient mathematics is *exemplary* for the total design of human knowledge in later times" (126/121).

With this last statement, Klein has in mind "the intimate connection between the mode of '*generalization*' of the 'new' science and its character as an '*art*'" (126/122), the "most characteristic expression" of which "is to be found in the symbolic formalism and calculational techniques of modern mathematics." He notes that the understanding of mathematics as an art "is heralded

at the end of the Middle Ages by an increasing interest in the practical, i.e., applied mathematical disciplines" (128/124), and that the latter, in the guise of "logistic and mensuration (arithmetica et geometria *practica*)" (129/125), now takes precedence over "'theoretical' arithmetic and geometry (arithmetica et geometria *speculativa*) such as especially Boethius, using Neopythagorean or Neoplatonic sources, passes on to the Middle Ages" (128–29/125). These practical disciplines "are consistently understood as '*artes*'" (129/125), and "to learn them means to master the corresponding 'rules of the art.'" The goal of the arts, then, is "'[*a*]*rtful procedure*,' the 'practica' or 'praxis' of calculating and measuring." The confluence of two events is responsible for the connection between the art character of these practical disciplines and the symbolic formalism and calculational techniques of modern mathematics. First, these disciplines "succeed in gaining recognition as part of the 'official' science" on the basis of "precisely their character as 'arts,'" which is "thought to lend them their true theoretical dignity." Second, the structure of mathematical objects is transformed by the symbolic abstraction operative in the mathematical thought of Vieta, Stevin, and Descartes, such that, in place of the determinate objects intended by premodern mathematics, modern mathematics intends the "*possibility* of making" (127/123) them "determinate," and does so precisely on the basis of letters that symbolize the value of numbers, letters that are operated on in accordance with the rules of calculation.

Because, on Klein's view, "this whole process" (129/125) "is mainly initiated by the reintroduction and assimilation of the *Arithmetic* of Diophantus," and because, as we have seen, interpreting the meaning of this work—"backwards," as it were—from the conceptual level of modern mathematics makes it impossible to describe the precise nature of the conceptual transformation of ancient arithmetic and logistic presupposed by this level, Klein endeavors, "first of all" (20/6), to "see the work of Diophantus *from the point of view of its own presuppositions*."

§ 93. Diophantus's *Arithmetic* as Theoretical Logistic

Klein writes that the standard interpretation of Diophantus's Ἀριθμητικά is based on the latter's use "of a series of abbreviations for unknowns and their powers which enter into the calculation itself" (130/126) of "the solution of those computational problems which are known today as determinate and indeterminate equations of the first and second degree" (129/126). Diophantus's manner of presenting these calculations lends it the quality of a "new abstract 'logistic'" (132/128), one that, in contrast to the "old 'concrete' study" of ἀριθμοί, is such that he "could come to be called—always with cer-

tain reservations—the 'inventor' or 'father' of our present-day algebra" (130/126). On Klein's view, however, "the opposition of 'abstract' and 'concrete' is insufficient for characterizing a work of this kind" (135/129); and the understanding of "*all* the signs which Diophantus uses" (150/146) as anything more than "merely word abbreviations" is only possible on the basis of "externally transposing" (131/127) "the content of Greek mathematics" "into another mode of presentation," rather than "by comprehending it in the one way which seemed comprehensible to the Greeks."

The inquiry into precisely how Diophantus himself understood the "being" (135/130) of the ἀριθμοί—which, "without further ado," are "simply . . . called 'abstract'" by the proponents of the standard interpretation of Diophantus—is crucial for Klein's purpose of "making explicit the particular character of Greek conceptuality, whose peculiar transformation in the sixteenth and seventeenth centuries is equivalent to the 'introduction of a completely new means of expression for mathematical thinking'" (131–32/128). Thus, while "It is, to be sure, characteristic of Diophantus that he calculates with ἀριθμοί which are nothing but *amounts of pure monads*" (135/129–30), the key question for Klein is whether their being is understood "as independent and therefore incapable of division, or as the result of reduction to 'neutral' items which merely indicate the character of the 'measure,' and thus easily admit further partition" (135/130).

Klein's answer to this question is that "which of these two possibilities is here realized cannot be in doubt." This is clear to him because "The *Arithmetic* of Diophantus is . . . a theoretical work . . . that does not scruple to introduce fractional parts of the unit of calculation into the inquiry" (138–39/133), which is something that "*is possible only if the ontological understanding of the μονάς is Peripatetic in character*" (139/133). At the same time, however, "this handling of fractional parts of the unit clearly does not diminish the 'theoretical' character of the whole work, although this ought to be the case in light of Neoplatonic conceptions" (138/132).[88] Thus, when "we consider the essential content of the Diophantine work as it appears in the formulations of the problems themselves, the thought that this is a discipline to which the Platonic definition of logistic . . . is immediately applicable can hardly be avoided" (139/133). Klein's analysis of the signs that comprise Diophantus's nomenclature will show that they have as their concern "the 'kind,' the εἶδος of definite amounts" (147/143), although "we are, to be sure, no longer dealing with the original Pythagorean-Platonic εἶδος which is responsible for the characteristics of a 'kind' because it gives unity

88. See §§ 54 and 59 above.

to, and thereby makes possible, the being of definite amounts."[89] Notwithstanding this difference, Klein will show that

> in Diophantus too the εἶδος of a definite amount means—as in the Neoplatonic mathematicians—the characteristic of its kind which it shares with other definite amounts, or by which it is, in turn, separated from them so that a classification of definite amounts can be obtained, except that this characterizing εἶδος is here understood rather in the Peripatetic-Euclidean manner, as a mere property of the various definite amounts.

Yet Diophantus's signification of and solution to calculational problems "*in terms of εἴδη themselves*" (149/144) does not mean, according to Klein, "that the εἴδη *as such are definite amounts*." Thus, although "Diophantus' calculation with the *sign* of the magnitude sought, where this sign is introduced into the very process of solution as it develops step by step" (144/139), is clearly similar to the calculation with signs characteristic of modern algebra, "the *object meant* is in each case a definite amount of monads" (149/145). Hence, for Klein, the answer is negative to "the important question whether Diophantine logistic may not contain within itself the possibility of a symbolic calculating technique" (144/139). Indeed, Klein stresses that unless this is recognized, the "true problem" (150/147) "in a historical study" of the origin of modern symbolic algebra will remain unrecognized. He therefore characterizes this problem as follows:

> How was it possible at all that in the face of a *conceptually self-sufficient and complete* calculating procedure, such as the Diophantine *Arithmetic* is *par excellence*, the idea of a symbolic algebra was conceived? Or, more exactly: *What transformation did a concept like that of ἀριθμός have to undergo in order that a "symbolic" calculating technique might grow out of the Diophantine tradition?*

Klein begins to desediment the non-symbolic character of Diophantus's theoretical logistic by addressing the question of what we know about its originality, given that "the *form* in which this material is presented" (130/126–27) by Diophantus "cannot be documented before him" (130/127). The *content* of Diophantus's theoretical logistic, however, can be so documented, and the fact that "The *material* assimilated in Diophantine problems is to be found already" (130/126) in earlier works provides "reason for the claim that he must have been related to his predecessors somewhat as Euclid was to the authors of the *Elements* . . . , whose works were completely eclipsed by the Euclidean compilation" (130/127). The relation to earlier Greek sources, however, does not prevent Klein from holding that "it can hardly be denied" (131/127) "[t]hat the science of Diophantus exhibits certain non-Greek traits." Notwith-

89. See § 64 above.

standing these traits, Klein writes, "whatever the case for the more remote prehistory may be—the conceptuality of a work which developed, as this one did, on Hellenistic soil must first of all be understood on the basis of *Greek* presuppositions." Moreover, regarding "the origin of modern algebra and its formal language" (151/147), the fact that the "*Arithmetic* of Diophantus may, to be sure, itself refer back to a pre- and *non*-Greek, perhaps even a 'symbolic,' technique of counting" changes nothing regarding the fundamental role of both this work and Greek science in this origin. This is the case, for Klein, because "it was precisely the *direct* assimilation of Diophantus' work and of its concepts which was crucial" for this origin.

Turning now to Klein's desedimentation and reactivation of these concepts, his analysis of Diophantus's text establishes four interrelated and interdependent points that rule out its standard interpretation as a work whose conceptuality, although rudimentary, is commensurate and therefore continuous with the conceptuality of modern algebra and symbolic mathematics in general.[90] The first is that the "numbers" operative in Diophantus's logistic concern monads. The second is that the precise nature of this concern with monads is the reference to there always being so and so many—that is, a definite amount—of them. The third is that all the signs that Diophantus uses, including his sign for an indeterminate multitude of monads, 'ς', are word abbreviations for concepts that are only provisionally indeterminate, for they are indeterminate only from the standpoint of the determinate solution. The fourth and final point is that Diophantus's use of the εἶδος concept is completely instrumental, which means that its significance is likewise instrumental and therefore not ontological.

§ 94. The Referent and the Operative Mode of Being of Diophantus's Concept of Ἀριθμός

That the *ἀριθμοί* operative in Diophantus concern monads Klein establishes on the basis of both Diophantus's explicit statement to this effect and the

90. Klein refers to the following as representatives of the standard interpretation: Thomas L. Heath, *Diophantus of Alexandria: A Study in the History of Greek Algebra* (Cambridge: Cambridge University Press, 2d ed., 1910); Hermann Hankel, *Zur Geschichte der Mathematik im Altertum und Mittelalter* (Leipzig: Teubner, 1874); Friedrich Hultsch, "Diophantos," in A. Pauly and G. Wissowa (eds.), *Real-Encyclopädie der klassischen Altertumswissenschaft* (Stuttgart: Metzler, 1894–1980); Otto Neugebauer, "Studien zur Geschichte der antiken Algebra I," *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, Abteilung B, II (1936), 1–27; G. H. F. Nesselman, *Algebra der Griechen* (Berlin: Reimer, 1842); Paul Tannery, *La géométrie grecque* (Paris: Gautiers-Villars, 1887); and H. G. Zeuthen, *Die Lehre von den Kegelschnitten im Altertum* (Copenhagen: Höst & Sohn, 1886).

meaning of the letter he uses to designate ἀριθμοί that are presupposed as known. Klein reports that in "'Definition I' (2, 14 f.)" (136/130) of his *Arithmetic*,[91] Diophantus states that "'all definite amounts consist of a certain multitude of monads'. . . ." In addition, Klein observes that "in accordance with 'Definition II' (6, 6–8)," "the definite amounts which are presupposed as 'known' (the determinates – οἱ ὡρισμένοι – 6, 6 f.) in the calculation" are designated by a "letter which gives the definite amount of monads," a letter that "usually has . . . the sign for μονάς ($\mathring{M}$) as a prefix; for instance: $\mathring{M}\overline{\delta}$ (four units)."[92] Klein concludes from this that "the use of the sign itself was the result *of the meaning of the word ἀριθμός in common speech, namely: a definite amount of definite objects*—in the extreme case this might be a definite amount of units of measurement, or of 'pure' or 'neutral' monads" (136/131).

Klein also finds it significant for establishing the referent proper to the ἀριθμός concept operative in Diophantus that its linguistic usage "also occurs throughout the *Metrics* of Heron" (137/131) and in "the second book of Pappus . . . where the system of calculation and the nomenclature of Apollonius are described." Klein notes that this linguistic usage "is, incidentally, the basis both of Apollonius' system of nomenclature and of Archimedes' exposition in a lost work addressed to Zeuxippos as well as in the *Psammites*." Specifically, both of these works "depend on the introduction of *units* of a higher order; they are called by Archimedes 'definite amounts ⟨formed⟩ analogously *to those based on monads*' (ἀριθμοὶ ἀπὸ μονάδος ἀνάλογον . . .), by Apollonius simply 'analogous definite amounts' or 'analogues' (ἀνάλογοι [ἀριθμοί] or τὰ ἀνάλογα . . .)." This linguistic usage allows "unusually large definite amounts of simple monads to be pronounced, written down, or made objects of calculation without much difficulty," and in this manner represents "nothing but the consistent development of the Greek mode of thought and speech" (137/132). In the case at hand, what is developed is the "ordinary system of counting" (137/131), where "the *myriad*, the unit-ten-thousand (μυριάς), forms a new and higher unit, and so, actually, does the *chiliad*, the unit-thousand (χιλιάς), if the numerical adjectives thousand, two thousand, three thousand (χίλιοι, δισχίλιοι, τρισχίλιοι) and their written form are taken into account." Hence, Klein's point is that Diophantus's sign '$\mathring{M}$', like the myriad, chiliad, etc., has a determinate referent: the units whose amount composes the ἀριθμός in question.

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91. Diophantus, *Arithmetica* (Ἀριθμητικά), in *Diophantus* (Greek and Latin), ed. Paul Tannery (Leipzig: Teubner, 1893–1895).

92. According to Heath, the "use of the letters of the alphabet as numerals was original with the Greeks" (*A History of Greek Mathematics*, 32). Thus, α = our numeral 1, β = our numeral 2, γ = our numeral 3, δ = our numeral 4, etc.

Regarding the mode of being of ἀριθμός in Diophantus, Klein holds that "In the Diophantine text calculation with fractional parts of the unit of calculation, i.e., the division of this unit, is simply admitted without comment" (137/132), which attests that an Aristotelian understanding of the ontological status of the μονάς is operative therein. That is, it attests to the fact that "we are dealing with 'neutral' monads obtained 'by abstraction' (ἐξ ἀφαιρέσεως)" (139/133).[93] In this respect, Diophantus's *Arithmetic* is therefore "altogether irreconcilable with that of the Neoplatonists" (137/132).[94] Yet the unit's partitioning does not detract from the "'theoretical' character of the whole work" (138/132), as it does in Neoplatonism. Moreover, because it poses and solves all its problems "in terms of 'pure' monads," together with the "strictly 'methodical,' i.e., transferable to other cases of the same type," nature of each solution (ἀπόδειξις), it is consistent with the Platonic definition of theoretical logistic as the part of mathematics that concerns "all those *relations*, i.e., ratios (λόγοι) among 'pure' units, on which the success of any calculation depends" (35–36/24)."[95]

According to Klein, "by a fraction Diophantus means nothing but *a definite amount of fractional parts* ***of the unit of calculation***, or a single such fractional part.[96] The 'magnitude' of such a fractional part corresponds to 'how many' (i.e., what multitude of) partitions the μονάς undergoes" (142/137). Therefore, "*In its character as a measure*, the μονάς itself is not affected by any partition." This can be seen, on the one hand, where Diophantus in "'Def. V' (8, 11 f.)" (143/138) maintains that "When a fractional part of a unit is multiplied as many times as the homonymous definite amount indicates, the whole original unit is reconstituted." Likewise, it can be seen, "On the other hand, where the fractional parts of an original unit may be transformed into a new unit by *changing the measure* . . . , which is precisely what is done by Diophantus in problems I, 23, 24, 25; III, 14; VI, 2 and 16." Thus, as we saw in § 56 above in the Peripatetic account of abstraction, "What is divided is only a bodily, or perhaps a geometric, i.e., linear, measuring 'item,' which *is* in itself divisible" (143/137–38). For Klein, then, "Diophantus' use of 'fractions' should by no means lead us to conclude that on the level of Diophantine logistic the concept of ἀριθμός comprehends the whole of what we call the 'realm of rational number'" (142/136).

93. See § 87 above.

94. See § 56 above.

95. See § 53 above.

96. The phrase '*the unit of calculation*, or a single such fractional part' renders "***der Recheneinheit*** bzw. einen einzelnen solchen Bruchteil," which was not included in the English translation.

Fractions, let alone "negative definite amounts," are not ἀριθμοί for Diophantus—the former, because they are parts of the unit (μονάς), which is not a definite amount; the latter, because they are "impossible." Regarding the latter, Klein notes that whenever an equation "leads to a 'negative' or 'irrational' solution," Diophantus "in each case" introduces "a 'restrictive condition' (προσδιορισμός)." Such equations are therefore "'impossible' (ἀδύνατος) or 'not enunciable' (οὐ ῥητή – i.e., ir-rational), just as it is 'absurd' (ἄτοπον) to state such solutions, particularly to state an 'ir-rational definite amount' (ἀριθμὸς οὐ ῥητός . . .)—an irrational ἀριθμός is precisely not an ἀριθμός at all" (143–44/138).

§ 95. The Ultimate Determinacy of Diophantus's Concept of Unknown and Indeterminate Ἀριθμοί

Having established that Diophantus's concept of definite amount follows "directly from that aspect of the ἀριθμός which implies that it is a 'definite amount *of something*' (ἀριθμός τινος . . .)" (144/139), Klein next establishes that "the other aspect of the ἀριθμός, namely that the objects meant by it are precisely *so and so many*, is also preserved in Diophantus, as indeed in the whole of Greek mathematics." He does this by showing that the significance of "the use of the sign for the unknown in the Diophantine text itself" (145/140) is *ultimately determinate* in a sense that decisively rules out what has been its misinterpretation, since Vieta, as a sign for the *possible* determinacy of its referent, which is to say, as a symbol whose meaning is inseparable from a symbolic calculating technique. The sign in question, 'ς',[97] represents the "special Diophantine sense of an *unknown* definite amount," in which "the ἀριθμός is defined as 'having in itself an *indeterminate* multitude of monads' . . . (6, 4 f.)." However, and this is what decisively settles the issue on Klein's view, "The multitude of monads which the unknown definite amount contains is . . . indeterminate only 'for us' (πρὸς ἡμᾶς)." This is because, for Diophantus, "The whole point of each problem lies precisely in this—that a *completely determinate amount of monads completes the solution of each problem*."

For Klein, then, in Diophantus the "'indeterminate' solution forms only a stage preliminary to the final one" (140/134), which, as final, "is

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97. Klein follows "Heath's typography in using the final sigma to represent this sign" (*GMTOA*, 145/140); see Heath, *Diophantus of Alexandria*, 37. Klein also maintains that, "as Heath has convincingly shown" (*GMTOA*, 150/146), this sign "represents nothing but a ligature for αρ (= ἀριθμός)."

throughout concerned only *with finding wholly determinate definite amounts* (and under certain circumstances also wholly determinate fractional parts of the unit of calculation) *such as have to each other a certain particular relation given by the problem*." Thus, even though "Diophantus does know problems and solutions of a 'general' kind (in his terms: 'in the indeterminate ⟨solution⟩' . . .), namely those which leave their object '*indeterminate*'" (139/134), Klein maintains that "it becomes apparent *that such problems and solutions have a merely auxiliary character*" (140/134). Specifically, the "unknown" (145/140) is "to be understood as an 'indeterminate multitude' (πλῆθος ἀόριστον) only from the point of view of the completed solution, namely as 'provisionally indeterminate,' and as an amount *which is about to be exactly determined in its multitude*." Moreover, this connection to the determinate that belongs to the "indeterminacy" of Diophantus's problems and solutions "in the indeterminate solution," as well as to his "indeterminate multitude," is also shared by his "concept of the ἀριθμοὶ ἀόριστοι (indeterminate definite amounts)." That is because only in connection with geometrical "*figures* 'similar' to one another (i.e., given only in shape and not determinate size)" (145/140) does this concept become "transparent" (141/136), figures that, once their problematic proportions are correctly determined and "taken together with the further conditions of the problem" (141/136), permit a "univocal solution" to the problem to follow "immediately."

Among other things, the peculiar "indeterminacy" at issue in Diophantus's solutions and concepts means that "When the unknown or its sign is introduced into the process of solution, it is precisely *not* indeterminacy in the sense of 'possible' determinacy which is meant" (145/140), the latter being precisely what, as we shall see, is meant by the signs used in a symbolic calculating technique.[98] Rather, what is meant is always a provisional indeterminacy, the provisionality of which is established by the exactly determined multitude that comprises for Diophantus the complete solution to the problem. For Klein, then, there is "no sharp boundary between 'the determinate' (οἱ ὡρισμένοι – 6, 6 f.) and the 'indeterminate' definite amounts (ἀόριστοι)" (146/142) in Diophantus. Moreover, when "definite amounts are marked off by brief names indicating their kind" (146/141), and when each of these names, in turn, is "rendered by a 'sign' (σημεῖον)," such that each kind may be formulated in terms of its "root" relation to "'an indeterminate multitude of monads' . . . and its sign, ς," "it is by no means their original character to be merely powers of an unknown or 'indeterminate' definite amount."

98. See §§ 94 and 103 below.

Klein establishes the latter by "attending to the way in which these terms are introduced by Diophantus himself" (145/140). The "quadratic and cubic definite amounts and furthermore those definite amounts which result from 'squaring' the quadratic, from multiplying a quadratic with a corresponding cubic (i.e., one which has the same root as the quadratic), and finally from the 'quadrature' of a cubic definite amount" (145/140–41) are listed by Diophantus "*as falling under* (ἐν τούτοις – 2, 17)" (145/140) "the series of definite amounts going to infinity." The names of these definite amounts, "δύναμις, κύβος, δυναμοδύναμις, δυναμόκυβος, κυβόκυβος, i.e., square, cube, square-times-square, square-times-cube, cube-times-cube" (146/141), indicate "their kind," and are called by Diophantus "the elementary constituents of 'arithmetical' science (4, 12–14)." Furthermore, he says that "the texture of most 'arithmetical' problems is knit (4, 7–10)" "by interweaving these and their roots in various ways" (145/141). In the solutions that follow Diophantus's signifying each of the aforementioned kinds of definite amounts, respectively, as "Δ^{Υ}, K^{Υ}, $\Delta^{\Upsilon}\Delta$, ΔK^{Υ}, $K^{\Upsilon}K$" (146/141), Klein stresses that "no unknown or indeterminate definite amount is so much as mentioned." When "the expression ἀριθμός in the sense of an 'indeterminate amount of monads' (πλῆθος μονάδων ἀόριστον) and its sign, ς, as well as the sign for the immutable μονάς" are finally introduced, such that "within the course of each solution the definite amounts rendered by Δ^{Υ}, K^{Υ}, $\Delta^{\Upsilon}\Delta$, ΔK^{Υ}, and $K^{\Upsilon}K$ have as their root (cf. 2, 19 f. . . .) precisely the ς for each case, it is by no means their original character to be merely the powers of an unknown or 'indeterminate' definite amount. For even as powers of the ς they represent only transformations of a *determinate* amount of monads." Thus, Klein maintains that "it is not permissible simply to call the expressions δύναμις, κύβος, δυναμοδύναμις, δυναμόκυβος, κυβόκυβος 'powers of the unknown.'"

Klein contends that "Exactly the same thing holds true also for the fractional parts homonymous with these, namely the 'fraction named after a 'definite amount,' 'square,' 'cube,' etc. . . . ⟨i.e., the reciprocals of the terms named earlier⟩ (. . . 6, 9–21)." That is, these are not "powers of the unknown"; rather, each is a fractional part of the unit that, when multiplied by its homonymous definite amount, "yields *one*" (146/142). Moreover, this "is exclusively based on the fact that this holds for the multiplication of *any* definite amount with its homonymous fractional part." Indeed, "That the multiplication of several fractional parts of the unit again yields fractional parts which are homonymous with the product of the definite amounts homonymous with the original fractional parts ('Def. VII') is something that likewise holds true of *all definite amounts*, and, *therefore* also those special multitudes of monads which occur within the course of the solution

and which are, therefore, at that point still 'indefinite'" (146–47/142), establishes the following for Klein: At issue in all of Diophantus's "rules for multiplication" (147/142) are precisely *not* rules in the modern sense, namely, "rules by which certain relations among newly introduced magnitudes, *and thus the magnitudes themselves*, are originally determined." On the contrary, "they are . . . the rules of calculation already in use within the *ordinary* realm of definite amounts, which are here, in deference to the unfamiliar notation, merely explicitly confirmed." Thus, Diophantus suggests that "these rules can be inferred from the nomenclature itself: 'They will be clear to you because they are pretty well expressed in the name itself' (. . . 6, 24 f.), for the relevant expressions show directly the multiplicative components of the definite amounts which are meant" (147/142–43).

§ 96. The Merely Instrumental, and Therefore Non-ontological and Non-symbolic, Status of the Εἶδος-Concept in Diophantus's Calculations

The nomenclature at issue here concerns, of course, "the 'kind,' the εἶδος, of definite amounts" (147/143), with which Diophantus calculates by means of designative signs. Thus, "all definite amounts which can be rendered with the aid of the signs ς, Δ^{υ}, K^{υ} . . . ς^{χ}, $\Delta^{\upsilon\chi}$ . . . represent, in respect to their multiplicative composition, one εἶδος each, while all definite amounts which are written with the aid of the sign $\mathring{M}$ likewise form an εἶδος by themselves (cf. e.g., 114, I f.: 'the rest of the classes . . . of the unknowns and units' – τὰ λοιπὰ . . . εἴδη τῶν ς καὶ τῶν $\mathring{M}$)." Characteristic of this, Diophantus thus holds "the sign χ" (148/143)—"which denotes reciprocals (6, 20 f.)" (146/141)—"to be a sign 'distinguishing the class' (διαστέλλουσα τὸ εἶδος – 6, 21) in that it allows the εἶδος in question (e.g., $K^{\upsilon\chi}$) to be distinguished from the homonymous εἶδος (K^{υ})." Moreover, "with reference to the classes enumerated, the division of one definite amount by another of another class is mentioned: 'the partitionings of the aforementioned classes' (. . . 14, 2); and practice in the additive, subtractive, and multiplicative combination of definite amounts of the same or different classes is recommended (14, 3–10)" (148/143–44). Diophantus's concern with the εἴδη of ἀριθμοί is also apparent in "the general rule (14, 11–23) for the treatment of equations of the first order (and of pure quadratic and pure cubic equations, etc.)" (148/144), which "has precisely this object—to order and combine the definite amounts *according to their class membership* until finally both sides (μέρη, i.e., parts) of the equation are reduced to definite amounts of *one class*, that is, 'until one class becomes equal to one class' (. . . 14, 14)." Finally, Diophantus speaks of his problems "ex-

plicitly as 'having their material for the most part concentrated *in the εἴδη as such*' (. . . 14, 25–27)." Klein thus considers it to be "of the greatest significance that here and throughout *the εἴδη themselves are directly mentioned*, and not the definite amounts, each of which belongs to a certain εἶδος" (147/144). This means that Diophantus's "presentation and solution of the problems takes place essentially *in terms of the εἴδη themselves*," εἴδη that, as "a 'characteristic of the kind' of each determinate definite amount," are manifestly *not* themselves definite amounts. In this connection, Klein writes:

> Here, in the realm of mathematics, we thus find mirrored, albeit in a disjointed fashion which is no longer appropriate to the original phenomenon, the relation between εἶδος and single object—in the language of the schools, between "secondary" (δευτέρα) and "primary substance" (πρώτη οὐσία). On its home ground, that is, within Greek conceptuality, this relation is taken as a "matter of course"; in Aristotelian ontology it is forcefully brought to the fore: As the single object is *just that* which we name in its εἶδος, and is therefore accessible to "scientific" treatment through it alone, so the definite amounts which the Diophantine problems are supposed to yield are available to the "scientific" grasp only in their "eidetic" properties; and this is precisely because in each εἶδος a completely determinate amount of monads is always *meant*. (149/145)

According to Klein, then, both Diophantus's explicit statements and his calculative practice make it necessary to "distinguish strictly between *procedure* and the *object*; while the procedure is applied to the εἴδη which are as such independent of each 'multitude of monads' (πλῆθος μονάδων) and in this sense 'general' (καθόλου), the object meant is in each case a determinate amount of monads" (149/144–45). Thus, for Klein, notwithstanding the vast difference between "Diophantine *Arithmetic* and the Neoplatonic 'arithmetical' science" (149/145), a difference rooted, as we have seen, in the former, but not the latter, permitting the calculation with fractions, they are "altogether compatible" insofar as each "deals with the multitude of monads essentially only in the medium of the εἴδη." However, this compatibility should not be allowed to obscure what fundamentally distinguishes Diophantus's εἴδη from those of the Neoplatonists: according to Klein, it is "their purely *instrumental* significance." Thus, unlike the Neoplatonic εἴδη, which, as we have seen, are inseparable from the ontological positing of both their indivisibility and their somatic independence, Diophantus's placing of "[a]ll definite amounts, which in this sense[99] have one and the same εἶδος" (147/143), into "one and the same 'class,'" "has therefore no ontological significance" (148/143). Its significance is rather taxonomical, "exactly as, for instance, all hundreds belong to one and the same τάξις, although it may be that

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99. That is, in the Aristotelian sense of the εἶδος as merely the non-independent property of ἀριθμοί.

the same definite amount is included in some other respect in another class." Hence, according to Klein, "Within the calculating procedure itself the εἴδη are used as special units of calculation exactly in the same sense in which units of a higher order occur in Archimedes and Apollonius[100] and in which the myriad is used in other places (cf. in Diophantus himself, 332 f.: 1 second-degree myriad, and 8-thousand 7-hundred 4-ty and 7 first-degree myriads and 4-thousand 5-hundred and 60 monads [187,474,560 in our notation]— β′ Mʸ α καὶ α′ ͵ηψμζ καί M̊ ͵δφξ = δευτέρων μυριάδων α καὶ πρώτων ͵ηψμζ καὶ μονάδων ͵δφξ)" (149/145). That is to say, they are used as "units of calculation" that, "in every single case" (150/146) "mean a definite amount of units." And Diophantus's signs are also used in this way. Thus, "the sign for the ordinary unit, M̊, is introduced as a complete parallel for the sign ς [the unknown amount of units]: 'and its ⟨i.e., the definite amount's⟩ sign is ς. There is also another sign, the invariant sign of determinants, the monad . . . M̊ (. . . 6, 5–8)."

Klein thus shows that, notwithstanding "the facility with which Diophantus carries out the multiplications of expressions which are composed of definite amounts of different 'kinds,' the matter-of-course fashion in which he handles such expressions in general . . . , and, finally, the purely instrumental use he makes of the εἶδος concept" (151/147), Diophantus's *Arithmetic* cannot be accurately characterized as a symbolic logistic. Yet Klein nevertheless acknowledges that "an inner tension between the 'material' treated and the character of the concepts forced on it" is evident in this *Arithmetic*. As a consequence, he holds that there is "some justice" (150/151) for Léon Rodet's division of 'algebra' into "two types," "namely 'l'algèbre des abbréviations et des données numèriques' and 'l'algèbre symbolique,'"[101] maintaining that "It is to the first type, that of the algebra of abbreviations and numerically given constituents, that the Diophantine '*Arithmetic*' unquestionably belongs."

Before turning to his account of Vieta's realization of the "fundamental transformation of the conceptual foundations" (152/148) of Diophantus's—and, of course, with this, of ancient Greek—arithmetic and logistic, Klein briefly sketches the lineage of his "*direct* assimilation" (151/147) by the tradition leading up to and through Vieta, Stevin, and Descartes. First, Rafael Bombelli, "under the influence of the impression made by a reading of the Diophantine manuscript" (151/148), "changes the 'technical' character

100. See § 91 above.

101. Léon Rodet, *Sur les notations numériques algébriques antérieurement au XVI siècle* (Paris: Ernest Leroux, 1881), 69.

of his manuscript" (152/148), *L'algebra, opera di Rafael Bombelli da Bologna*, "which was probably already written by 1550."[102] Moreover, the "third book of his *Algebra*, which appeared in a new version in 1572, now contains also (in modified, partly 'generalized' form) the majority of the problems in the first five books of Diophantus. Here the original verbal 'cloaking' of the problems, which had been the practice in previous 'algebraic' works, is abandoned in favor of a 'pure' form taken over from none other than Diophantus." Second, "In 1575 there appears the first Latin translation of Diophantus by Xylander."[103] Third, "in 1577 the algebraic work of Guillaume Gosselin"[104] appears, "which includes a treatment of the equations of Diophantus." Finally, fourth, "Diophantus is put into modern 'symbolic' form in 1585 by Stevin and in 1591 by Vieta." Klein notes that even though "Stevin, who depends directly on Bombelli for his terminology and symbolism, already makes the new '*number*' *concept as such* which is the basis of the 'symbolic' procedure completely explicit" (152/148–49), it is "Vieta who, by means of the introduction of a general mathematical symbolism" (152/149), transforms Diophantine logistic into a symbolic logistic and thus should be credited with inventing modern algebra. We now turn to Klein's desedimentation and reactivation of this epochal event.

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102. Rafael Bombelli, *L'algebra parte maggiore dell'aritmetica divisa in tre libri* (Bologna: Giovanni Rossi, 1572).

103. Diophantus, *Res arithmeticae* = Diophanti Alexandrini Rerum Arithmeticarum libri sex, quorum primi duo adiecta habent Scholia, Maximi (ut coniectura est) Planudis. Item Liber de Numeris Polygonis seu Multangulis. Opus incomparabile, verae Arithmeticae Logisticae perfectionem continens, paucis adhuc visum. A Guil. Xylandro Augustano incredibili labore Latinè redditum, et Commentariis explanatum, inque lucem editum, ad Illustriss. Principem Ludovicum Vuirtembergensem (Basel: Episcopius & Nicolai, 1575).

104. Guillaume Gosselin, *De arte magna seu de occulta parte numerorum quae et Algebra et Almucabala vulgo dicitur* (Paris: Gilles Beys, 1577).

Chapter Twenty-two

Klein's Account of Vieta's Reinterpretation of the Diophantine Procedure and the Consequent Establishment of Algebra as the General Analytical Art

§ 97. The Significance of Vieta's Generalization of the Εἶδος-Concept and Its Transformation into the Symbolic Concept of Species

What is at stake for Klein in Vieta's "analytical art" is the birth of both "the *modern concept of* '*number*,' as it underlies symbolic calculi" (183/176), and the expanded—in contrast to ancient Greek science—scope of the generality of mathematical science itself. Klein writes that the former "heralds a general conceptual transformation which extends over the whole of modern science" (183/175), while the latter lends the "treatment" (πραγματεία; 187/179) at issue in the ancient Greek mathematical idea of a "'general treatment' [καθόλου πραγματεία]" "a completely new sense" "within the system of 'science.'" The generality of this new sense will concern both the *method* and the *object* of science in what will come to be known as 'universal mathematics' (*mathesis universalis*). This transformation of the basic concept and scope initially of mathematics and then of "*the system of knowledge in general*" (193/184) "concerns first and foremost the concept of ἀριθμός itself" (183/175). As a result of "its transfer into a new conceptual dimension" (194/185), namely, into that in which both "the concept of 'number'" and "that which it means" (183/176) are "*symbolic in nature*," a transfer that "becomes visible" (194/185) "for the first time in Vieta's 'general analytic,'" there follows "a thoroughgoing modification of the means and aims of science." Klein maintains that what this modification involves is "best characterized by a phrase . . . in which Vieta expresses the ultimate problem, *the* problem proper, of his

'analytical art': 'Analytical art appropriates to itself by right the proud *problem of problems*, which is: TO LEAVE NO PROBLEM UNSOLVED'."

§ 98. The Sedimentation of the Ancient Practical Distinction between 'Saying' and 'Thinking' in the Symbolic Notation Inseparable from Vieta's Concept of Number

Central to Klein's view of these matters is that the usual explanation of the development of the modern symbolic concept of number "by a reference to its ever increasing 'abstractness'" (183/175) is a "facile and easily misunderstood manner of speaking" (184/175–76), one that "leaves its true complicated structure completely in the dark" (184/176). That is because it belongs to the symbolic number's specific character to be indirectly related to the things or units that the ἀριθμός concept is directly related to, thereby rendering it inaccessible to the traditional Aristotelian measure of abstractness, which is calibrated in terms of a concept's degree of remoteness from or proximity to sensuously perceived things.[105] Klein elaborates the reason for this inaccessibility on the basis of the peculiar nature of the "abstractness" proper to the symbolic number concept, which he describes in terms of the direct reference of the letter sign employed in Vieta's *logistice speciosa* to "*the general character of being an amount* that belongs to every possible definite amount" (182/174). That is to say, Vieta's letter sign "*immediately means the general character of being an amount* that belongs to every possible definite amount—i.e., 'amount overall'—and only *mediately* the things or units that may be present in any particular definite amount." According to Klein, one important consequence of this is that "the '*being*' of the species [or εἶδος]" (182/175) to which the letters of Vieta's notation refer, "i.e., the 'being' of the objects of the 'general analytic,' is to be understood neither as independent in the Pythagorean and Platonic sense nor as obtained 'by abstraction' (ἐξ ἀφαιρέσεως), i.e., as 'reduced' in the Aristotelian sense, but as *symbolic*" (182–83/175) in the precise sense of "*formations whose merely possible objectivity is understood as an actual objectivity*" (183/175).[106]

Thus, for Klein, "As soon as the 'amount in general' is conceived and presented in the medium of the species as an objective *formation* in itself, that

105. See § 103 below.

106. The novelty of Klein's desedimentation of the peculiar "abstractness" at issue here, as well as the inherent difficulty of the issues it involves, is exemplified by the following, which is written by the distinguished historian of modern mathematics and science, Michael S. Mahoney—in his "The Beginnings of Algebraic Thought in the Seventeenth Century," in Stephen Gaukroger, ed., *Descartes: Philosophy, Mathematics and Physics* (Brighton: Harvester, 1980),

is, symbolically, *the modern concept of 'number' is born*."[107] When Klein compares this concept of species with "the Pythagorean and Platonic concept of the εἶδος of an ἀριθμός, which as such first makes the unified being of each definite amount, as an amount of something, possible" (183/175), he maintains that "we may say that the *ontological* independence of the εἶδος, having taken a detour through the *instrumental* use made of it by Diophantus . . . here finally arrives at its *symbolic* realization." Furthermore, Klein maintains that this comparison reveals that "sedimented" (185/177) in the notation that is inseparable from the symbolic realization of the εἶδος is the "distinction between 'dicere' (saying) and 'cogitare' (thinking), which antiquity knows well enough in practice but never as a principle." Klein suggests[108] that this distinction is grounded in the "close relation of διάνοια and λόγος" (74 n. 1/235 n. 81) that is manifested in the "original comportment of human delibera-

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141–55—in connection with the status of Vieta as "the founder of the theory of equations, one of the greatest achievements of the seventeenth century, if not the greatest of all" (144): "Greek mathematics was intuitional and strongly dependent on physical ontology," such that "the concept of number as the thing counted, that is, as a collection of counted units, derived from this basically physical ontology of Greek mathematics" (143); and, "In this respect, the algebraic mode of thought can be characterized as an abstract mode of thought, in contrast to an intuitive one" (142). Apart from Mahoney's interpretation of the "abstractness" at issue here in precisely the terms that Klein derides as "facile," it is interesting to note that later in his article Mahoney cites Klein's book in support (!) of his (Mahoney's) claim that "Descartes also frees the concept of number from its classical intuitive foundations" (146).

107. As we mentioned above (§ 92) and shall elaborate in detail below (§ 103), Klein's desedimentation of Vieta credits his reinterpretation of the Diophantine εἶδος concept with the transformation of the "*concept of* number," specifically with the realization of something that within the compass of the structure and self-understanding proper to ancient Greek mathematics was an impossibility: arrogating a *numerical* interpretation to the εἶδος of number (number as *Anzahl*), i.e., the *concepts* related to number. The numerosity at issue in *this* reinterpretation of the Diophantine εἶδος concept, which involves (as we just noted above and shall discuss further below) the notion of 'number in general', and thus what Klein refers to here as the birth of "*modern concept of number*," is (as we shall see in § 103 below) a state of affairs Klein keeps distinct from the separate but related issue of the transformation of the "*number* concept." Involved in this latter state of affairs is the gradual identification by "the everyday understanding" (186/178) of "definite amounts of definite objects" with "ordinary numbers" in the precise sense of the coincidence of "these amounts with the 'numeral' *sign* as such." As we shall see in our discussion of Klein's desedimentation and reactivation of "the new 'number' concept'" (204/195) in Stevin (see §§ 107–110 below), it is precisely the identification of the 'number concept' with the 'number sign' or numerals used in calculation that "once and for all fixes the ordinary understanding of the nature of number, for which being able 'to count' is tantamount to knowing how to handle 'ciphers'" (205/197), and thus makes possible the extension of the "number *concept*" to include zero, one, fractions, along with the "numbers" traditionally designated as "'absurd' or 'surd' or 'irrational' (i.e., un-speakable)" (204/195).

108. This suggestion becomes apparent when the references (in n. 135 [p. 185] in the German and n. 264 [p. 278] in the English) to earlier parts of *Origin of Algebra* are considered in connection with his present discussion of this distinction.

tion" (74/73), which, as we have seen,[109] he maintains "is revealed immediately in *speech* as it exhibits and judges things," "prior to all science." On Klein's view, the close relation at issue here also points to their distinction, albeit one that is not recognized as such in Greek theory but only in their practice. He enlists as evidence for this not only Apollonius's and Archimedes' attempts to show that even large definite amounts can be expressed in language,[110] but also various accounts in Plato's dialogs. In the *Sophist*, this involves διάνοια and λόγος, which are the "'same' [ταὐτόν]" (74 n. 1/235 n. 81),[111] save for the former being "'voiceless'"; in the *Theaetetus*, their relationship is characterized by the status of λόγος as either a likeness (εἴδωλον; 74 n. 1/236 n. 81) of διάνοια or its making it "'apparent . . . through the voice'" (74 n. 1/236–37 n. 81). In the case of saying large definite amounts, it is the obvious importance for both Apollonius and Archimedes of *saying*—and not just writing—what is *thought* that indicates for Klein a practical recognition of the distinction between διάνοια and λόγος. That is, their very endeavor to show that *even* very large definite amounts can be spoken points to their tacit awareness that *saying* such definite amounts, which readily enough can be *thought*, is something of an issue. In the case of the Platonic distinctions, the practical distinction between διάνοια and λόγος is attested both by the Stranger's and Theaetetus's lack of an investigation of the former's statement regarding the exception to their being the same and the unproblematic way in which the λόγος is maintained to make "apparent" διάνοια and be its "likeness." In each case, the implicit distinction is manifestly *not* subject to an investigation aimed at providing an "account of the reason why" (λογισμὸς αἰτίας), which is to say, a theoretical investigation and therewith recognition of the distinction—hence the practical significance of this distinction for Klein.

Klein's claim that the practical distinction between 'saying' and 'thinking' is sedimented in the innovation of the symbolic notation that both defines and makes possible modern mathematics should not be taken, however, as amounting to the claim that with its sedimentation this distinction is now known as a principle by mathematics. On the contrary, his point is that this very distinction is no longer encountered even at the level of practice by the modern science that this notation makes possible. Harbored within this distinction is the defining ontological problematic for Greek science,[112] how to conceive of the relation between its concepts (i.e., εἴδη) and the objects (i.e., sensuously perceived things or intelligible units) to which they refer. That is

109. See § 63 above.
110. See § 91 above.
111. See *Sophist* 263 E.
112. See Part II, § 28 above.

because the signs belonging to the art[113] of symbolic calculation *are not directly related* to the things or units to which the concepts of Greek science are directly related, being instead directly related *to concepts*, to the *species* of these things and units, and only (as we shall see) indirectly related to the things and units that these *species* originally are the *species* of in Greek science.[114] Moreover, these signs refer to concepts in such a way that *the conceptuality of the concepts to which they refer becomes inseparable from the significative status of the signs themselves*: sign and concept, as we shall also see, are therefore no longer distinct.[115] The consequence of all this for Klein is that the Greek practice of *saying*, or attempting *to say*, what it is that is *thought* in the concepts of science *becomes superfluous* for the art of calculating with signs, that is, for *symbolic calculus*. Hence, the practical distinction between *thinking* and *saying* what it is that the concepts of science refer to *is lost* when signs—or, more precisely, *symbols*—become inseparable from science's concepts. This is forcefully exhibited in the words of Diophantus's first editor, Bachet de Méziriac, written in 1621, which Klein quotes: "'Who, in fact, when he hears the number 6 does not at once *think* of six units? *Why then is it necessary to say 'six units' when it suffices to say 'six'*?'" (185/177). On Klein's view, the sedimentation of the distinction here

> holds both for the formal language of algebra and for the representation of the definite amounts of definite objects themselves in this sphere: The "definite amounts" [i.e., the "definite amounts of definite objects themselves" just referred to] are now conceived as "ordinary numbers," and for the everyday understanding, which in effect determines the actual formation of concepts, these definite amounts coincide with the "numeral" *sign* as such, especially in the course of specific calculational operations. (185–86/177–78)

§ 99. The Decisive Difference between Vieta's Conception of a "General" Mathematical Discipline and the Ancient Idea of a Καθόλου Πραγματεία

Because Vieta's "species serve as the objects" (187/179) of his "'general' analytic," and because, moreover, they do so "*in their numerical character*," Klein maintains that "Vieta's conception of a 'general' mathematical discipline and the ancient idea of a καθόλου πραγματεία, a 'general treatment,'" are decisively

113. Which is, of course, a practice, albeit one with a distinctively modern character, the very character of which, in contrast to ancient Greek practice, is precisely what is involved in Klein's suggestion that the latter's practical distinction between διάνοια and λόγος is sedimented in the former's symbolic notation.

114. See § 103 below.

115. Ibid.

different. The latter idea concerns "the *general theory of proportions* of Eudoxus as transmitted in the fifth book of Euclid" (162/158), with the generality of its theory being rooted in the fact that its demonstrations "are identical neither with arithmetical nor with geometric nor with astronomical theorems" (163/159). To the theory itself here "belong, above all, the theorems of the theory of ratios and proportions." Klein holds that "already for Aristotle the Eudoxian theory of proportions, together with the 'common notions' (κοιναὶ ἔννοιαι), formed the classical example of a discipline that proceeds 'generally' without being bound to a *specific sphere of objects*: 'Among the mathematical ⟨sciences⟩ . . . geometry and astronomy *deal with a nature of a certain kind, while the general science is common and about all things*' (. . . *Metaphysics* E 1, 1026a 25–27)" (162/158). Likewise in this connection, Klein cites the *Posterior Analytics*: "'Just as it used to be proved separately that a proposition can be alternated {cf. Euclid V, Def. 12 and Prop. 16} insofar as it is a proportion of definite amounts and lines and solids {the last with reference to astronomy, cf. the previous quotation and *Metaphysics* M 2, 1077 1–4; 12}, *so it now can be shown for all in one demonstration*; but because all of these—definite amounts, lengths, time, solids—did not have any one name and differed from one another in species they were taken separately, but *now* {using the model of Eudoxus} *it is proved generally*; for it belongs to them ⟨to be alternable⟩ not insofar as they are lines or definite amounts, *but insofar as they are in general supposed to have this property*.' (. . . A 5, 74 a 17–25)" (163/159).

Klein also enlists testimony from Proclus regarding "'one science' (μία ἐπιστήμη) which gathers all mathematical knowledge into one," a science whose "'common theorems' (τὰ κοινὰ θεωρήματα)" can, "it is true, be studied 'in definite amounts and magnitudes and motions' (ἐν ἀριθμοῖς καὶ ἐν μεγέθεσι καὶ ἐν κινήσεσι), but which are identical neither with arithmetical nor with geometric nor with astronomical theorems."[116] Proclus characterizes this science as involving theorems "'*about proportions*, their compositions and divisions, their conversions and alternations, furthermore about *all ratios*, such as their multiples, their superparticulars,[117] their superpartients[118] and their opposites, *and the theorems established generally and in common for the equal and unequal simply*' (. . . Proclus, *in Euclid*., 7, 22–27)" (163/159–60). Klein notes that by the last mentioned theorems, "Proclus

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116. Proclus, *In primum Euclidis elementorum librum commentarii*, ed. Gottfried Friedlein (Leipzig: Teubner, 1873), 7 ff.; 18 ff.

117. A ratio in which the antecedent contains the consequent once with one aliquot (contained an exact amount of times in something else) part over.

118. A ratio in which the antecedent contains the consequent once with any amount of aliquot parts over.

means theorems which are not tied to 'figures or definite amounts or motions (of heavenly bodies)' (ἐν σχήμασιν ἢ ἀριθμοῖς ἢ κινήσεσιν) but in which 'a certain common nature' (φύσις τις κοινή) is grasped as such, 'itself by itself' (αὐτὸ καθ' αὑτό)" (163/160).

Klein locates "the crucial difference" between the "generality" at issue in the ancient "general theory" and in Vieta's "general analytic" "precisely" in the "'numerical' character of the species,"[119] which, "while preserving the position of this πραγματεία within the system of science, lends it a completely new sense." The new sense concerns the shift in the very conceptuality proper to the "generality" of mathematical science that occurs when the meaning of the species of the numbers conceived as "definite amounts of definite objects," species whose status (as εἴδη) is non-numerical for Diophantus as well as for all ancient science, assumes with Vieta a numerical character. As we have seen,[120] for the ancients the εἴδη of ἀριθμοί are responsible for their delimitation as so and so many, *without themselves being so determined*—that is, without the εἴδη being numerical in this precise sense. For Vieta, the species are not conceived as numerical in the sense of a definite amount of definite objects, which is the salient characteristic of the ancient ἀριθμός, but they are nevertheless taken to be numerical in the transformed sense of "the general character of being an amount that belongs to every possible definite amount."[121] With this shift, not just the method of mathematics (as is the case with the ancients) but also its *object* is now general, which is a state of affairs that, according to Klein, contrasts sharply with ancient science, where "the existence of a '*general object*' is by no means a simple consequence of a 'general theory'" (166/161).

Klein appeals to both the Aristotelian and the Platonic traditions, as well as to Diophantus's peculiar blending of them, for evidence in support of this last statement. Aristotle himself leaves little room for doubt, as the following quote cited by Klein shows. "'The *general* propositions in mathematics {namely the 'axioms', i.e., the 'common notions'}, but also all theorems of the Eudoxian theory of proportions, *are not about separate structures*, which exist *outside of and alongside* the {geometric} magnitudes and definite amounts, but are precisely about these; not, however, insofar as they are such

119. Klein's claim here, that it is precisely Vieta's numerical interpretation of the species that is crucial for the transformation of the object of mathematics into a general object, seems not to have been appreciated by the few commentators who have addressed this issue. See below (nn. 146, 149, and 153), where we discuss in detail the various ways in which Klein's claim has been either overlooked or distorted.

120. See § 60 above.

121. See § 103 below.

as to have a definite amount or to be divisible {into discrete units}.' (. . . *Metaphysics* M 3, 1077 b 17–20)" (166/161–62). Hence, "There cannot be a specific mathematical object which is 'neither a {definite} amount {of monads} nor {indivisible, geometric} points, nor an {arbitrarily divisible geometric} magnitude, nor a {determinate period} of time' (. . . *Metaphysics* M 2, 1077 a 12)" (166/162). Moreover, that this is "not some special Aristotelian dogma but rather a *generally granted* premise of Aristotelian argumentation" is made evident for Klein when Aristotle writes (following the previous quote): "'and if this is {granted to be} impossible . . .' (εἰ δὲ τοῦτο ἀδύνατον . . . – 1077 a 12 f.)." Klein likewise finds in Proclus support from the Platonic tradition for his claim that in ancient science it does not follow from the generality of mathematical theory that its object is general. Klein writes that this tradition makes itself "*heard* one last time" in Proclus, who "never assigns a special *mathematical* object to the 'general treatment' (καθόλου πραγματεία)" of mathematics.[122] Finally, in Diophantus there is "[t]his same fundamental conception of the objects of mathematics," "insofar as in his problems and solutions he admits . . . only *definite* amounts of monads" (166–67/162).

§ 100. The Occlusion of the Ancient Connection between the Theme of General Mathematics and the Foundational Concerns of the "Supreme" Science That Results from the Modern Understanding of Vieta's "Analytical Art" as *Mathesis Universalis*

The position of the ancient idea of a general treatment that is preserved within the system of science by the numerical character of Vieta's species concerns its relationship to what Klein, within the context of ancient science, calls "the theme of the 'supreme' discipline" (187/179). For the ancients, this discipline is "characterized Platonically as 'dialectic' or in Aristotelian terms as 'first philosophy.'" For the early moderns, it is understood "as the general 'doctrina quantitatis,' the 'universa Mathesis' in Descartes' sense" (192/183). The completely new sense of this treatment that follows from the numerical character of its object, however, does not concern for Klein simply the contrast between this object's indeterminateness or generality and the determinateness or non-generality of the object of ancient science. Rather, it concerns

122. Klein considers the possibility that "This may, to be sure, be seen as an attempt to understand the possibility of any 'ratio,' 'proportion,' and 'harmony' on the basis of a 'common property' (κοινόν) of a *primordial kind*." He notes, however, that "as soon as this attempt is realized," as in Plato's theory of ἀριθμοὶ εἰδητικοί, "the realm of the properly 'mathematical' has been far superseded."

the very sense of the "generality" of the general treatment in each science's methodical relation to its objects. In the case of the ancient πραγματεία, a distinction in this regard is drawn between the mathematician's and the philosopher's use and cognition of the general propositions that compose the axioms and general theory of proportions in the καθόλου πραγματεία. However, in the general treatment of the *mathesis universalis* of modern science, under which heading Vieta's analysis comes to be understood, the distinction between the philosopher's and the mathematician's use and cognition of its general object is *not* made. It is not made, according to Klein, because the "universal science of Vieta . . . in his own eyes, as in those of his contemporaries" (192/184), is "the complete realization of the ancient καθόλου πραγματεία, of the 'general theory of proportion,'" the latter being understood, however, *in accord with the mathematical use and cognition of its objects.* Hence, Klein concludes, the ancient relating of the καθόλου πραγματεία to a still higher philosophical discipline or science is occluded by the modern understanding of Vieta's symbolic discipline's relation to the mathematical science of the ancients. As we shall see, this occlusion is highly significant for Klein, because it means, among other things, that "From now on the fundamental *ontological* science of the ancients is replaced by a *symbolic* discipline whose ontological presuppositions are left unclarified" (193–94/184). This occlusion is embedded in the very Latin term that came to be associated with Proclus's reference to the supreme science to which, as we have seen above, the ancients connected the καθόλου πραγματεία of mathematics: *mathesis universalis.* It is so embedded, as we have already suggested and shall discuss in more detail below, because the referent of Proclus's words was understood by the moderns to be mathematical, albeit mathematical in the sense of the inherently numerical characteristic of Vieta's art of analysis, and therefore *not* to be philosophical in the sense of the ancient ontological preoccupation with the problematic relation of general or universal concepts to the sensible and intelligible beings of which they are the concepts.

Klein substantiates his portrayal of the close connection in ancient science between the axioms and propositions belonging to the general theory of proportions and the "supreme" discipline with a discussion of how, for Aristotle, "they form not only the stock of examples of the manner in which 'first philosophy' {πρώτη φιλοσοφία} treats the theme peculiar to it . . . , but they also belong directly to this very theme itself" (187/179). This is clear for Klein from the way in which Aristotle

> poses, and answers affirmatively, the question whether the examination of "being" and the examination of mathematical axioms are to be referred to

> one single science, namely πρώτη φιλοσοφία as such: "We must say whether the concern with the so-called axioms of mathematics {cf. *Posterior Analytics* A 10, 76 b 14 f.} and with 'being' belongs to one or to different sciences. *It is clear that the examination of these things belongs to one [science], which is that of the philosopher too*," and also: "these too are the study of him who comes to know 'being' as 'being.'" (. . . *Metaphysics* L 3, 1005 a 19–22; . . . ibid. a 28 f.). (*GMTOA*, 187–88/179–80)

Klein thus maintains that for Aristotle not only does first philosophy examine and study both general propositions and being, but also the cognition of being as being is something that concerns the very study of the general propositions themselves. Regarding the distinction between the ways in which the mathematician and the philosopher deal with the subject matter of first philosophy, Klein writes:

> Whereas the mathematician makes use of "general" propositions with reference to those objects which happen to be before him, the "philosopher" treats them as *primary*, that is to say, not as they can be derived from lines or angles or definite amounts but as they are *common to all* that is countable or measurable in their *generically determined being-character as "quantitative."* (187/180)

Klein supports his account of this distinction by quoting Aristotle, who writes: "'Since even the mathematician uses the common notions *in a special way* it should belong to first philosophy to investigate their *origins*.' (. . . *Metaphysics* K 4, 1061 b 17 ff.)." The same passage in Aristotle also provides an illustration of a "'common notion applicable to all that is quantitative' (κοινὸν ἐπὶ πάντων τῶν ποσῶν)," namely, "the axiom: 'If equals are subtracted from equals the remainders are the equal'. . . ."

Klein observes that Proclus also connects the study of these common notions to knowledge of being as being and likewise distinguishes between a mathematical inquiry into such notions and an inquiry "far *superior* in rank" (188/181). Klein writes in this connection:

> Similarly Proclus, referring to the Aristotelian doctrine and stressing the general theory of proportion, asks who would be the man that would, in contrast to a "geometer" and an "arithmetician," demonstrate, for instance, the proposition "that alternation [of the middle members] again produces proportion" . . . using *either* geometric magnitudes *or* definite amounts (*in Euclid*. 9, 2–8), i.e., what man might study alternation and the like in "general" and "in itself"; "Who then comes to know alternation, be it in magnitudes, be it in definite amounts, in his own behalf . . . ?" (. . . 9, 8 f.) His answer is that this manner of inquiry belongs to an independent science which is far *superior* in rank to geometry and arithmetic: "the knowledge of these things belongs to a science prior by far" (. . . 9, 14 ff.; cf. 8, 24–26); and the "ascent from more partial to more universal understandings" . . . is that by which "we climb up to the very science of 'being' as 'being'" (. . . cf. *Republic* 534 E). (188/180–81)

Klein reports, "In the sixteenth and seventeenth century these words of Proclus acquired tremendous significance, since, as they came to be widely known, first through Grynaeus' edition of the Euclid commentary (1533), and then chiefly through Barocius' translation (1560), they were normally understood as a reference to the *Mathesis universalis*" (188/181). Thus, when Proclus "mentions 'common theorems which are both general and have arisen from one science *which comprehends all mathematical knowledge together in one*'" (Proclus, *in Euclid*., 7, 18–19), he is understood to be "referring precisely to this 'superior' science." Barocius's marginal notes make this clear, according to Klein, where in reference to Proclus's subsequent passage, which says of this science that "'It reveals in the reasoning proper to it *the truth about the gods* and the vision of things that are.' (. . . 20, 5 f.)," Barocius writes "*Divina Scientia*." Klein notes here, however, that "Proclus himself is, to be sure, of the opinion that this μία καὶ ὅλη μαθηματική (one and whole mathematics – p. 44, 2 f.) only ἐφάπτεται τῆς τῶν πρώτων θεωρίας (touches on the study of first things – p. 19, 24)" (188 n. 139/278–79 n. 268). This is because "'Dialectic,' as θριγχός (copingstone) of all the sciences (cf. Plato, *Republic* 534 E), is said to hold an even higher rank; the highest rank, finally, belongs to ὁ νοῦς αὐτός (intellect itself – pp. 42–44; cf. p. 9, 19–23)" (188 n. 139/279 n. 268). Thus, Proclus too, on Klein's view, both connects and yet maintains a distinction between the universality of the general theory of proportion and all mathematical axioms and the "supreme" science.

It is Descartes who, by inquiring (in the *Regulae*) into what the ancients might have understood by the name *mathesis* (189/182) when they referred to arithmetic and geometry, "'but also astronomy, music, optics, mechanics, and many other sciences'" as the "*parts* of mathematics," used the term "'*Universal Mathematics*'" (190/183) to characterize this *mathesis*. Descartes identified its referent as the "'general discipline'" that not only "is none other than the 'true science'" but also "'is nothing else than that which is called by the barbaric name of *algebra*.'" Klein maintains that it was precisely this latter interpretation on Descartes's part that paved the way for the common understanding, already mentioned, of the universal character of Vieta's analysis as the *mathesis universalis* in Descartes's sense. By retracing Descartes's "reflections which lead him to recognize in Pappus and Diophantus the traces of a 'true science'" (189/181), reflections in which he "is still reminded of Proclus' thinking in the Euclid commentary," Klein shows that and how the ancient distinction between the universal mathematical science and the discipline still more superior—dialectic or first philosophy—that is still retained in Proclus comes to be occluded in Descartes's reflection.

Thus, after asking what exactly it is that the various parts of mathematics are *parts of*, Descartes "continues, clearly attacking Proclus or Barocius: 'Here it is certainly not sufficient to look at the *origin of the word*'" (189/182). He reasons that "'since the name 'Mathesis' means no more than ⟨scientific⟩ *discipline*, these [i.e., the aforementioned parts of mathematics following arithmetic and geometry] might indeed, with as good a right as geometry itself, be called mathematical.'" Yet Descartes rejects identifying the object of *mathesis* with objects studied by these various parts. He does so on the grounds that "attentive observation finally makes it apparent that precisely *all* those things, and those *only*, in which 'order' and 'measure' (ordo et mensura) can be observed, deliver the object of *mathesis*, and that it is, however, emphatically of no importance whether one deals with numbers, figures, heavenly bodies, tones, etc." (189/182). Klein cites (190/182–83) in its entirety the passage in Descartes where this is established:

> And thence it became clear that there ought to be some general science which would explain everything that could be investigated in respect to order and measure when these are not ascribed to any special material, and that this same science was named—*using a word not newly appropriated but old and of accepted usage—Universal Mathematics*, since in it was contained everything on account of which other sciences are called parts of mathematics.

Klein therefore maintains that by understanding the meaning of the *mathesis*, the discipline or science of the ancients, in terms of a general science so articulated and named *mathesis universalis*, Descartes accomplishes two portentous things in, as it were, a single stroke. First, the ancient science (dialectic and first philosophy) recognized by both the Platonic and Aristotelian traditions to be connected with and yet supreme or superior to the mathematical science whose "general treatment" deals with common notions or axioms and the general theory of proportion is collapsed into the modern, algebraic understanding of the latter. That this is Descartes's understanding is most apparent in Rule IV of the *Regulae*, where, in reference to the *mathesis universalis*, he says explicitly, "*I am persuaded that this is more powerful than all other knowledge passed on to us by human agency, inasmuch as it is the source of all others*" (193 n. 149/283 n. 279). Second, with this, the "Mathesis universalis" (193/184), now understood "as algebra, is first and last an 'art of finding' (ars inveniendi) and thus, above all, a '*practical*' art." This means, among other things, that "Even if Descartes in no way consciously continues Vieta's work, yet the 'general algebra' he has in mind is precisely that 'new' and 'pure' algebra *which Vieta first established as the 'general analytic art'*" (191–92/183). Thus, Francis van Schooten, "in his notes to Vieta's *Isagoge*," characterizes it "as the general 'doctrina quantitatis,' the

'universa Mathesis' in Descartes' sense: 'Everything which comes within the scope of 'mathesis' always enjoys the name of *quantity* and is precisely that which becomes apparent only through *equations and proportions*. Thus, also *Vieta*'s Analysis must come under this name as *being of the greatest possible universality*'" (192/183–84).

For Klein, then, the "universal science of Vieta, which is in his own eyes, as in those of his contemporaries, the complete realization of the ancient καθόλου πραγματεία, has . . . inherently *numerical characteristics*: its object is, its generality notwithstanding, 'arithmetically' determined" (192/184). Because of this, however, Klein maintains that despite the difference between the object of Vieta's science and that of the ancient discipline, it has something in common with the great attempt in antiquity to conceive, numerically, the objects of the "supreme" science. Thus, on the one hand, Klein contrasts in the sharpest possible terms the meaning of "generality" in ancient and modern mathematics. In the modern understanding, generality is taken to extend to the "object" of mathematics, which, as we have seen, is something he stresses would be impossible in the ancient understanding. On the other hand, he claims that Vieta's arrogation of "arithmetical" qualities to this object has something in common with Plato's "attempt to grasp the 'supreme' science 'arithmologically,'" because by ascribing numerical qualities to science's supreme concepts, both Plato and Vieta "broke through the limits set for the λόγος" (192–93/184). Plato, as we have seen, accomplished this with his unwritten theory of ἀριθμοὶ εἰδητικόι, which exhibit an "arithmological" structure that, paradoxically, cannot be counted.[123] Vieta, as we have suggested and shall see in greater detail below, likewise realizes this with an "'arithmetical' interpretation of 'general magnitudes.'" This interpretation "leads to a special—an 'algebraic'—mode of cognition or, more exactly, to the conception and project of a *symbolic* mathematics" (193/184) whose "frame of reference" and "internal completeness" not only are *not* established by Vieta—and are therefore "yet to come"—but also are "still lacking even in Descartes' Mathesis universalis."[124]

Despite the characteristic of transcending, as it were, the limits set for the λόγος common to both Plato's theory of ἀριθμοὶ εἰδητικός and Vieta's an-

123. See § 72 above.

124. Given Klein's overall argument in *GMTOA* that "the fundamental *ontological* science of the ancients is replaced by a *symbolic* discipline whose ontological presuppositions are left unclarified" (193/184), it is questionable whether he thought that in the science that this discipline "slowly broadens into," i.e., mathematical physics, such clarification to this day has been forthcoming. For it would seem that the very structure of this science's knowledge, a structure that "is henceforth governed by the symbolic 'number' concept" (194/185), pre-

alytical art, it is nevertheless important to note that they differ in how they realize this characteristic. As we have seen,[125] in Plato it is realized on the basis of an "arithmological" account of something that, properly speaking, cannot be *counted* and therefore cannot be *unambiguously* spoken about from the "point of view of ordinary predication" (95/99), namely, the conditions that, in the guise of the greatest kinds, are responsible for *being and non-being*. In Vieta it occurs because something that is not, strictly speaking, numerical—that is to say, something that is not a definite amount of definite things, namely, the species of the numbers that *are* numerical in this strict sense—is nevertheless interpreted "arithmetically," *as something that manifests this twofold determinacy*, *albeit* "*in general*."[126] To gain a better understanding of Klein's position on the nature, limits, and significance of Vieta's transformation of the ancient concept of species in this manner, we now turn to Klein's detailed account and desedimentation of Vieta's critical appropriation of ancient logistic.

§101. Vieta's Ambiguous Relation to Ancient Greek Mathematics

According to Klein, "It is important to be clear about the fact that modern mathematics is guided from the outset by cosmological-astronomical interests. This is true not only of Vieta, but of Kepler, Descartes, Barrow, Newton, etc. In this respect the 'new' science repeats the course of ancient science" (155/152). Indeed, Klein maintains that "*All the mathematical investigations of Vieta are closely connected with his cosmological and astronomical work*" (153/151). The way in which this course was repeated by these thinkers, however, "betrays, from the outset, a different conception of the world, a different understanding of the world's being, than that which belonged to the ancients" (155/152). On Klein's view, the founders of modern science "were not, for the most part, themselves aware of their own conceptual presuppositions,"[127] a consequence of which was "a tension within the science that they founded." We have seen that for Klein this situation has its root in the symbolic discipline Vieta founded, with its unclarified ontolog-

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cludes *in principle* the clarification of its "frame of reference" and "internal completeness" in a manner that would not transcend "the limits set for the λόγος."

125. See § 80 above.

126. See § 106 below.

127. Mahoney writes in this connection, referring to Pierre de Fermat and Descartes, that they "treated old problems by means of a new symbolic algebra, without themselves being clear on the extent to which the new means had changed not only the techniques of solution but also the very manner of posing problems" (Mahoney, "Beginnings of Algebraic Thought," 141).

ical presuppositions, replacing the supreme ontological science of the ancients. Klein points out that the "legacy" of the foundational orientation of the latter "gradually came into conflict with the new ontological understanding borne by the modern mode of cognition," a conflict that "led in the nineteenth century to a new 'formalized' foundation of infinitesimal analysis and that today has its effect in the struggle over the principles of mathematical physics." Indeed, he holds that this conflict "[i]n germ is already present in Vieta."

Klein speculates that "References by Peletier[128] and Petrus Ramus,[129] as well as Xylander's translation, must certainly have introduced Vieta to Diophantus' *Arithmetic*, which he undoubtedly came to know also in the original" (154/151). From Vieta's "study of the Diophantine work eventually grew his symbolic algebra, whose fundamental characteristic he sketched out programmatically in his work *In artem analyticen Isagoge* (*Introduction to the Analytical Art*), which appeared in 1591." Unlike many of his contemporaries, however, "Vieta's comprehensive humanistic education does not lead him into open and explicit opposition to the traditional science" (155/152), although "he resembles them in turning, by preference, toward the neglected or unknown sources of ancient literary tradition." Nevertheless, "He wishes to be in every respect the loyal preserver, rediscoverer and continuator of our ancient teachers." As a result, in his "mathematical writings he is always concerned not only with borrowing his terms directly from ancient terminology (or at least, when he must invent new terms, to match it as closely as possible) but also with interpreting all 'innovations,' once they have been introduced, as a mere development of the tradition. All 'innovation' is for him, as for so many of his contemporaries, '*renovation*'" (156–57/153).[130] Klein quotes Vieta's "*Letter Prefatory* to the *Isagoge*" (157/153) at length to illustrate this peculiar self-understanding, which, because it "wishes in every respect to be the loyal preserver, rediscoverer and continuator of our ancient teachers," is "not, for the most part," itself "aware" of its "own conceptual presuppositions":

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128. Jacques Pelletier (also spelled Peletier) du Mans (1517–1582) was a humanist, poet, and mathematician of the French Renaissance.

129. Petrus Ramus, *Scholarum mathematicarum, libri unus et triginta* (Basel: Episcopius & Nicolai, 1569).

130. See Mahoney, who writes that Descartes "on the one hand holds his algebraic universal mathematics to be a reconstruction of those general methods that underlay Greek mathematics and that the Greeks meanly withheld from later generations, and . . . on the other hand praises himself for having created a mathematical method that the Greeks had never possessed" (Mahoney, "Beginnings of Algebraic Thought," 141).

> Those things which are new are wont in the beginning to be set forth rudely and formlessly and must then be polished and perfected in succeeding centuries. Behold, the art which I present is *new*, but in truth *so old*, so spoiled and defiled by the barbarians, that I considered it necessary, in order to introduce an entirely new form to it, to think out and publish a new vocabulary, having gotten ridden of all its pseudo-technical terms, lest it should retain its filth and continue to stink in the old way.

Vieta thus "claims to have been the first to have discovered" (158/154), "[b]eyond the mystery that surrounds it and recalls the dark art of the alchemists," "the 'previously buried genuine gold' (aurum fossile et probum) of the *ancient* mathematicians" contained in this *Ars magna* (great art), "which they guarded jealously, and whose possession allows Vieta to solve not only, as people did before him, 'this and that problem' singly, but precisely to manage problems of this kind in any desired amount—'by tenths and twenties' (decadas et eicadas)."

Despite Vieta's self-understanding as the "faithful preserver and interpreter of the traditional doctrine" (165/161), Klein substantiates his view that there is a "crucial difference with respect to the ancient καθόλου πραγματεία" and Vieta's analytical art by reactivating 1) Vieta's understanding of the ancient geometrical analysis and Diophantus's arithmetical procedure "as completely parallel procedures," 2) Vieta's "use of the εἶδος-concept in Diophantus," and 3) the influence on Vieta of "Proclus' position on the general theory of proportion" (165–66/161). On the basis of the reactivation of these three aspects of Vieta's "point of view" (165/161), Klein desediments "the fundamental *reinterpretation* which the ancient mode of conceptuality experiences in modern mathematics and which reaches its most characteristic expression in the transformed understanding of 'ἀριθμός'" (166/161)—a reinterpretation that "appears in the conception which Vieta has" (165/ 161) of the "*object*" of mathematics' καθόλου πραγματεία.

§102. Vieta's Comparison of Ancient Geometrical Analysis with the Diophantine Procedure

According to Klein, "The point of departure for Vieta's 'renovation'" (161/ 157) of the ancient *Ars magna* is located in his bringing together aspects of Pappus's geometrical employment of the concepts of analysis and synthetic ἀπόδειξις with Diophantus's arithmetical procedure of calculating with unknown definite amounts in his *Arithmetic*. Vieta's *Isagoge* draws on both of these sources in its opening chapter, where he reports, beginning with Pappus, that "There is in mathematics . . . a special procedure for discovery, 'a certain way of investigating the truth' (veritatis inquirendae via quaedam)

which, so it is claimed, was first discovered by Plato"[131] (159/154). Although Vieta proceeds to provide definitions of this procedure that he attributes to Theon of Alexandria, Klein remarks that they "also occur in Pappus in a modified and clarified form, namely at the beginning of his seventh book" (159/155), the "Latin edition" (158 n. 86/259 n. 214) of which "appeared in 1588–1589." Klein speculates, moreover, that "Vieta had, without doubt, access to Pappus manuscripts before that time." Given the importance of Vieta's appropriation and conjoining of this procedure with that of Diophantus's for his *ars analytica* (analytical art), Klein reconstructs both Vieta's account of it and that of what is doubtless his source.

Vieta's account reports that

> Theon of Alexandria gave this procedure the name of "analysis" and defined it precisely, namely as a process beginning with "the assumption of what is sought as though it were granted, and by means of the consequences [proceeding to] a truth [which in fact was already granted]" . . . , just "as in converse" (ut contra) he defined "synthesis" as a process beginning with "the assumption of what is granted and by means of the consequences [proceeding to] the conclusion and comprehension of what is sought. . . ." (159/154–55)

And Pappus says:

> Analysis, then, is the *way* from what is sought, taken as admitted by means of a [previous] synthesis . . . but in synthesis, *going in reverse*, we suppose as admitted what was the last result of the analysis, and, arranging in their natural order as consequences what were formerly the antecedents, and connecting them with one another, we arrive at the *completion of the construction* of what was sought; and this we call synthesis.[132] (159 n. 90/260 n. 218)

Klein identifies an additional context for the ancient method of analysis and synthesis "[i]n a *scholium* to Euclid," where he notes that "it is shown with reference to the first five theorems of the thirteenth book how the 'synthesis' results in each case from the preceding 'analysis' *by means of conversion* (analysis and synthesis both proceeding 'without drawing the figure' – ἄνευ καταγραφῆς[133])" (159–60/155). Specifically, "'*Analysis*, then, is the taking of what is sought as admitted and ⟨going through⟩ the consequences from this

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131. Klein maintains that "[t]he assertion that Plato was the discoverer of the 'analytic' method . . . loses its strangeness when understood in the original context of teaching and learning characteristic of Platonic philosophy" (159 n. 90/260 n. 218). See n. 134 below for an elaboration of this issue.

132. Pappus, *Mathematicae collectiones*, ed. Friedrich Hultsch, 3 vols. (Berlin: Weidmann, 1876–1878), II: 634, 11 ff.

133. Euclid, *Opera omnia* (Greek and Latin), ed. Johan L. Heiberg and Heinrich Menge (Leipzig: Teubner, 1883–1916), 366, 4 and 368, 16; henceforth cited as 'Heiberg-Menge'.

to something admitted to be true, while *synthesis* is the taking of something admitted and ⟨going through⟩ the consequences from this to something admitted to be true.' (. . . Heiberg-Menge, IV, 364 f.)" (159 n. 89/259 n. 217).[134]

In addition to mentioning "the aforesaid procedure [discussed in the *scholium* to Euclid quoted above] with reference to the so-called *Treasury of Analysis* (ἀναλυόμενος τόπος)" (160/155), Pappus "emphatically stresses the relationship of conversion" and, moreover, "distinguishes *two kinds, two* γένη, *of analysis*: 'the one is for searching for the truth ⟨i.e., *zetetic*, from ζητέω, to search⟩, which is called *theoretical*, and the other is for supplying what is required ⟨i.e., *poristic* from πορίζω, 'to supply'⟩, which is called *problematical* (. . . Hultsch, II, p. 634, 24–26)." The distinction concerns "the application to be made of the 'analysis'—whether it is to be applied to the discovery of the *proof of a* '*theorem*' or the *solution* (i.e., construction) *of a* '*problem*.'" Corresponding to the conversion of each type of analysis for Pappus is a different type of synthesis. In the case of theoretical analysis, the synthesis "represents a direct ἀπόδειξις," while in the case of the problematical, "it consists first of a geometric construction (κατασκευή), or sometimes a porism (πορισμός, i.e., the production or finding of something already implicit in the figure; Hultsch, II, p. 650, 16 ff.), upon which the ἀπόδειξις then follows" (160/155–56). Klein notes, however, that

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134. Klein sees "the concepts of the ζητούμενον (the sought) and the ὁμολογούμενον (the admitted)" to be at work in the "Socratic question-and-answer game of the Platonic dialogues" (159 n. 90/260 n. 218); e.g. in the "*Meno* 79 D, where they are probably employed with a view to the general 'geometrical' background of the dialogue." Klein holds that while "the purity of this mathematical *synthetic* procedure [i.e., the procedure in Proclus and the *scholium* to Euclid] is *not* to be found in dialectic" (160 n. 90/260 n. 218), "the 'analytical' power of Socratic conversation" is nevertheless rooted in "the truly Socratic way of 'recollection' (ἀνάμνησις) which is the very subject of the *Meno*" (160 n. 90/261 n. 218). It is rooted in the way "the *word* which *designates* the unknown or the thing sought (ζητούμενον) is thus always used as if the thing designated were something already known and admitted (ὁμολογούμενον)" (160 n. 90/260 n. 218). The "course of *this way to the truth*," however, is prepared by the conversational *impurity* of Plato's dialogs, which rather than begin, like the mathematical synthetic procedure, "with definitions, axioms and postulates," instead begin "with 'opinions' (δόξαι) which presuppose what is 'sought' as 'known.'" Thus, by means of what Socrates calls to Theaetetus's attention in the *Theaetetus*, that "'for a long time now we have been infected by *impure* conversation' (196 E . . . ; cf. also *Meno*, 75 C–D)," the *sought* is presupposed as *known* "in order to arrive, by means of the refutation of these opinions as 'false opinions' (ψευδεῖς δόξαι), at the '*true*' or '*right* opinion' (ἀληθὴς or ὀρθή) which is sleeping in the soul and which must, once it is found, be fixed by means of an exact '*account of the reason why*' (λογισμὸς αἰτίας) in order to become '*knowledge*' (ἐπιστήμη; cf. *Meno*, 98 A)." Indeed, when Theaetetus asks Socrates, "'But in what manner will you converse if you refrain from these ⟨i.e., words designating things sought⟩ (. . . *Theaetetus*, 196 E–197 A) (160 n. 90/261 n. 218), Socrates replies: "'*In no manner, at least while I am who I am*.'"

> in elucidating the difference between "theoretical" and "problematical" analysis, Pappus at both times calls the synthesis simply an "ἀπόδειξις": "And in reverse, the proof is the converse of the analysis." (636, 5 f.: καὶ ἡ ἀπόδειξις ἀντίστροφος τῇ ἀναλύσει, also 636, 12 f.: καὶ πάλιν ἡ ἀπόδειξις ἀντίστροφος τῇ ἀναλύσει.). (160/156)

As we shall see,[135] the use of the term ἀπόδειξις to refer simply to the synthesis, without differentiating between its direct and indirect modes, modes that are differentiated by their "theoretical" or "problematical" objects, is something that Vieta will take over when he transforms the Diophantine arithmetical procedure based, in part, on his comparison of it with Pappus's presentation of geometrical analysis.

Klein reports that Diophantus also called ἀπόδειξις "the conversion of each solution, namely the 'test proof' which is intended to show that the definite amounts found do, indeed, fulfill the conditions, that they 'do the problem'" (160/156). Thus, "the words 'and the proof is clear' (καὶ ἡ ἀπόδει-ξις φανερά) form the conclusion of a whole series of his problems" (160–61/156). It is important to stress, however, that the ἀπόδειξις in Diophantus does not deal with geometrical theorems and problems, as is the case of the "character of the analysis [and synthesis] intended by Pappus" (161/157), but rather, as we have seen, with "*determinate* amounts of monads" (166–67/162). Moreover, Klein maintains that "the comparison which can be drawn between the role of 'analysis' in geometry on the one hand and in the Diophantine *Arithmetic* on the other" (165/161) reveals that "the relationship between ('problematical') analysis and synthesis which is traditional in geometry undergoes a significant change in the *Arithmetic* of Diophantus" (167/162). Significantly for Klein, Vieta was aware of both 1) the different objects in the "purely geometric" (161/157) application of Pappus's procedure for discovery and the arithmetical application of Diophantus's "art" (177/170), and 2) the different role of the analytic procedure employed by each in the discovery of these objects. Furthermore, Klein finds that the point of view characterizing Vieta's analytical art was explicitly informed by a "comparison [of Diophantus's analysis] with *geometric* analysis" (167/163). This comparison "considers together" (161/157) the "states of affairs" at issue in both types of analysis in a manner that understands them as "completely parallel procedures" (167/163). This understanding, Klein maintains, "causes Vieta to go beyond Diophantus" in his formulation of the possibilities for the use of the Diophantine procedure of "calculation '*ending in the indeterminate*' (the solution ἐν τῷ ἀορίστῳ)."

The comparison of geometrical and Diophantine analysis reveals that each deals, albeit in different ways, with what is sought "*as with something al-*

135. See § 103 below.

ready given or 'granted' (concessum)" (161/156). For Diophantus, this is because "The construction of an *equation* means nothing but to put the conditions of a problem into a form which enables us to ignore whether the magnitudes occurring in the problem are 'known' or 'unknown.'" As such, the "indeterminate" magnitudes in question are dealt with "as with something already given or granted." For geometrical analysis, this is the case because in such analysis the "given" relations between "given" magnitudes are "understood only as a '*possible* givenness'" (168/164). Thus, this "'possible givenness' appears in geometrical analysis in the fact that the construction which is regarded as already effected (the 'quaestitum tanquam concessum') does not need to use the 'given' magnitudes as unequivocally determinate *but only as having the character of being 'given*.'"

In Diophantine analysis the "test proof" (ἀπόδειξις) confirms that the "consequences (consequentia) to be drawn from an equation" (161/156), which "finally lead, by means of computation, to the finding of the definite amount sought," yield the "'true' definite amount which is only then, at the end, 'granted as true' (verum concessum)." In geometrical synthesis, the ἀπόδειξις of a theorem or the construction of a figure takes place only when, respectively, "the state of affairs in question [in the theorem] has *actually* been 'derived' solely from the 'given' relations between the 'given' magnitudes" (168/164), or when the figure in question "has actually been drawn using the magnitudes 'given' with *just these determinate dimensions*." Because in both geometrical and Diophantine analysis the object sought is treated in a manner that is *not* unequivocally determinate, their analytical treatments—precisely in this regard—are *general*. Thus, it sometimes happens that the ἀπόδειξις, which "in accordance with the fundamental Greek conception of the objects of mathematics" is "obliged to 'realize' this general procedure in an *unequivocally determinate object*," yields either "an 'impossible' definite amount" (161/156) from "the final computation" or a geometrical solution "'agreed to be impossible.'" In both cases, "the problem itself is taken to have been badly posed, that is, impossible." When this occurs, the problem is understood to be "in need of a ⟨condition of possibility⟩ (διορισμός), or in Pappus' words (636, 15 f.), an 'additional specification for when, and how and in how many ways the problem will be possible' (προδιαστολὴ τοῦ πότε καὶ πῶς καὶ ποσαχῶς δυνατὸν ἔσται {καὶ} τὸ πρόβλημα); Diophantus calls this condition a προσδιορισμός" (161/156–57), a "further condition."

Continuing the comparison between Diophantus's analytical procedure and geometrical analysis reveals that because Diophantus's problems and solutions only admit *determinate* definite amounts, "*the last step of the* [Diophantine] *analysis*, namely the final computation which furnishes the definite

amount sought, *is at the same time also the first step of the synthesis*—the final *computation* actually corresponds to the geometric '*construction*'" (167/162–63). This state of affairs thus contrasts with the "solution of *geometric* problems" (167/162), that is, with the finding of the magnitudes sought, in which "the required *construction* forms the *first part of the synthesis*." Subsequent to the required construction, "the ἀπόδειξις which follows has to use the relations between the 'given' magnitudes, relations which are themselves 'given' *from the beginning*, together with those brought to light by the construction, to prove that this construction satisfies the conditions of the problem." The contrast that comes to light here is that the generality of Diophantus's arithmetical procedure both furnishes the object sought, that is, the "unknown," and establishes via the ἀπόδειξις that this object solves the problem "analytically," while the generality of the geometrical procedure furnishes the object and establishes the ἀπόδειξις "synthetically." Diophantus's procedure is able to furnish its object *analytically* because unlike geometrical analysis, which is unable *actually* to furnish the givenness of its object, Diophantine analysis can do precisely this, for it is guided from the start by the "presupposition" (167/163) that this object is a determinate amount of monads. Thus, "The conversion of the process of solution in Diophantus . . . corresponds only to the second part of the 'problematical' synthesis in geometry" (167/163), that is, to the ἀπόδειξις that follows the construction of the figure in question. According to Klein, this means "that the relationship between ('problematical') analysis and synthesis which is traditional in geometry undergoes a significant change in the *Arithmetic* of Diophantus" (167/162). It does so because the object sought in the Diophantine procedure, that is, the unknown amount of monads, is determined analytically. This means that even though the general analytical procedure for furnishing this object starts out as indeterminate, in the precise sense that the equations employed by this procedure disregard the distinction between known and unknown magnitudes, the solution to these equations is nevertheless both analytic and determinate. In other words, because of Diophantus's presupposition that the "condition of possibility" for the solutions to his equations must always be "*numerically* determinate" (167/163), Diophantus's general procedure—in contrast to the general procedure in geometry—is able to solve them analytically.

§103. Vieta's Transformation of the Diophantine Procedure

Klein finds that Vieta's transformation of the Diophantine procedure begins with his understanding of "the 'analytical' manner of finding solutions in Diophantus and geometric ('problematical') analysis . . . as completely *parallel*

procedures." One important consequence of this understanding is "that a sharper line must be drawn between the *transformations* of equations and the *computation* of the definite amounts sought than what Diophantus generally does draw." That is, the analytic assumption (in the Diophantine procedure) of what is sought (the unknown) as if it were granted, and the equally *analytic* conversion of this assumption in the computation of the definite amount sought are treated by Vieta as if they corresponded, respectively, to the distinct operations of analysis and synthesis in geometrical analysis. In the synthesis belonging to the "problematical" analysis in geometry, the ἀπόδειξις *follows* the construction of the figure, with the construction itself proceeding according to the conversion of the analysis that treated the object sought—which is now the constructed figure—exclusively as though it were already granted. Hence, the ἀπόδειξις in the geometrical procedure of finding solutions (of a problem) composes the *second* part of the synthesis that functions as the conversion of the analysis. Because in Diophantus's arithmetical finding of solutions the first step of the synthesis is also the last step of the analysis, *both* the calculation with the species of the unknown definite amounts and the computation (the ἀπόδειξις) of the "true," and thus *actually* granted definite amounts, belong to the analysis. By understanding these two different "analytical" procedures to be parallel, Vieta takes "the calculation ending '*in the indeterminate*' (the solution ἐν τῷ ἀορίστῳ . . .), *which Diophantus himself uses only as an auxiliary procedure* . . . ," to be "*the true analogue to geometric* ('problematical') *analysis*." That is, Diophantus understands calculation in the indeterminate mode to be only one part of the analysis belonging to his arithmetical procedure, a part, moreover, he understood to be inseparable from its other analytical part, the computation of the "true" definite amounts. Vieta, however, using the (problematical) geometrical criterion for differentiating analysis from synthesis, namely, the connection to the unknown (what is sought) that is characteristic of analysis and the computation with known magnitudes that is characteristic of synthesis, interprets Diophantus's procedure of ending calculations in the indeterminate as an independent *analytic* procedure, one that is therefore not necessarily connected with the computation of the "true" definite amounts. With this, there emerges the kernel of the distinction between 1) "an indeterminate solution," which "permits any amount of 'determinate' solutions on the basis of arbitrary numerical assumptions" to be realized, and 2) a solution based on the actual computation of the definite amount sought, which solves "this and that" problem singly. However, "For Diophantus there is only a limited possibility for the employment of this procedure, because it still remains connected with his determinate *numerical* presuppositions." It is therefore precisely Vieta's comparison of this

procedure with geometrical analysis and its capital result of separating the indeterminate from the determinate in Diophantus's arithmetical procedures that cause Vieta "to go beyond" him.

The question that confronts Vieta, then, is how the "possible givenness" that characterizes the magnitudes in geometrical problematical analysis, whose possibility is rooted in their having the character of being given, can "be transferred to 'arithmetical' analysis" (168/164). According to Klein, it is "[o]bviously in this way—that the definite amounts [*Anzahlen*] 'given' in a problem are also regarded *only in their character of being given*, and not as just these determinate definite amounts." This means that "to assimilate the arithmetical to the geometrical analysis completely, the 'given' definite amounts must be allowed a certain indeterminateness which should [according to the Greek conception of mathematics], in fact, be limited only by 'the condition of possibility' (διορισμός) of the problem." Klein maintains that "Here the Diophantine model presents itself" (168/165) to Vieta, since "just as Diophantus represents the unknown definite amount—although it is 'in itself' likewise 'determinate' . . . —by its εἶδος, its '*species*,' which leaves the question of '*how many*?' provisionally indeterminate, so *every definite amount can be expressed by its* '*species*'" (168–69/165). Indeed, Vieta realizes that "As soon as the εἴδη of the unknown and its powers appear in Diophantus . . . as new units of calculation—the authentic computation, however, being effected in terms of determinate definite amounts—the 'calculation' exclusively utilizing the 'species' of definite amounts ought now to be entirely transferred into the domain of the 'indeterminate.'"

Klein maintains that "This crucial last step is also taken by Vieta" (169/165), and that it is done so "in the consciousness of merely confirming a practice long in use by the ancients (and found especially in Diophantus), although not sufficiently clarified by them." Moreover, it is precisely "[i]n the course of taking this last step" that Vieta will be "forced to reinterpret the tradition at essential points." Vieta will thus view Diophantus's *Arithmetic* "exclusively as an 'artful' procedure" (177/170), by which he understands a *logistice speciosa* (169/165), that is, a logistic that calculates solely in species and therefore "has but a small interest in the determinate results of solutions" (178/171). Klein writes that "With this interpretation of Diophantine *Arithmetic* . . . Vieta prescribed to historical research the approach which governs it to this day" (178/171), an approach that is evident above all "in the matter-of-course acceptance within the modern consciousness of the revolution in the ancient mode of forming concepts and of interpreting the world, which first took tangible shape when Vieta founded his 'general analytic.'"

Klein observes that although Vieta's "'logistice speciosa' is originally very closely connected with the Diophantine procedure" (169/165), the latter forming "the 'arithmetical' analogue to geometric analysis," Vieta also understands it "as the most comprehensive possible 'analytic' art, indifferently applicable to definite amounts and to geometric magnitudes." Indeed, it is as a consequence of this that in Vieta "the εἶδος concept, the concept of 'species,' undergoes *a universalizing extension despite its tie to the realm of definite amounts of*" (169/166). Vieta thus "devotes the 'logistice speciosa' to the service of '*pure*' algebra" (169/165), whose "*general procedure*" allows "the species, or as Vieta also says, the 'forms of things' (formae rerum . . .), to be seen simply as representing 'general' magnitudes" (169/166). Vieta's "extension of the Diophantine εἶδος concept" leads him to concentrate "his *reflection on the procedures*" (170/166) of analysis that involve "'seeking {the truth}" or that are "'productive {of the proposed theorem}'" procedures that he, borrowing from Pappus's text, terms respectively "*zetetic* (ζητητικόν {sc. τἀληθοῦς})" and "*poristic* (ποριστικόν)," thus significantly, on Klein's view, bypassing Pappus's "expressions 'theoretical' (θεωρητικόν) and 'problematical' (προβληματικόν)." While for Pappus, as we have seen, the difference between the latter two types of analysis was rooted "in the kind of object presented in a 'theorem' and in a 'problem,'" Vieta "is interested less in the 'truths' themselves than in the *finding of* '*correct finding*.'" The basis, then, of Vieta's "general definition of the 'analytical art'"—whose name, by the way, he formulates "*on the basis of Pappus' exposition*" (161/157) of analysis—"as the 'theory for finding {what is sought} in mathematics {in general}" (170/166), is rooted for Klein in the fact that Vieta "no longer differentiates between 'theorems' and 'problems,' or, more exactly," in the fact that "*he sees all theorems as problems*."

§ 104. The Auxiliary Status of Vieta's Employment of the "General Analytic"

Klein writes, "Vieta explicitly notes that properly speaking these alone, 'zetetic' and 'poristic,' are intended in Theon's definition of analysis." Vieta defines 'zetetic' "as the procedure 'through which the equation or the proportion is found which is to be constructed with the aid of the given magnitudes with a view to the magnitude sought,'" and 'poristic' "as the procedure 'through which by means of the equation or proportion the truth of the theorem {!} set up {in them} is investigated'" (170/167).[136] However, to these

136. Klein reports that Vieta elucidates the meaning of his definition of the poristic procedure as the way that "is to be taken when a problem is given which does not fit immedi-

two types of analysis, which belong to the analytical art, "Vieta adds still a third" (170/166), which he defines "as the procedure 'through which the magnitude sought is *itself* produced out of the equation or proportion set up {in canonical form}'" (170/167).[137] Depending on whether the magnitude to which it leads is arithmetical or geometrical, this third type of analysis is called, respectively, "*rhetic* (ῥητική) with respect to the definite amounts to which it leads and which can be expressed by the ordinary numeral names of our *language*" (172/167) and "*exegetic* (ἐξηγητική) in respect to the geometric magnitudes which it makes directly available to *sight*."

This third and final stage "in the solution of an equation, which, as we have seen . . . is actually already a part of the synthesis," is "nevertheless understood by Vieta as an analytical procedure" (172–73/167). Indeed, according to Klein, "'Synthesis' in Vieta generally takes second place to 'analysis,' although in geometric problems he frequently makes use of it and recognizes its traditional priority" (172 n. 105/268 n. 235). Hence, Vieta "says expressly that the results of the analysis have to brought 'under the order of the art' (in artis ordinationem) according to the 'laws' (leges) *κατὰ παντός, καθ᾽ αὑτό, καθό - λου πρῶτον* (i.e., in school language: predicated 'of every instance of its subject,' 'essentially,' 'commensurately with the universal')." Klein reports that for Vieta the "'law of every instance'" is something that "'*essentially*' demands that every 'rule of the art' (artis decretum) be 'of the same genus and a member of the same body as it were'. . . ." Thus, "such results 'as are demonstrated and firmly established by *zetetic*' . . . must be subjected to 'synthesis,' 'which is commonly considered the logically tighter way of demonstration' . . . ; this means that 'the tracks of analysis are thus repeated'. . . ." Vieta, however, "significantly adds, '*this is itself also analytical*' (quod et ipsum analyticum est)," as well as "'not troublesome, on account of the species calculation introduced' . . . by him."[138]

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ately into the systematic context, i.e., which occurs by chance or incidentally" (171 n. 104/265 n. 233). In Vieta's words: "'But if something *unfamiliar* is *discovered*, or some *chance finding*, the truth of which must be weighed and investigated, is proposed for proof, then the way of poristic must first be tried'" (171 n. 104/266 n. 233).

137. For Vieta's understanding of this, see § 106 below.

138. Klein says here that "the equivocality of the corresponding Latin term 'resolutio' should be noted" (172 n. 105/268 n. 235) in connection with Vieta's "preference for 'analysis.'" Specifically, it "means (1) 'reverse solution' in Pappus' sense (*ἀνάπαλιν λύσις* – Hultsch, II, 634, 18), (2) 'resolution' into the fundamental elements . . . , and, finally, (3) simply 'solution'" (172–73 n. 105/268 n. 235). Descartes, Klein also notes here, likewise considers "analysis" to be "far more essential than 'synthesis'" (173 n. 105/268–69 n. 235), since the latter "'does not teach the manner in which the thing was *found* (quia modum quo res fuit inventa non docet) (*Secundae responsiones*, Ad.-Tann., VII, p. 156)," while the former "'*is the true and best way of teaching*' (quae vera et optima via est ad docendum) (ibid.)."

Klein reports that Vieta considers "rhetic" and "exegetic" to be "the most important part of the 'analytic'" art (175/168), since—unlike the other two parts, "zetic" and "poristic," which "consist essentially of 'examples' (exempla)"—each of these "comprises a series of 'rules' (praecepta)." Because of "[t]his organization of the 'analytic'" (175/168), Klein maintains that the "'general analytic' is understood [by Vieta] as nothing more than the indispensable auxiliary *means* to the solution of geometric and numerical problems." Thus, in the final stage of analysis, "the 'analyst' must become either a 'geometer' or a 'logistician,' i.e., a calculator, in the ordinary sense" (173/167), depending on whether the problem is geometrical, which would require exegetic analysis for its solution, or logistical, in which case rhetic analysis would be required.

Klein holds that, for Vieta, the analyst, "[a]s *geometer*," "does his work according to the 'solution' of an 'other, though similar' problem, insofar as he, to be sure, in the construction 'synthetically' 'repeats,' in reverse order, the *other* 'given' magnitudes, namely those traversed though the 'pure' algebraic analysis (resolutio)." Moreover, "In doing this he may thus 'hide' his preceding purely algebraic 'analytic' work, pretending to solve the problem in a directly 'synthetic' manner, and only later—for the assistance of the calculator, as it were—handling it analytically by reading the equation off from the synthetic construction." Indeed, Klein quotes Vieta precisely to this effect. "'Thus the skillful geometer, though an expert analyst, dissimulates this and presents and explicates his problem as a synthetic one, as if he thought ⟨only⟩ about how to accomplish the work; thereafter, to help the logistician, he constructs and demonstrates the theorem in terms of the proportion or equation recognizable ⟨in the problem⟩' (*Isagoge*, Chap. VII)" (173 n. 108/269 n. 237). And further: "The 'geometrical solution' (effectio Geometrica) is thus effected in such a way that 'it does not derive and justify the synthesis from the equation but the equation from the synthetic construction—while the very synthesis speaks for itself' (. . . Chapter VII)" (173/168).

The analyst as logistician "finds numerical solutions—be it exactly, be it by approximation procedures—which may be either simple equations (resolutio potestatum purarum – analysis of *pure* powers, i.e., simple equations whose unknown has the same exponent everywhere), or composite equations, or any others desired (resolutio potestatum adfectarum – solution of *conjoined* powers, i.e., *impure* equations whose unknown has different exponents for different occurrences)" (174/168). And "in every case" (174/168), in a remark that Klein will show is directed at Diophantus, the analyst as logistician "needs not to forget to make clear, by means of a specimen, the *artful device* employed" (175/168).

On Klein's view, by maintaining that "'rhetic and exegetic . . . must be considered to be most powerfully pertinent to the establishment of the art,'" namely, the general analytic of "pure" algebra, Vieta therefore conceives of the latter as "first of all a 'technique,' to use a modern expression—it does not aim at opening up a domain of true states of affairs, progressively attaining an overview of its scope, nor at the solution of a determinate amount of problems; rather it intends to be an *instrument* for the solution of *problems in general*" (175/169). Klein has shown this on the basis of his reactivation of Vieta's *logistice speciosa*, which, as we have seen above, establishes that Vieta understands it, despite its arithmetical genealogy and hence arithmetical analog in Diophantus, to have a scope wider than merely numerical analysis insofar as it treats "magnitude" (*magnitudo*) in the most general sense, that is, in a sense that is "indifferently applicable to definite amounts and geometrical magnitudes." Klein's reactivation has also shown that notwithstanding precisely this "generality" of Vieta's art, Vieta nevertheless considers that aspect of it whose analysis is concerned with the solution of either numerical or geometrical problems—that is, rhetic and exegetic—to be its most important part. Klein's account of this understanding, then, provides the basis for his claim that the general analytic has an auxiliary status for Vieta. Nevertheless, because "'Its 'material' is not a single problem or a series of single problems but, as Vieta himself says . . . , the problem of being able to solve problems in general (problema problematum)," Klein maintains that it presents the *art of finding or the finding of the finding*" (175/169). Thus, Klein holds that the "'general analytic' is an 'organon,' an instrument in the realm of mathematical finding in the same sense as the Aristotelian 'logical' works, above all, the (*Prior* and *Posterior*) '*Analytics*' are an *Organon* in the realm of all possible knowledge whatever."

The "double function" (176/169) of Vieta's *logistice speciosa*, its being understood by him "on the one hand as the procedure of *general* 'pure' algebra and on the other hand as the procedure analogous to geometric analysis and directly related to the Diophantine *Arithmetic*" (176/170), has, according to Klein, its basis in Vieta's understanding of the whole analytical art as being "in the service of his cosmological and astronomical investigations" (176/169). As a consequence, Vieta is "concerned mainly with the numerical exploitation of solutions and therefore with 'rhetic' and 'exegetic'" analysis, which is one of the two reasons that for him "the 'general' algebra has the role of a merely auxiliary procedure" (176/170). The other reason for this auxiliary status is connected with "Vieta's assumption that Diophantus solved his 'arithmetical' problems with the aid of 'zetetic' as understood by Vieta himself, but that he considered it better to 'hide' this." Klein quotes

Vieta at length on this matter, no doubt in order both to substantiate and to highlight this reason's peculiarity:

> Zetetic was employed most subtly of all by Diophantus in those books which were written on the subject of arithmetic. *But he exhibited it as though it were founded only on definite amounts and not also on species—although he himself used them—so that his subtlety and skill might be more admired*, since things that appear very abstruse to one who calculates in definite amounts [i.e., practices logistice numerosa] appear very familiar and immediately obvious to one who calculates in species [i.e., practices *logistice speciosa*].

On Klein's view, then, "Vieta obviously sees the traces of the *true* Diophantine art, namely of 'pure' *analytic* conceived as a '*general*' *mathematical auxiliary technique*" (178/170), in "not only the instrumental use of the εἶδος concept in Diophantus but also in the 'indeterminate' [ἐν τῷ ἀορίστῳ] solutions which the latter used as an *auxiliary procedure*" (177/170). Because of this, that is, because Vieta understands his *innovation* of a *general* pure algebra as in truth a *renovation* of what he see traces of in Diophantus, "the revolution in the ancient mode of forming concepts and of interpreting the world which first took shape when Vieta founded his 'general analytic'" (178/171) is something that "comes to expression" in its "matter-of-course acceptance within modern consciousness."

§ 105. The Influence of the General Theory of Proportions on Vieta's "Pure," "General" Algebra

Having considered in detail Klein's reactivation of Vieta's comparison of geometrical analysis and Diophantine analysis, Klein's reactivation of the influence on Vieta of Proclus's position on the general theory of proportion needs to be considered. According to Klein, "Vieta's conception of a 'pure,' '*general*' algebra which will be equally applicable to geometric magnitudes and definite amounts is met half-way by the *general theory of proportions* of Eudoxus as transmitted in the fifth book of Euclid" (162/158) and in Proclus's comments on this book. In connection with his talk about "theorems which are not tied to 'figures or definite amounts or heavenly bodies' . . . but in which 'a certain common nature' . . . is grasped as such, 'itself by itself,'"[139] Proclus "also mentions the procedure of 'analysis' and 'synthesis' *common to all mathematical disciplines*: 'the road from things better known to things sought and the reversal [of the process, so as to go] from the latter to the former, which they call *analysis and synthesis*' (. . . 8, 5–8)" (163/160). Klein observes that "With this tradition in mind Vieta maintains . . . that every 'equation' (aequalitas) is

139. See § 99 above.

a 'solution of a *proportion* (resolutio proportionis), and correspondingly, every proportion is the 'construction of an equation' (constitutio aequalitatis)." Indeed, Vieta "always speaks of 'equations' and 'proportions' *together*," such that "'pure algebra is for him not only a 'general theory of equations,' but at the same time a 'general theory of proportions'" (163–64/160). As a consequence, Vieta forms "his 'stipulations for equations and proportions' (Symbola aequalitatum et proportionum – Chap. II), which are to serve as the general and firm foundations (firmamenta) 'by means of which the equations and proportions are obtained as conclusions' (quibus aequalitates et proportiones concluduntur – Chap. I)" (164–65/160), by amalgamating aspects of Euclid's *Elements* pertaining to "common notions" and "generalized" definitions and theorems. However, notwithstanding the fact that "[h]ere Vieta does indeed prove himself the faithful preserver and interpreter of the traditional doctrine" (165/161), the fundamentally different nature of *his* understanding of the *object* proper to the ancient "general treatment" that guides his analysis points to conceptual presuppositions that, as has been suggested repeatedly above and now will be explored in detail, can only be understood on the basis of his fundamental reinterpretation of the ancient mode of conceptuality whose most characteristic expression is the transformed understanding of ἀριθμός.

§106. Klein's Desedimentation of the Conceptual Presuppositions Belonging to Vieta's Interpretation of Diophantine Logistic

Klein's desedimentation of the conceptual presuppositions that make Vieta's interpretation of Diophantine logistic possible focuses on the following questions: "*What* does Vieta understand by the species which form the object of the 'general analytic' and *in which way* does he understand them?" (178/171). Klein's answer to the first question, as we have seen, concerns "the universal extension which the Diophantine εἶδος concept undergoes at Vieta's hands, an extension through which the species become the objects of a 'generalized' mathematical discipline that is identifiable *neither* with geometry *nor* with arithmetic" (179/172). Regarding the second question, Klein maintains that Vieta understands the species to retain a "direct connection with the 'logistice numerosa,' i.e., with 'calculation,'" and, with this, a direct connection to "a homogeneous field of monads presupposed by its original meaning." As a consequence, the way in which Vieta understands the species of his general analytic "is dependent upon 'definite amounts of definite objects'[140] and their

140. Because on Klein's view the ancient ἀριθμοί are precisely *Anzahlen*, i.e., definite amounts of definite objects, in essence his point here is that the way in which Vieta understood

relations." They are dependent in the sense that Vieta's "fundamental rules" (180/172) for calculating with species, what he calls "'*the canonical rules of species calculation*' (logistices speciosae canonica praecepta)," "correspond to the rules for addition, subtraction, and multiplication used for instruction in ordinary calculation." Even though the calculations governed by these rules operate on the species of unknown and known magnitudes, Klein stresses that Vieta's fundamental law for such calculations, the "'*law of homogeneity*'" (180/173), concerns "the fundamental fact that every 'calculation' [with species] finally depends on 'counting off' the appropriate basic units, which presupposes *a field of homogeneous monads*" (181/173–74).

Klein supports this last claim by reactivating an equation in Vieta's general analytic. Because the latter involves the representation of every species "by a *letter* (the vowels being assigned to the unknown, the consonant to the known magnitudes . . .), to which are joined the designation of the degree or genus, beginning with the second degree" (180/172–73), Klein's reactivation first considers Vieta's organization and presentation of the *genera* proper to known and unknown magnitudes. The *genera* of unknown magnitudes are organized by Vieta as "'a venerable series or scale of magnitudes ascending or descending from genus to genus by their own power in ⟨continuous⟩ proportion' (. . . Chapter I, end)" (179/172). Klein reports that for Vieta the *genera* of the unknown magnitudes "are therefore called 'magnitudines scalares' (*scalar* or ladder magnitudes)" (179 n. 120/274 n. 249), and:

> Their 'rungs' or 'degrees' (gradus) are:
>
> | | |
> |---|---|
> | Latus seu Radix | (Side or Root, i.e., [in modern notation] x) |
> | Quadratum | (Square, i.e., x^2) |
> | Cubus | (Cube, i.e., x^3) |
> | Quadrato-quadratum | (Squared-square, i.e., x^4) |
> | Quadrato-cubus | (Squared-cube, i.e., x^5) |
> | Cubo-cubus | (Cubed-cube, i.e., x^6) |
> | Quadrato-quadrato-cubus | (Squared-squared-cube, i.e., x^7) |
> | Quadrato-cubo-cubus | (Squared-cubed-cube, i.e., x^8) |
> | Cubo-cubo-cubus | (Cubed-cubed-cube, i.e., x^9). |

The continuous proportion (in modern notation) is arranged thus by Vieta: "$x : x^2 = x^2 : x^3 = x^3 : x^4 \ldots$"

> The *genera* of the known magnitudes (the 'magnitudines comparatae') are ordered correspondingly:

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the concept of species in his *logistice speciosa* was still dependent on the ancient ἀριθμοί. This is reflected in the English translation, which renders *Anzahlen* here as *arithmoi*.

Longitudo latitudove	(Length or breadth)
Planum	(Plane)
Solidum	(Solid)
Plano-planum	(Plane-plane)
Plano-solidum	(Plane-solid). (180 n. 120/274 n. 249)

One of Vieta's equations (180/173) thus looks as follows:

$$\frac{A \text{ cubus} - B \text{ solido } 3,}{C \text{ in } E \text{ quadratum}}$$

read: *A* cubed minus 3 times *B* solid divided by *C* times *E* squared

(in modern notation: $\frac{x^3 - 3b}{cy^2}$).[141]

Klein maintains that while Vieta was not the first to use a notation that employed letters for both known and unknown magnitudes, "The term 'symbolum,' used for letter signs as well as for connective signs, originated with Vieta himself" (183 n. 128/276 n. 257). As for the meaning of the term itself, Klein relates that "By a 'symbolum' Vieta doubtless understands a 'contractual *stipulation*,' which corresponds to the judicial concept of the Greek σύμβολον" (165 n. 98/264 n. 226). In addition to being governed by the "*canonical rules of species calculation*," Klein stresses that Vieta's "'first and eternal law of equations and proportions' (prima et perpetua lex aequalitatum seu proportionum)" (181/173), the "'*law of homogeneity*' (lex homogeneorum), . . . according to which only magnitudes of 'like genus' can be compared (i.e., can appear in the same equation) with one another,[142] must be kept in mind throughout" (180–81/173). Thus, "According to it, only such magnitudes can be related by way of addition or subtraction as belong to the same or the corresponding 'rung,' although this does not hold for multiplication and division" (181/173). On Klein's view, Vieta's "law of homogeneity" is not connected with the ancient Greek definition of a *ratio* as something that "can only exist between 'homogeneous' magnitudes," since it "certainly is not unusual for ancient mathematicians to 'compare,' for instance, a ratio of *lengths* with a ratio of *planes*, and to bring both into one '*proportion*.'" Rather, as mentioned above, Vieta's law is "concerned with the fundamental fact" that the known and unknown "magnitudes" united in the equations of his *logistice speciosa* "represent, each and every one, 'definite

141. The '3*b*' corrects the '3^b' printed in the English translation.

142. According to Klein, this law "says, in modern terminology, that all numbers of an equation must have the same dimension" (181/173).

amounts of monads'" (181/174).[143] In fact, "Vieta himself says '*in the case of numbers* the homogeneous elements of equations are *units*'" (181 n. 126/ 276 n. 255). Since unlike "Diophantine 'logistic,'" where "this demand is fulfilled as a matter of course, because it already operates within such a field of 'pure' monads," Klein points out that "For the 'logistice speciosa' this fundamental presupposition needs to be especially stressed." According to him, this is why "Vieta, in contrast to the 'ancient analysts' (veteres Analystae), expounds the 'lex homogeneorum' as the foundation of the 'analytical art'" (181–82/174).

Klein thus maintains that "the concept of species is for Vieta, its universality notwithstanding, irrevocably dependent on the concept of 'ἀριθμός.' *The character of 'ἀριθμός' as a 'definite amount of . . .' is preserved in it* [i.e., Vieta's concept of species] *in a peculiarly transformed way*" (182/174). Klein therefore desediments the way in which this "numerical" character of ἀριθμός is, as it were, "preserved" in Vieta's concept of species. He does this by desedimenting the non-Aristotelian "abstractness" proper to the "symbolic" number concept operative in Vieta's concept of species[144] as follows:

> While every ἀριθμός means *immediately the things or the units themselves* whose "definite amount" it exhibits exactly, Vieta's letter signs [first of all mean precisely this concept of an amount as a determination insolubly related to things or units, and thus it][145] *immediately mean the general character of being an amount* that belongs to every possible definite amount—i.e., "amount overall"—and only *mediately* the things or units that may be present in any particular definite amount. (182/174)[146]

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143. See n. 153 below.

144. See § 94 above.

145. The clause in brackets renders the German original, "meint zunächst einmal das Buchstaben-Zeichen bei Vieta eben diesen *Begriff* der Anzahl als einer auf Dinge bzw. Einheiten unablöslich bezogenen Bestimmung," which is not translated in the English translation.

146. Caton's gloss on this passage is instructive, as it exemplifies what was noted above (see n. 110) regarding the difficulties Klein's commentators have in following his desedimentation of Vieta's "numerical" interpretation of the species. After quoting the English translation of this passage, into which he inserts, after the word "ἀριθμός," the following in brackets: "[number sign]," he writes: "Klein means to say, I take it, that the letter sign signifies an indeterminate number of other signs (numbers), which in turn signify things or beings (units, monads). But the main thing is to understand correctly in what way the letter signifies the number signs, for therein lies the secret of the letter as a sign of an *indeterminate magnitude* (i.e., a wholly new, un-Greek number conception), and hence as a true variable or symbol" (Caton, 223).

There are three problems with Caton's view of the matters expressed here. First, Klein nowhere associates or otherwise connects 'number sign' with ἀριθμός. Indeed, his account of the non-theoretical—which is to say, practical—distinction between 'saying' and 'thinking' (see § 94 above) rules out precisely the *separation* of the 'expressive' (i.e., saying) and the 'conceptual' (i.e., thinking) that is implicit in Caton's apparent equation of ἀριθμός, or the ἀριθμός concept, with 'number sign'. Second, Klein nowhere suggests that Vieta's letter sign signifies other signs (indeed, signs that *are* numbers!), and that this—namely, the letter signifying num-

Klein maintains that "the language of the schools permits this state of affairs to be expressed as follows: the letter sign designates the intentional object of a 'second intention' (intentio secunda), namely a concept which itself directly means another *concept* and not a being."[147] However, with respect to the peculiar abstractness of Vieta's concept of species, what "initially brings about the decisive turn" is that "now this general character of being an amount or, what is the same thing, this 'general amount' in its indeterminateness, that is, its *merely possible* determinateness, is accorded a certain independence which permits it to be the bearer of 'calculational' operations." This occurs when Vieta's letter signs, together with the *genera* of the unknown and known magnitudes—for instance, '*A* cubus', '*B* solido'—are interconnected "according to unambiguously manifest rules," rules that assign "the homogeneous field that is supposed to comprise the basis of the construction of any equation." When these letter signs *themselves* become the focal point of calculational operations, Klein holds that "The 'rung' designation, which taken independently corresponds to the Diophantine εἶδος, *thus transforms the object of the* intentio secunda, namely the 'general amount' meant by the letter sign, *into the object of an intentio prima*, of a '*first intention*,' namely into a 'being' which is accessible in a direct apprehension and whose counterpart in the realm of ordinary calculation is, for instance, 'two monads,' 'three monads,' etc." (182/174–75).[148] What is involved here for Klein, *when*

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ber signs—has anything to do with the signification of an indeterminate magnitude. Klein's passage, restored to its entirely, makes this clear, since it *explicitly* states that what Vieta's letter sign means is the "*concept* of an amount [*Anzahl*]," a concept that is said to be, moreover, "insolubly related to things or units" and therefore manifestly *not* to other signs. Third, as we shall see below, Vieta's letter as a sign—in the novel sense of a symbol—of an indeterminate magnitude does not *signify* anything apart from itself. This (the identification of "the object represented with the means of its representation") is the "secret" of Vieta's letter sign, and it means that the concept of an indeterminate magnitude, in the guise of Vieta's species, collapses into his letter sign, such that what is meant by both is, *in precisely this sense*, "*symbolic* in nature" (183/176).

147. It is important to note here that Klein nowhere suggests in this passage, or elsewhere, what Caton maintains he does, namely, that this "analysis" (225) is to be found "in the *self-conscious* thought of Vieta." Rather, Klein's analysis (in the language of the schools) of the transformation of the ancient concept of species and therewith 'number' that occurs in the modern interpretation initiated by Vieta occurs within the context of Klein's own analysis, i.e., his *desedimentation* of Vieta's thought.

148. With the exception of Caton, all of the discussions (with which I am familiar) of Klein's account of the origin of symbolic cognition completely elide the "decisive turn" in connection with this origin that Klein attributes to the "direct apprehension," as the object of a "first intention," of Vieta's letter sign. Thus, in the following gloss, Caton is very much to the point: "The letter sign takes on 'independence' in the sense that it becomes a *computational element* thanks to 'precise rules' for its manipulation. It thus becomes an object of the first intention, namely, as a visible sign, with which computations may be performed" (Caton, 223–24). (See also § 45 above.)

the letter sign itself is literally perceived as something that exists independently of that which it functions to designate as a sign, which is to say, *when as consequence of this it functions as a symbol*, is precisely the *interpretation* of what, for Diophantus and the ancient tradition is the "*merely possible objectivity*"

John O'Neill, in an extensive discussion of the origins of modern algebra that relies heavily on Klein, has this to say about the role of signs in Vieta: "The logistic of species by contrast [to the logistic of numbers] operates on species defined as the 'forms of things,' in symbols such as letters. The objects of study become symbols that can refer to any quantity or magnitude, geometric or arithmetic"; see *Worlds without Content: Against Formalism* (London: Routledge, 1991), 112. O'Neill thus understands Vieta's letters, and perhaps letters per se—without more ado—*already to have the status of symbols*: "They [the objects of Vieta's logistic] are symbols, types of letter or marks" (109). The issue for Klein, however, is precisely how Vieta's letters, in contrast to the letters used by ancient Greek mathematics, e.g., the letters used by Diophantus, acquire the status of symbols. Klein's pointed expression of *how* this occurs, in terms of the conceptuality of "first" and "second" intentions, is nowhere mentioned by O'Neill. No doubt the reason for this is that O'Neill thinks that his reference to the species, as being "taken [by Vieta] to be the forms of things like letters," already explains the origin of Vieta's symbols.

Joseph Gonda's discussion of Klein's account is much closer to the mark, insofar as he characterizes "the decisive and culminating step" as occurring, "according to Klein, when the letter sign is treated as independent." However, he inexplicitly unpacks this to mean that "the letter sign, because of its indirect reference to, say, things or units, is accorded the status of a first intention, but—and this is critical—all the while remaining identified with the general character of a number, i.e., a second intention." See Joseph Gonda, "On Jacob Klein's *Greek Mathematical Thought and the Origin of Algebra*," *Interpretation* 22/1 (1994), 111–28, here 119. Here we have Vieta's letter sign becoming a "first intention" (NB: Klein refers to its becoming the *object* of a first intention) because of its indirect reference to "things or units," i.e., to objects of "first intentions." Klein, however, does not mention anything about Vieta's letter sign referring indirectly to things or units—this is something he attributes to the *concept* 'number in general'; rather, he explicitly states that the letter sign becomes the *object* of a first intention, and that it does so on the basis of the "independence" accorded to it "which permits it to be the bearer of 'calculational operations.'" Which is to say, with Caton, that it "becomes an object of the first intention, namely, as a visible sign."

Carl Page's discussion of Klein's account—or, perhaps more precisely, his Klein-inspired account—of the "reconstitution of the mathematical as *symbolic*" also makes no mention of the role of letter signs in this reconstitution; see his "Symbolic Mathematics and the Intellect Militant: On Modern Philosophy's Revolutionary Spirit," *Journal of the History of Ideas* 57 (1996), 233–53, here 242. This is perhaps a consequence of the question he poses, which is "what do the symbols of modern mathematics signify?"—a question that is radically different from the one that Klein seeks to answer by means of his desedimentation of Vieta's original constitution of a mathematical symbolism. As we have seen, the latter concerns "how the calculation with letter signs comes to have a symbolical significance." Page's question thus seems to presuppose that a symbol's mode of being is already sufficiently transparent so as to permit what is questionable about symbolic mathematics to shift from the problem of the original constitution of the letter signs that it employs as symbols to the question of the referent or referents of these symbols themselves. Thus, when Page considers Klein's "[s]ymbol-generating abstraction," he does so in terms of "what is common in the notions that render determinate, first-order objects accessible" (243), that is, the "generic" notions apprehended in "a reflective, second-order intentionality" being "reconceived in a first-order mode," and

(183/175) of the εἶδος or species, *as* an "actual objectivity." In the understanding that guides this interpretation, "the 'general amount' is both conceived and represented symbolically in the medium of species as an objective formation in itself," and therefore "is itself—as is what is meant by it—*symbolic* in nature" (183/176). This means that the letter signs (together with the designations of their genera) designating indeterminate (conceptual) objects, namely, the *species* of both unknown and known magnitudes, acquire their "numerical" significance on the basis of Vieta's "stipulations." Notwithstanding their indeterminate conceptual and thus objective status, however, these signs—as well as the concept of the indeterminate objects they designate—are nevertheless perceived as determinate objects *qua* their being unambiguously manifest as readily perceivable letters. Klein writes in connection with this that "the possibility of being able to see a 'number' in the isolated letter signs 'A' or 'B' is, however, obviously possible only through the syntactical rules which Vieta gives, in the fourth chapter of the *Isagoge*, in contrast to the operating rules of the 'logistice numerosa'" (183–84/176). In Klein's judgment, the symbolic nature of Vieta's concept of species is most readily apparent "in the species of the first degree, where the designation of the 'rung' is not appended to the letter sign and thus, so to speak, collapses into it" (183/176).[149] Thus, instead of designating the rung as *latus seu radix*

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thus "turned into general objects that enter the same field of mathematical operations inhabited by the original objects." Page's only mention of "letters" assumes that their status is already symbolic, without raising or otherwise addressing the issue of *how* they have acquired such a status: "When letters appear in simple algebraic equations . . . they represent relations between sets of possible numbers." How, exactly, the generic notions apprehended by "second-order intentionality" are "reconceived in a first-order mode" is something that Page does not address. Klein, as we have seen, not only addresses it, but also, in his account of the sedimentation (in the symbolic notation that makes modern mathematics possible) of the practical ancient Greek distinction between 'saying' and 'thinking' (see § 94 above), he provides *his* answer to the question that Page also raises (247)—though without referring to Klein's attempt to answer it (see § 100 above): "How is it that mathematics which used to guard the vestibule to philosophy has now, in the guise of a *mathesis universalis*, usurped its inner sanctum?"

149. Klein's analysis here of the origin of the "meaning" of mathematical symbols, an analysis that we have shown occurs within the context of his desedimentation of the "matter-of-course acceptance within modern consciousness" (178/171—see also § 104 above) of the symbolic procedure made possible by such symbols, will remain inaccessible to anyone whose conceptual horizon is determined (and, therefore, limited) by such an acceptance of their "meaning." This is exemplified in the following account, where Mahoney writes: "what should be understood as the 'algebraic mode of thought'? It has three main characteristics: first, this mode of thought is characterized by the use of an operative symbolism, that is, a symbolism that not only abbreviates words but represents the workings of the combinatory operations, or, in other words, a symbolism with which one operates" (Mahoney, "Beginnings of Algebraic Thought," 142). Not only does he make no attempt here to define or otherwise account for what an 'operative symbolism' is, but the very fact that it is explained in the manner of a

(side or root), which, as we have seen, for Vieta represents the first "rung" or "degree" of the *genera* of unknown magnitude, as *A latus* or *A radix*, Vieta's notation simply designates it as 'A'.

These considerations lead Klein to conclude that Vieta's syntactical rules "therefore represent the first modern *axiom* system; their systematic connection is what initially 'defines' the object to which they refer" (184/176). Because, however, "these rules are directly read off the 'calculation' with determinate amounts of monads," Klein maintains "This means that, ultimately, it is only possible for the transformed mode of the species to retain an amount or quantitative character [*Anzahl Charakter*] and therewith to become a 'number,' namely to become an object of 'calculational' operations, *because the ancient 'determinate amounts of monads' are themselves also interpreted as 'numbers,'* which means that they are conceived from the point of view of their symbolic representation."[150] Two factors are therefore involved in the concept of species in Vieta's sense being able to retain the general character of number in the Greek sense: 1) the *reinterpretation* of the Greek ἀριθμός as 'number', that is, as a sign whose numerical status is characterized *not* by its reference to a definite amount of things (or monads) but by its representation of the general character of being just *this* one number

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logical tautology (i.e., "an operative symbolism" = "a symbolism with which one operates") raises the suspicion that the author does not really see anything problematic in what a symbolism is because *what it is* is for him apparently *self-evident*.

150. In his 1932 talk, "The World of Physics and the 'Natural' World," Klein characterizes this conception of the ancient ἀριθμοί from the standpoint of their symbolic representation as precisely that which allows Vieta to take the step "from the numerical coefficient (the term 'coefficient' stems from Vieta himself) to the literal coefficient" (*WP*, 25). The step from "*2x* to *ax*" becomes possible "because the concept of 'two' no longer refers, as it did for Diophantus, directly to an object, viz., to two pure monads." Rather, it "already has a 'more general' character," insofar as "'Two' no longer means in Vieta 'two definite things,' but the general *concept* of twoness in general." With this, the numerical sign '2', "no longer means or intends a determinate number of things, but the general number-character of this one number." Klein stresses that it is precisely Vieta's understanding of the sign '2' as already meaning the general character of a single number and *not* a definite amount of things (or monads), which permits the introduction of the "symbol 'a'" as something that "represents the general numerical character of each and every number." The "sign 'a'" therefore "represents 'more' than the sign '2.'" Nevertheless, Klein maintains that "The symbolic relation between the sign and what it designates is . . . the *same* in both cases." That is, what is the same is that in each case the sign not only designates a "general concept," but also that the sign and what it "designates" *literally* coalesce in the letter sign: 'two' becomes indistinguishable from '2', 'number in general' (i.e., any possible quantity) becomes indistinguishable from 'a'. Hence, Klein writes: "the replacement of '2' by 'a' is in fact only 'logically required' here." To return to what Klein emphasizes is "the decisive thing," just as Vieta identifies his species symbols with Diophantus's signs for the unknown, he (Vieta) likewise identifies the symbol '2' "with the sign employed by Diophantus" for 'two'. And this means that "The concept of twoness is at the same time understood as referring to two entities."

(e.g., the sign '2' would represent 'twoness'); and 2) the *interpretation* of the determinate amounts of monads (ἀριθμοί)—which underlie the operating rules of the *logistice numerosa* (i.e., the rules from which the syntactical rules of the *logistice speciosa* are derived)—as being "at the same time" numbers in the symbolic sense.[151] In addition to making possible the numeric status of the symbolic notation that involves the species of unknown and known quantities in the formal language of algebra, the reinterpretation of the Greek ἀριθμός as a number therefore allows for "the representation of the definite amounts of definite objects themselves in this sphere [i.e., the formal language of algebra]" (185/177). Klein stresses that "This reinterpretation has to this day remained the foundation of our understanding of ancient 'arithmetic' and 'logistic.'"

It is upon the basis of his desedimentation of Vieta's concept of species, then, that Klein advances the thesis that the "numerically" indeterminate status proper to this concept[152] prepares the way for numbers, as "'definite amounts'" (186/177–78), to be "conceived as 'ordinary numbers'" (186/178). This conception, moreover, means "for the everyday understanding, which in effect determines the actual formation of concepts," that "these definite amounts coincide with the 'numeral' *sign* as such, especially in the course of specific calculational operations" (185–86/177–78).[153] Klein also maintains

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151. How exactly the general concept of 'twoness' (in the example at hand) *and* the reference to two (again, in the example at hand) entities is to be understood as belonging "at the same time" to *number* is something that, according to Klein, modern set theory first tries to clarify. He writes: "Modern *set theory* first tries to separate these two constituents, to clarify what 'at the same time' means" (*WP*, 25).

152. In other words, Vieta's concept of species is "numerically" indeterminate when the measure for numerical determinacy is ἀριθμοί, namely, "numbers" that are "definite amounts of definite objects," which is to say with Klein: *Anzahlen*.

153. O'Neill argues that "Klein's thesis concerning the effect of the symbolic nature of the logistic of species on the concept of number is this: that the dependency of the logistic of species on the prior logistic of number for an understanding of the arithmetical operations it employs comes to transform the concept of number itself" (O'Neill, *Worlds without Content*, 116). Moreover, he argues that the consequence of this transformation is that the traditional account of numbers "disappears and a symbolic understanding appears, that allows for the extension of the number-concept to include the 'algebraic' numbers, the absurds and imaginaries" and he judges that "Klein's arguments for this position are weak." Before addressing what O'Neill presents as Klein's weak arguments, it should be pointed out that his statement of Klein's thesis misrepresents it. It is *not* the reliance of the logistic of species on the logistic of number that Klein singles out as decisive for the transformation of the traditional concept of number. Rather, it is because the numbers (*Anzahlen*) in the latter are conceived from the perspective of their symbolic representation, as "objects" capable of being represented in the formal language first invented by Vieta, that Klein maintains that the symbolic sphere constituted by this formal language can, as Klein writes, "retain a numerical [in the sense of definite amounts of definite objects] character," albeit one that is now general. O'Neill overlooks this point entirely, namely, that for Klein the transformation of the ancient ἀριθμός into the mod-

ern symbolic number is emphatically *not* something that involves "an extension of *the* number-concept" (my emphasis). By understanding the transformation in just these terms, O'Neill misses the sense of Klein's account of the sedimentation that is coincident with the origin of the modern symbolic number. The reason "the traditional account of number according to which numbers refer to multitudes of units disappears" (O'Neill, *Worlds without Content*, 116) is therefore because the "number" in this account is understood to "coincide with the 'numeral' *sign*." And this means, for Klein, that "ordinary numbers," as it were, become symbolic, and therewith among "numbers" (i.e., symbolic numbers) fractions, irrationals, negatives, etc., can now be included. However, Klein does *not* attribute, here or elsewhere, *this* reinterpretation to Vieta (see directly below), but rather claims, in effect, that one of the conditions that made it *possible* was Vieta's symbolic concept of the species. Investigating the actual reinterpretation or "*gradual change in the understanding of number*" (184 n. 130/277 n. 259—see directly below), which he here attributes to "the everyday understanding," is something according to Klein "whose ultimate roots lie too deep for discussion in this study."

Turning now to O'Neill's account of Klein's arguments, he holds that "They are premised on a particular interpretation of Vieta's law of homogeneity which cannot be sustained" (O'Neill, *Worlds without Content*, 116). As we have seen above, Klein's interpretation of this law concerns what he claims is the "irrevocable" dependency of Vieta's concept of species, its universality notwithstanding, on the concept of ἀριθμός. This law for Klein, then, concerns a radically different issue than that of the transformation of the concept of number to which O'Neill attributes it. This is apparent, because what is at issue in Klein's interpretation of the law of homogeneity is *not* the transformation of the concept of number but *the transformation of the concept of species*. For Klein, what is involved here is the shift from the species' non-numerical status in ancient Greek mathematics to its "numerical" status in modern mathematics, a shift that occurs with its connection to "'the arithmetical' interpretation of 'general magnitudes'" (193/184; see § 95 above) in Vieta's *logistice speciosa* and in the symbolic cognition belonging to the *mathesis universalis* made possible by his analytical art.

As for the problem O'Neill finds in Klein's interpretation of Vieta's law of homogeneity, he claims that Klein's thesis is that it "is introduced to ensure that one is comparing multitudes of like units, to indicate 'the particular homogeneous field underlying each equation which is constructed'" (O'Neill, *Worlds without Content*, 117). The problem with this, according to O'Neill (117–18), "is that the law of homogeneity is not relevant to the problem of comparing like units. The only connection between the law of homogeneity and homogeneous units is verbal. Changes in the unit are a consequence only of the use of fractions, but the law of homogeneity has nothing to say on fractions. The question it deals with is the genus of numbers, the relations between linear, plane, solid numbers and the like. And these in the arithmetical domain refer to arrangements of the units, not the units at all. Klein's account is based on the odd assumption that the different genera of number are made up of different units."

Three brief points will suffice here to establish that O'Neill's objections are untenable. First, Klein does not maintain, as O'Neill claims he does, that the law of homogeneity indicates the "particular homogeneous field" underlying the construction of equations. Rather, he maintains that what indicates this field are the "unambiguously manifest rules" (*GMTOA*, 182/174) for calculational operations. He maintains this because Vieta's law of homogeneity is not one rule for calculation among others, but rather, the "fundamental presupposition" (181/174) of all the rules for calculation in his *logistice speciosa*. Second, when considered in relation to the homogeneous units or monads that compose the basic field to which all of the calculations with species ultimately refer for Vieta (because the analytical art governing such calculations remains an auxiliary procedure—see § 101 above), what Klein holds is stipulated by the "law of homogeneity" is *not*, as O'Neill maintains, that only homogeneous units of numbers can be compared, on the putative assumption that "the different genera of number are made up of different units." On the contrary, Klein holds that unlike the case in Diophantus's

that this reinterpretation of the traditional ἀριθμός concept "was supported by the 'Arabic' positional system of ciphers, which had been spreading in the West since the twelfth century and whose 'sign' character is much more pronounced than that of the Greek or Roman notation" (184 n. 130/277 n. 259). However, he maintains that "it would be a mistake to attempt to understand the origin of the language of symbolic formalism as the final consequence of the introduction of the Arabic sign language." Rather, it is the case that "[t]he acceptance of this sign language in the West *itself presupposes a gradual change in the understanding of number as a definite amount of definite objects*," a change, moreover, "whose ultimate roots lie too deep for discussion in this study." What Klein does discuss is how "after Vieta and under his immediate influence (as in indeed already before him) the '*numeri* algebraici (or 'algebrici,' or 'cossici') are posited along with the 'numeri simplices' (or 'vulgares')" (184/176). This is to say, Klein discusses the change in the *concept* of number that occurs as a result of Vieta's invention of symbolic calculi but *not* the "gradual change" in the understanding of *number itself* that was one of the factors involved in the change of its concept. Before Vieta, evidence for the change in its concept can be seen in, "above all, Chuquet" (184 n. 130/277 n. 260), "who says . . . by 'nombres' we are now to understand not only one, and all the fractions ('tout nombre rout' – 'every broken number'), but also those magnitudes marked by a 'denomination' (namely an integral component)." After Vieta, "The new 'num - ber' concept which already controlled, although not explicitly, the algebraic expositions and investigations of Stifel,[154] Cardano,[155] Tartaglia,[156] etc., can now

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calculation with the species of number, where it is known in advance of the solution of any equation that the genus of the units involved in the solution will be homogeneous monads, in the case of the solution of Vieta's equations this has to be stressed, because the very indeterminacy of their solutions does not render it at all apparent that the ultimate referent of the species involved in each solution is, as Vieta believes, number in the traditional sense of ἀριθμός. Third, and finally, the notion of "the different genera of number" introduced by O'Neill into this whole discussion is problematic. The genera in Vieta's *logistice speciosa* concern the genera of unknown and known magnitudes (see this section, above), *not* different genera of number. In other words, the genus of number, understood as a field of homogeneous units, is something that remains constant so long as the traditional sense of number as ἀριθμός is in effect. What does change, and this is what is reflected in Vieta's ordering of the genera of unknown and known magnitudes, is the determinate amount of such units. And this state of affairs is precisely what is reflected when Klein reports that "Vieta himself says . . . '*In the case of numbers* the homogeneous elements of equations are *units*'" (181 n. 126/276 n. 255).

154. Michael Stifel, *Arithmetica Integra* (Nuremberg: Johannes Petreius, 1544).

155. Gerolamo Cardano, *Artis Magnae, Sive de Regulis Algebraicis Liber Unusi* (Nuremberg: Johannes Petreius, 1545); see *The Great Art, or The Rules of Algebra*, trans. T. Richard Witmer (Cambridge, Mass: MIT Press, 1968).

156. Niccolò Fontana Tartaglia, *Quesiti et invenzioni diverse* (Venice: Venturino Ruffinelli, 1546).

also be used to justify speaking of 'fractions' as 'non-integral rational numbers,' or 'irrational' numbers, etc." (186/178). Indeed, Klein reports that Vieta himself "calls improper fractions 'monades non purae'" (186 n. 137/278 n. 266) or "'numeri fracti,'" while he calls 'irrational' numbers, "depending on ancient terminology—'numeri asymmetri,' in contrast to the 'numeri symmetri.'" However, "like Diophantus, Vieta knows no negative numbers, if for no other reason than the fact that they cannot be represented in the geometric 'exegetic,' [because] the parallelism of geometric and arithmetical analysis must always be preserved."

For Klein, then, even though the new concept of species introduced by Vieta "actually realizes the fundamental transformation of the conceptual foundations" (152/149) of the Greek ἀριθμός, the "new '*number*' *concept as such* which is the basis of the 'symbolic' procedure" (152/149) that is invented by Vieta with this new species concept is not made "completely explicit" by him, but by Simon Stevin.[157] Nevertheless, because Vieta's species are "comprehensible only within the language *of symbolic formalism*" (183/175), that is, in what "is fully enunciated first in Vieta as alone capable of representing the '*finding of finding*,' namely 'zetetic,'" Klein credits him with making possible "the most important tool of mathematical natural science, the *formula*, . . . which, above all, opens up a new way of 'understanding' that was inaccessible to ancient ἐπιστήμη."

157. See § 96 above, where Klein's first suggestion of this is mentioned.

Chapter Twenty-three

Klein's Account of the Concept of Number and the Number Concepts in Stevin, Descartes, and Wallis

§ 107. Stevin's Idea of a "Wise Age" and His Project for Its Renewal

In contrast to Vieta, whose conservatism regarding the tradition, as we have seen, bound him to the traditional ἀριθμός-concept (i.e., a determinate amount of monads), even as his *logistice speciosa* invented the modern concept of number (i.e., the concept of 'amount in general'), Klein writes that Simon Stevin (1548–1620) "decidedly prefers novel approaches and unusual theses" (195/186). Stevin thus "lightly pushes aside the science of the schools and has little respect even for the authority of the Greeks" (198/190). Indeed, regarding the latter, "he always draws invidious comparisons between Arabic and Greek science" (199/190) on the basis of "the Arabic *ciphers and positional system*," which "appears to him to be immeasurably superior to the Greek notation." Notwithstanding his harsh judgment of the Greeks, however, Stevin nevertheless "is possessed by the idea of a 'renewal'" (196/186) of a "'wise age,' the 'siècle sage,' which once existed and which must be brought back" (196/186–87). Klein quotes at length Stevin's account of this age:

> We call the wise age that in which men had a wonderful knowledge of science which we recognize without fail by certain signs, although without knowing who they were, or in what place, or when. [. . .] It has become a matter of common usage to call the barbarous age that time which extends from about 900 or a thousand years up to about 150 years past, since men were for 700 or 800 years in the condition of imbeciles without the practice of letters and sciences—which condition had its origin in the burning of the books through troubles, wars, and destructions; afterwards affairs could, with a great deal of labor, be restored, or almost restored, to their former state; but although the aforementioned *preceding* times could call themselves

> a wise age in respect to the barbarous age just mentioned, nevertheless we have not consented to the definition of such a wise age, *since both taken together are nothing but the true barbarous age* in comparison to that unknown time at which we state that it {i.e., the wise age} was, without any doubt, in existence.[158] (196/187–88)

For Stevin, then, the "'barbarous age' . . . extends 'from the beginning of the Greeks to the present' (p. 108, col. 2)" (196/188). Klein reports that for Stevin the "'signs' that in earlier times a 'golden age' (aurea aetas) of science actually existed" include the following: 1) "The traces of a perfected astronomical knowledge in Hipparchus and Ptolemy" and "'certain writings in the *Arabic* tongue' (p. 107, col. 1)." 2) "*Algebra* . . . , which represents one of the strangest 'vestiges' of the 'wise age' (p. 108, col. 1)" (197/188), and of which no trace can be found "in the writings of the Chaldaeans, the Hebrews, the Romans, and even the Greeks, for, as Stevin expressly adds, '*Diophantus is modern*' (ibid.)." 3) "The books of Euclid" (197/189), which "pass on to us 'something admirable and very necessary to see and to read, namely *the order in the method of writing on mathematics* in that . . . time of the wise age' (p. 109, col. 2)." 4) "Information concerning the height of clouds, which appears in an *Arabic* work and which Stevin does not hesitate to trace back to the science of the 'wise age.'" And, finally, 5) "'Alchemy,' which was unknown to the Greeks and whose most expert representative Stevin sees in Hermes Trismegistos!"

Klein holds that Stevin's "general plan for the gradual recovery of the knowledge of the 'wise age'" (197–98/189) "represents the first project for 'organized research' (II, p. 110 ff.)" (198/189) and that it is based "[o]n the presupposition that the human ability to know has not changed since that time" (197/189). Klein reports the following four points to Stevin's plan. 1) "Many *observations* (especially in astronomy, 'alchemy,' and medicine) must be made, and this must be done by *many* people living at many different points of the earth and belonging to nations as different as possible" (198/189). 2) These observations and their "(primarily mathematical) exploitation" are to be communicated in "each man's own mother tongue," and thus "not learned Latin, which is accessible to few." 3) The languages used to accomplish this communication, however, must be up to the task, since "not all languages are fit for this purpose." Languages in which "words are very easily *compounded*" are singled out by Stevin as being especially suited to it, "which is why even now so many mathematical terms of Greek origin are in

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158. Simon Stevin, *Geographie*, in *Les Oeuvres Mathématiques de Simon Stevin de Bruges*, ed. Albert Girad (Leiden: Elsevir, 1634), 106, col. 2.

use" (198/189–90), though, on his view, "Greek is far surpassed by *Flemish*" (198/190) in this respect. And, finally, 4) "In every scientific presentation and in all teaching activity the *right order* must be preserved," which means for Stevin "the procedure of the mathematical disciplines is exemplary: 'I have not noted any better ⟨order⟩ for the matter of mathematics than that of the wise age' (p. 110, col. 2)."

§ 108. Stevin's Critique of the Traditional Ἀριθμός-Concept

Klein finds that Stevin's contribution to the project of this "renewal" was guided by his seeing "in the notion of a wise age, once a fact and now to be reestablished, the basis for the necessity of a thorough investigation of traditional views both for the sake of the *primeval* truths which might be contained in them and also to test the conventional *concepts* for their reliability and usefulness." Beyond Stevin's "invidious comparisons between Arabic and Greek science," which tout the superiority of the former's "*ciphers and positional system*" and "which he is inclined to view as the heritage of the 'wise age'" (199/190), he also "undertakes a fundamental critique of the traditional 'ἀριθμός concept,'" that is, of 'number' conceived as a definite amount of definite objects. The basis of this critique is precisely "this Arabic cipher system," and its point of departure is "the concept, decisive indeed, of the 'one,' the 'monad,' the 'unitas.'"

Klein avers that "the symbolic character of the new 'number' concept fully appears" (199/191) in a syllogism used by Stevin to critique the traditional exclusion of the unit as a number. After stating that "'Arithmetic is the science of numbers' (Arithmetique est las science des nombres)"[159] and that "'Number is that by which *the quantity* of each thing is revealed' (Nombre est cela par lequel s'explique *la quantité* de chacune chose)," Stevin states "'*that the unit is a number*' (*que l'unité est nombre*)." Klein reports that "He declares . . . that he has read all the 'old and new philosophers' who have pronounced on this question and has spoken with many scholars, not because he had any doubts with respect to this assertion—'no, definitely not, since I was as assured of this as if nature herself had told me with her own mouth'—but in order to be armed against all objections (p. 2ʳ)." For Stevin, the general claim "that the unit is not 'nombre,' but only 'its principle or beginning,'" is something that, according to Klein, he considers to be "completely false." Klein reproduces his argument for this as follows:

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159. Simon Stevin, *Arithmetique*, in *Oeuvres Mathématiques*, p. 1ᵛ.

> the part is "of the same material" . . . as the whole; the unit is a part of a multitude of units; consequently the unit is "of the same material" . . . as a multitude of units; "but the 'material' of a multitude of units is 'number' . . . ; therefore the 'material' of the unit, and thus the unit itself, is "number." He who denies this last step can be compared to someone who denies "that a piece of bread is bread." (199–200/191)

Klein maintains that "The decisive premise" (200/191) of this syllogism "is the one in which the 'material of a multitude of units' is equated with 'number.'" This can be seen insofar as "Stevin here simply accepts the classical definition of *numerus* as 'a multitude consisting of units' . . . , but he understands this conceptual determination as the 'material' of the thing to be defined, in the same sense in which one speaks of the material (materia) of water or of bread" (200/191–92). Thus, for Klein, it is "[o]nly on the basis of such an interpretation" (200/192) that "the first premise of the syllogism, according to which the 'part' is of the same material as the 'whole,'" is "relevant."

Klein does not think, however, that this means that "Stevin commits a paralogism." For "The fundamental *presupposition* which underlies his understanding—without, to be sure, him being able to see through it as such—is precisely the identification of the mode of being of the *object* with the mode of being of the *concept* related to the object." This means nothing less, according to Klein, than that "the one immense difficulty within ancient ontology, namely, to determine the relation between the 'being' of the object itself and the 'being' of the object in thought, is here (and elsewhere) accorded a 'matter-of-course' solution[160] whose presuppositions and the extent of whose significance are simply bypassed in the discussion." Klein stresses that "The *consequence* of this solution is the *symbolic* understanding of the object itself, an understanding in which its *actual* objectivity is posited as identical with the mode of being of a '*general* object.'" Moreover, he maintains that this symbolic understanding amounts to an understanding "through which the object of an '*intentio secunda*' (second intention), namely a concept as such, is transformed into the object of an '*intentio prima*' (first intention)."

§ 109. Stevin's Symbolic Understanding of *Numerus*

Klein shows that "Stevin himself confirms" that "Such a symbolic understanding of *numerus* or *quantitas* is precisely what is *presupposed* in the syllogism." Stevin does so when he relates that the "'absence of the necessary apparatus, namely of *ciphers*' (*Geographie*, II, p. 108)" (200/192–93) is "[t]he true reason for the [Greek] assertion that the unit is not 'number' but

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160. See § 104 above.

the *principium*, the ἀρχή of 'number'" (200/192). This absence "explains immediately why they were not arithmeticians" (200/193), as they misinterpreted as "'units'" (201/193) "a sign *called* a 'point' ('·') that they had inherited from the 'wise age' and which was identical with the present-day sign '0'" (200–1/193). They thus represented the units "with the aid of such points" (201/193), as "'*these points were used* ⟨by the Greeks⟩ *among their ciphers*' (p. 108)" (201 n. 166/288 n. 296). So the Greeks, on Stevin's view, were "misled by their false interpretation of the *arithmetic sign* '·' (the 'poinct Arabique')" (201/193) to "conceive of 'one' as the *principium* of 'number' *and not itself 'number*,'" whereas "In truth, not the unit but the nought is the *principium* of *number*—it, and not the unit, is the true analogue to the geometric point." This false interpretation, moreover, gave "the whole 'barbarous age' its characteristic 'unarithmetical' stamp."

Klein holds that it is because Stevin "transfers the role of ἀρχή, which had heretofore been assigned to the unit, to the *nought*: 'zero is the true and *natural* beginning' (. . . *Arithmetic*, p. 4r)" (202/193), that it is "so essential" "[t]hat the unit itself must be 'number.'" Indeed, it is precisely "his regard for the *sign notation*" that "determines him to do this," as "he completely identifies the *nought* with the sign '0,' whose full significance can be conceived *only within the cipher-system as a whole.*" And, what is more, this state of affairs "holds not only for the nought, but for all 'quantities' represented by 'ciphers.'" In fact, Klein notes that "This is also the reason for the well-known shift in meaning of the word 'cifra' or 'chiffre,' which was borrowed from the Arabic" (202 n. 168/289 n. 298) and which meant "at first only nought" and then "it gradually comes to be the common designation for all ten 'ciphers.'" Thus, "As 'arithmetician' Stevin no longer has in view amounts of units which are determinate in each case but the *unlimited possibility of compounding ciphers* according to definite 'rules of calculation'" (202/193). Klein emphasizes that this means that Stevin "no longer knows 'definite amounts of units' but only 'numbers' expressed in ciphers" (202/194), which is to say, "he conceives the 'quantities' with which he is dealing in a symbolic way."

§ 110. Stevin's Assimilation of Numbers to Geometrical Formations

One consequence of Stevin's symbolic understanding of numbers for Klein is that it "immediately leads to a far more complete assimilation of 'numbers' to geometric formations than was ever possible for the 'definite amounts of definite objects' and 'magnitudes' of antiquity." On the one hand, this is because "the symbolic understanding makes 'number,' as we

have seen, appear as a 'material' comparable to the material of bread and water." On the other hand, "the continuous extension of a line is compared to an arbitrarily continuable lining up of ciphers which yield continually new 'numbers'" (203/194) in Stevin's writings. Regarding the comparability of 'number' to material in the sense of bread and water, Klein quotes Stevin's view that "'The community and similarity of magnitude and number is so universal that it almost resembles identity'" (202/194).[161] Regarding the comparability of the continuous extension of a line and the lining up of ciphers, Klein reports that Stevin maintains that just as "a line $\overline{AB}$ is continuously lengthened by the addition of a point C in such a way that a new line $\overline{AC}$ results, then, by the same right, one may say *that the number 6, by the 'addition' of 0, continuously increases to the number 60!*" (202–3/194). Consequently, "This means that numbers too are continuous structures and not 'discrete,' as the 'barbarous age' asserted" (203/194), which is exactly what Stevin "formulates . . . explicitly: '*that number is not at all discontinuous quantity*' (p. 4^{v})."

The similarity of magnitude and number extends to the status of their "principles," respectively, the point and the nought. Therefore, "Just as a line is not lengthened by the addition of a point, so also 'number,' e.g., 6, is not increased by the addition of nought, for 6 + 0 = 6" (202/194). Thus, "neither the nought nor a point are 'parts' of a 'number' or a line" (203/195). Moreover, "neither can infinitely many points 'taken together' form a line, nor can infinitely many zeros 'together' form a 'number.'" However, just as "the unit is as much 'part' of a 'number,' and thus itself 'number' . . . , as a smaller line is part of a larger and is certainly itself 'line'" (203/195), so too "the 'parts' of the unit are, in turn, also 'numbers,' namely 'fractional numbers,' which decrease infinitely." Thus, not only "the unit is divisible as a matter of course," according to Stevin, but also, as a consequence of this, there "arises the *complete* correspondence of geometric magnitudes and 'numbers.'" Given the truly revolutionary implications of this breakdown of the "ancient view of the 'discreteness' of definite amounts of definite objects" (203/195), Klein quotes Stevin at length on this matter:

> As to a continuous body of *water* corresponds a continuous *wetness*, so to a continuous *magnitude* corresponds a continuous *number*. Likewise, as the continuous wetness of the entire body of water is subject to the same division and separation as the water, so the continuous number is subject to the same division and separation as its magnitude, in such a way that *these two quantities cannot be distinguished by continuity or discontinuity*. (*Arithmetic*, p. 5^{r})

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161. Klein holds that with this move, the "symbolic interpretation of geometric formations is herewith accomplished" (202 n. 169/289 n. 299), although "it becomes clearly visible only in Descartes."

Hence, "The 'units' of a 'number' are not 'disioinctes' but 'conioinctes' (p. 5^v)," not discontinuous but continuous.

Klein points out that it is on the basis of "the new 'number' concept" (204/195) that Stevin is able to address the consequences of the "schism between the *actual* understanding of 'number' and the understanding clinging to the *traditional* 'number' concept, as a 'definite amount of definite objects'" (204/195–96), consequences that Klein maintains include rejecting the "traditional appellations of 'absurd' or 'surd' or 'irrational' (i.e., un-speakable) numbers" (204/195). According to Klein, Stevin "attacks" (204/195) these traditional appellations with the thesis "'that there are no absurd, irrational, irregular, inexplicable, or surd numbers' (*Arithmetic*, p. 33 and p. 202, Thesis IV)" (204/196), a thesis that Stevin supports with the claim that "'incommensurability does not cause the incommensurable terms to be surds,' which is immediately obvious for incommensurable lines and planes. For example, √8 is a 'root' (racine)" and "'[e]very 'root' is a 'number' (p. 30; cf. p. 25)." As a consequence, Stevin "rejects the Diophantine (or Anatolian) terms 'side,' 'square,' 'cube' (latus, quadratum, cubus), etc." He does so, however, while still retaining "the distinction between 'arithmetical' and 'geometrical' numbers." Hence, he understands "'[a]n arithmetical number . . . as one expressed without adjective of size' (p. 6^r)," to which he contrasts "'roots,' 'quadratic' numbers, 'cubic' numbers, etc.," which he calls "'nombres Geometriques' (p. 6^v; 9 ff.)." Moreover, Stevin holds that "'any arithmetic *numbers whatsoever* can be squared or cubed numbers, etc. (p. 30),'" while "the 'geometric numbers'" (205/196), "insofar as their 'absolute,' i.e., numerical value is not known," "enter algebraic computations as indeterminate 'quantities.'" Consequently, Stevin maintains that "'The beginning of quantity is every arithmetic number or any *radical* whatever' (Def. XIV)," which means that "just as 0 is the 'beginning' of 'arithmetic numbers, so any arbitrary 'arithmetic' number is the 'beginning' of these algebraic 'quantities.'" Finally, Klein holds that "Stevin is also the first mathematician who understands the subtraction of a 'number' as the addition of a 'negative number'" (205/197). For Klein, Stevin's theses here and the conclusions he draws from them demonstrate that "Stevin is merely assimilating the *concept* of 'number' to *operations* on 'numbers' already long established." And a significant consequence of this is that "He thus once and for all fixes the ordinary understanding of the nature of number, for which being able 'to count' is tantamount to knowing how to handle 'ciphers.'"

§ 111. Descartes's Postulation of a New Mode of "Abstraction" and a New Possibility of "Understanding" as Underlying Symbolic Calculation

According to Klein, "It is an open question how much particularly Descartes owes" to Stevin. Nevertheless, Klein judges that Descartes's "apprehension of 'numbers' is far more tradition-bound than Stevin's, although, on the other hand, he sees through their conceptual structure with more clarity" (205–6/197).[162] Klein focuses his account of "the 'number' concept of Descartes" (207/197) on his "early *Regulae ad directionem ingenii* (*Rules for the Direction of the Mind*, ca. 1628), because it is apparently here that the original intentions of Descartes and the specific characteristics of his conceptual mode receive their clearest expression." Klein's account singles out Descartes's two main, interrelated achievements as follows: First, "he identifies 'algebra' understood as symbolic logistic with geometry *interpreted by him for the first time as a symbolic science*" (217/206). (Thus, for Klein, "Descartes does not, as is often thoughtlessly said, identify 'arithmetic' and 'geometry.'") Second, "Descartes' great idea now consists of identifying, by means of 'methodological' considerations, the 'general' object of this *mathesis universalis*—which can be represented and conceived only *symbolically*—with the substance of the world, with corporeality as 'extensio'" (207/197). On Klein's view, Descartes accomplishes the latter by connecting "two different trains of thought: (1) the conception of algebra as a 'general' theory of proportions, whose object, only symbolically comprehensible, acquires its specific characteristics from the *numerical* realm . . . , and (2) the identification of this 'symbolic' mathematical object with the object of '*true physics*'" (208/198).

Klein shows that "The connection of these two trains of thought is made possible by the 'methodical' concept of 'certain and evident cognition' (cognitio certa et evidens)." In his judgment, Descartes's "*original* identifica-

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162. Caton takes issue with this judgment, arguing that "to see through their conceptual structure as symbols, according to Klein's thesis, is identical with abandoning their conceptual structure as symbols" (Caton, 224). As we will see below, however, the "conceptual structure" that Klein maintains Descartes "sees through" concerns the relationship of the "pure" concept of *numerus*, as "'multitude in general'" (211/202), to the disregarding of "the monads counted, the units" (210 n. 184/295 n. 314) that is essential to "the special level of 'abstraction'" belonging to the "new 'number' concept." Stevin's comprehension of numbers rejects explicitly, as "unarithmetical," the traditional view of number as a "multitude consisting of units"; and, as we shall see, Descartes's "tradition-bound" apprehension does *not* simply reject outright this view of number in the account of the meaning of the "abstract" status of the "new" number concept he provides, which is why Klein can claim that "Descartes and, as far as we can see, only Descartes, struggles to fix the exact meaning of such an 'abstraction'" (210 n. 184/295–96 n. 314).

tion of the 'general mathematical object with extension having figure" (208 n. 180/294 n. 310)" results in "Descartes' 'method,' generally, developing from the need for a justification of the *place* which he assigns to algebra." Moreover, for Klein this means that "The point of view of 'methodical' cognition is thus secondary vis-à-vis" this original identification. However, because "everything depends on the *justification* of this identification, the 'method' gradually gains a more and more central significance, *in the course of which its rules are borrowed from the 'mathesis universalis' itself.*" Consequently, on Klein's view, "the road of 'inventio,' which the 'mathesis universalis' understood as 'general algebra' follows, is discovered as a way of cognition generally most appropriate to the human understanding.... In this sense the ... 'rules for the direction of the mind' ... are indeed identical both with the 'rules' of the 'mathesis universalis' and with those of the 'method' as such."

With regard to the *Regulae* themselves, Klein maintains that "In these formulations Descartes ... postulates—with an explicitness perhaps novel in the history of science—a new mode of 'abstraction' and a new possibility of 'understanding'" (210/200). In particular, Klein finds that Descartes's identification of the "general" object of the "'new' discipline" (207/197) of symbolic calculation, in the guise of contemporary algebra, with 'extension' understood as the substance of the world's corporeality, is an identification that "first gains symbolic mathematics that fundamental *place in the system of knowledge*, which it has never since lost" (207/198). Descartes's account of the "abstraction" that makes possible the symbolic representations that are essential for symbolic calculation, an "abstraction" that Klein calls "a '*symbolic abstraction*'" (212/202),[163] represents, according to Klein, Descartes's attempt to render perspicuous the precise cognitive status of these representations. Indeed, as we shall see below, Klein's presentation of Descartes's *Regulae* establishes a foundational link between the new mode of abstraction and the new possibility for understanding in Descartes's thought.

§ 112. The Fundamental Cognitive Role Attributed by Descartes to the *Imaginatio*

Klein traces the "fundamental role" (208/198) that Descartes attributes to "the *imaginatio* (= phantasia)" back to the "unmistakably *Stoic* origin" of the "Cartesian concept of knowledge," an origin evident in Descartes's "methodical" concept of "certain and evident cognition." To this concept there "corre-

163. The English translation renders *symbolische Abstraktion* as 'symbol-generating abstraction'. See n. 165 below for a discussion of the reason we translate it as 'symbolic abstraction'.

sponds 'apprehension' (κατάληψις), that is to say, the 'assent' (συγκατάθεσις) given to an '*apprehensional image*' (φαντασία καταληπτική)." This accounts, in part, for why "the *imaginatio* is, in the *Regulae*, always in the foreground." The prominence of the *imaginatio* in the *Regulae* and its role in cognition, however, does not signal for Klein that it has a foundational role in the genesis of the concepts in Descartes's "new" concept of knowledge. These concepts are "'abstract beings' (entia abstracta)" (210/200), and as such for Descartes they "are the products of the 'naked' or 'pure intellect' (intellectus purus), which is only called 'pure' insofar as the 'cognitive power' (vis cognoscens) that it exhibits is free from all admixture with 'images' or 'representations,' is entirely 'divorced from the aid of any bodily image' (absque ullius imaginis corporeae adjumento)," and performs its functions entirely "'by itself' (sola agit – Rule XII, 416, 4 and 419, 10)." The role played by the imagination concerns its "power" (211/201), a power distinct from the "pure intellect" that "must employ" it "in order to be at all able to get hold of what has been separated as such [from 'images' or 'representations']." Specifically, "the *imaginative* power . . . here enters the service of a faculty directed precisely toward something 'unvisualizable,' namely the 'pure intellect'" and the "abstract beings" that are its contents, which thereby enables the "pure intellect," "within the 'realm' of this 'alien' imaginative power," nevertheless to use it in order "to be able to represent 'visually'" that which the "pure intellect," in accord with the "performance of its functions," has "abstracted" from "*every immediate reference to the world*." Thus, on Klein's view, "*the imaginative power makes possible a* ***symbolic*** *representation of the indeterminate content which has been 'separated' by the 'naked' intellect*" (211/202). Klein unpacks all of this in great detail, beginning with Descartes's discussion of the precise nature of the "abstract beings" separated by the "pure intellect," next taking up Descartes's account of the relation between the "pure intellect" and the "'ideas' which the *imagination* offers it" (211/201), and concluding with Descartes's "reconciliation" (214/203) of "the traditional determinateness of the *numerus*, which Descartes, in contrast to Stevin, perfectly appreciates,[164] with the indeterminacy of the new 'algebraic' quantities."

Klein reports that "In Rule XIV (444 ff.), while treating of the role of the imagination, Descartes includes a discussion of the equivocality of certain concepts" (208/198), the point of which concerns the need, in order to preserve their truth by avoiding contradictions, to separate them completely from the imagination. Thus, "[i]n reference to the assertion that 'extension is not body' (extensio non est corpus)," Descartes "claims that *in this case* 'no special idea corresponds to this word "extension" in the imagination' . . . , that rather

164. See n. 162 above.

'this whole assertion is effected by the *pure intellect* which alone has the ability of *separating abstract beings* of that sort'" (208–9/198). Other assertions of this type include "'number is not the thing enumerated' (numerus non est res numerata) and 'a unit is not a quantity' (unitas non est quantitas)!" (209/199). Because "'All of these and similar propositions must be altogether divorced from the imagination in order to be *true*,'" "[i]f one were to try to 'represent' them to oneself by means of the imagination, one would necessarily arrive at contradictions—for in the imagination the 'idea' of extension cannot be separated from the 'idea' of body, nor the 'idea' of number from the 'idea' of the thing enumerated, nor the 'idea' of unity from the 'idea' of quantity."

Klein reports, however, that with respect to other propositions Descartes maintains "it is allowed *and even necessary to call to aid the power of the imagination*." The propositions Descartes has in mind are those in which

> the designated words, to be sure, are used with the same significance and in the same way as in those propositions that have been "abstracted" by the "naked intellect" from the "ideas" that are accessible to the power of the imagination, namely they refer to these "abstracted" formations, but, at the same time, they do not explicitly refer to the content of these separated "ideas," as their meaning refers rather to the matters themselves that follow from them, i.e., the matters as they appear in the "imagination."

Klein quotes Descartes to this effect:

> We must carefully note that in all other propositions in which these terms—although they retain the same meaning and are asserted in the same way as when abstracted from their subjects—do not [explicitly] exclude or deny anything from which they are not really distinct, we both can and *ought to call our imagination to aid*.

Thus, for example, when a proposition is about *numerus*, "we shall have to 'represent' to ourselves an object which can be measured by a 'multitude of units' (per multas unitates)," and we shall have to do so "even if the ('naked') intellect should mean precisely '*mere multitude*' (solam multitudinem), namely multitudinousness as such." That is because "we are not permitted to make the error of believing that the concept (conceptus) of *numerus* (to which corresponds the 'idea' of numerus in the 'imagination') excludes the *res numerata*, the 'ennumerated thing' itself." This error, which treats the "pure concepts" of the "naked intellect" as having no contact with the world, "is exactly what is not, and indeed cannot be, the case, in Descartes' opinion" (210/200). In the case of numbers, "if one does this, one eventually begins to ascribe deep secrets to 'numbers' and to hunt after phantoms." Of course, given Descartes's views about the "pure intellect," whose "constant presupposition is that *the 'pure' intellect in itself has no relation at all to the being of*

the world and to the things in the world" (212/202), it is not at all easy to grasp how the concepts proper to such an intellect nevertheless possess such a relation. The issue here, obviously, is "*the* insoluble problem of Cartesian doctrine" (212/203), namely, how "the relation between 'body' and 'soul' . . . is to be understood." However, on Klein's view, "For Descartes himself this difficulty is not a crucial one only *because he meets it originally in the realm of mathematics*, namely at that moment when it becomes important to reconcile the traditional determinateness of the *numerus*, which Descartes, in contrast to Stevin, perfectly appreciates, with the indeterminacy of the new 'algebraic' quantities" (213–14/203). Indeed, not only does Descartes meet it here, but he believes that on the basis of "the mediation of a special faculty, namely precisely that of the imagination" (212/202), both the mathematical problem of reconciling determinacy and indeterminacy and the ontological problem of reconciling body and soul can be achieved.

§ 113. Descartes on the Pure Intellect's Use of the Power of the Imagination to Reconcile the Mathematical Problem of Determinacy and Indeterminacy

Regarding the "pure" intellect's use of the imagination to reconcile the mathematical problem of determinacy and indeterminacy, Klein presents Descartes's thinking as follows:

> When, for instance, the "naked" or "pure" intellect separates from a multitude of units "represented" in the imagination (a "definite amount" of units, that is), their "multitudinousness" as such, i.e., the "mere multitude" (sola multitudo), the 'naked' indeterminate multitude to which simply nothing "true," nothing truly in "existence," and hence no "true idea" of an existent corresponds, it must employ the power of the imagination in order to be at all able to get hold of the thing separated.

It must do so because

> under those circumstances the *intellectus* or 'mind' (mens) is dealing only with itself, and only in this case can one speak of an '*intellectio*' and '*intellegere*,' "in that the mind, when it thinks, in a way turns *itself toward itself*" . . . and thereupon "beholds some one of the ideas which are within itself" (. . . Rule XII, 419, 8 ff.). (210/200–1)

The mind's act of beholding its concepts, the *actio intellectus* (211 n. 185/296 n. 315), "is called by Descartes an *intuitus*, literally, an insight: 'the unwavering grasp of a clear and attentive mind' (mentis purae et attentae non dubius conceptus) "consists in 'grasping – Rule III, 368, 18)." The concepts so grasped, namely, "'simple things' (res simplices)" (211/201), are either "'purely intel-

lectual' (pure intellectuales)," which belong to the "'spiritual'" realm, or "'communes,'" "which belong to the 'spiritual' as well as to the 'bodily' realm." The former include "'cognitio,' 'dubium,' 'ignorantia,' 'volitio,' etc. (cf. Rule III, 368, 21 ff.)" and the latter "'existentia,' '*unitias*,' 'duratio,' etc." Among the latter, Descartes also includes "the traditional 'communes notiones'—κοινὰ ἀξιώματα or ἔννοιαι—such as, for instance, the assertion that 'two magnitudes which are equal to a third are equal to each other,'" and "he expressly points out that the *intuitus* can, and even must, extend also to 'discursive' states of affairs, thus also to 'relations' and 'proportions'" (211 n. 185/296 n. 315).

Based on these considerations, Klein maintains that for Descartes the pure intellect, "which refers only to itself" and is thus "*bare of any immediate reference to the world*" (211/201), is appropriately characterized as "a faculty directed precisely toward something 'unvisualizable.'" However, Klein also holds that because for Descartes it "is also able to turn (or 'apply') itself to the 'ideas' which the *imagination* offers it, and can even 'separate' single constituents of these ideas [i.e., the res simplices]," "the intellect has, strictly speaking, already ceased to be 'pure,'" even though "it retains the ability *proper* to it—and *foreign* to the power of the imagination—of carrying out this kind of 'separating' operation." It is for this very reason, then, that "within the realm of the 'alien' imaginative power, *it must make use of this very imaginative power*" "in order to be able to get hold of what has been separated as such" from "images" or "representations," that is, in order to get hold of the *res simplices* as "abstract beings." Thus, for instance, "the *imaginative* power, which ordinarily allows us to be able to represent 'visually' 'five units' (perhaps as 'points'), here enters the service" of the "pure intellect," which "comprehends 'fiveness' as something separated from 'five' counted points or other arbitrary things—as mere 'multitude as such,' as 'naked' multitude" (211/201–2). In so doing, "*the imaginative power makes possible a* ***symbolic*** *representation of the indeterminate content which has been 'separated' by the 'naked' intellect*—this is what Descartes must emphasize" (211/202), according to Klein.

§ 114. Klein's Reactivation of the "Abstraction" in Descartes as "Symbolic Abstraction"

Klein maintains that "the 'abstraction' being spoken about here must accordingly be called '*symbolic abstraction*'" (211–12/202).[165] Before reactivating precisely how the "pure intellect," subsequent to its abstraction of concepts from the determinate images presented to it by the imagination, is

165. As we noted above (n. 163), in the English translation of Klein's *GMTOA sym-*

nevertheless able to use the imagination's "power" to render visible to it (the "pure intellect") "representations" of its otherwise invisible "pure concepts," Klein highlights both the novelty of this "symbolic abstraction" and its distinction from the "ancient mode of separation or 'abstraction' (ἀφαίρεσις)" (212/202). Moreover, he also signals its foundational significance in establishing what will become the opposition between "'intuition' and 'conception'" in post-Cartesian thought. Regarding the ancient mode of abstraction, from "here on" it "appears . . . as a 'direct' or 'imaginative' abstraction: The insights of the ancient mathematicians 'pertain more to the eyes and the imagination than to the intellect' (magis ad oculos et imaginationenem pertinent, quam ad intellectum – Rule IV, 375, 18 f.)." Indeed, on Klein's view, it is precisely the presupposition of the Cartesian "symbolic abstraction" that explains "why the ancient 'number' concept can at present[166] be characterized as having its 'intuitability.'" Regarding the opposition between 'intuition' and 'conception', Klein claims that Descartes's symbolic abstraction "alone gives rise to its possibility," as well as to the possibility, of "being able to posit 'intuition' as a *separate* source of cognition *alongside* of that belonging to the understanding."[167]

Klein's account of why Descartes thinks that the mediating "power of the imagination" makes possible the "pure" intellect's grasping of symbolic

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bolische Abstraktion is rendered as 'symbol-generating abstraction', in recognition of his characterization of the role that Descartes's account of abstraction plays in the generation of the symbols for the *mathesis universalis*, and the translator's belief that, properly speaking, this abstraction itself is not "symbolic." From the preceding, we have seen that one of the conditions responsible for the cognition of marks (algebraic letter signs, Descartes's figures) as symbols is the imagination's power to make visible "representations" of the indeterminate "ideas" that the "pure" intellect's power of abstraction has separated from the imagination's own determinate "ideas." Moreover, we have seen that another condition is that the "pure" intellect, notwithstanding this use of the "foreign" power of the imagination, must maintain its capacity to keep its indeterminate "ideas" separate from the determinate "ideas" offered to it by the imagination if the determinacy of the marks made visible by this same imagination's power are to be apprehended as "symbols." Because it is precisely the involvement of the "pure" intellect's abstractive power in maintaining the separation that is responsible for its apprehension of the algebraic letters and geometrical figures that belong to the *mathesis universalis* as "symbols" rather than as determinate images (and therefore determinate "ideas"), on my view it remains appropriate to characterize the abstraction involved as "symbolic."

166. Klein refers at this point in his text to his earlier discussion of the studies of Julius Stenzel and Oskar Becker, both of which attempt to draw the distinction between the ancient and modern number concepts on the basis of the former's "intuitability" (65/62). Klein's point, both in his earlier discussion and now, is that this basis for the misunderstanding of this distinction "is guided after all by *our* number concept," the presupposition of which obscures access to the non-conceptual, "one-over-a-determinate-many" structure of ἀριθμός.

167. The 'intuition' at issue here, of course, is *not* Descartes's *intuitus*, which, as we have seen, is "conceptual" or "intellectual" in a sense that rules out precisely the intuition as *Anschauung* that is involved in the opposition here between 'intuition' and 'conception'.

representations of its pure concepts in a new mode of abstraction, indeed, one that reconciles the "traditional determinateness of the *numerus* . . . with the indeterminacy of the new 'algebraic' quantities," begins with a discussion of the "figures" in the "*figural* representation" (214/203) characteristic of Descartes's geometry. Klein stresses that "Everything depends on understanding that the 'figures' with which the 'mathesis universalis' deals, namely 'rectilinear and rectangular planes' as well as 'straight lines' . . . have, as far as their mode of being is concerned, no longer anything to do with the 'figures' of what had up till then been the ordinary 'geometry.'" He quotes Descartes at length to support this (214/203–4):

> We easily conclude: here propositions must be just as much *abstracted from those very figures with which geometers deal*, if the inquiry involves these, as from any other subject matter; and for this purpose none need be retained besides rectilinear and rectangular plane surfaces, or straight lines, which we also call figures because by means of them we can imagine a subject which is in truth extended {namely in three dimensions} just as well as by means of a plane surface. (. . . Rule XIV, 452, 14 ff.).

In addition to emphasizing that the "figures" in Descartes's *mathesis universalis* do not represent the figures of ordinary geometry, Klein also points out that for Descartes the proportions and therefore equations with which it deals can, if a certain condition is met, "be understood as 'numbers'" (215/204). That is, proportions and equations can be understood as "multitude" in the sense belonging to "one of those numbers with which 'algebra' deals in setting up proportions among 'A,' 'B,' and 'C.'" The condition is that the "'common measure' (communis mensura)," specifically the "particular '*unit*' (unitas)," must be "known for each case" in the instance of the comparison of "'*continuous* and undivided magnitudes' (magnitudines continuae et indivisae), 'lines' and 'planes'" (214/204), which then permits "us to set up proportions (and thus equations)." Thus, "if that condition is met, these continuous magnitudes can immediately be understood as 'numbers'" (215/204), "numbers" that, moreover, "need no longer be referred to the 'common measure' because measuring (mensura) as such is here no longer the concern, but only 'arrangement' (ordo)." Klein quotes Descartes to this effect:

> [We must understand], too, that we are able afterwards to *arrange the multitude of units in such an order* that the difficulty which before was one of solving a problem of measurement now depends only on observing an order, and that the aid which our art gives us in this process is very great. (. . . Rule XIV, 452, 2–6).

Descartes's "figures," therefore, "appear as 'numbers' only through the '*mediating unit*' (mediante unitate)" (215 n. 189/299 n. 319), a "unit," as we

have seen, that is "itself understood as 'unit of measurement' (mensura)." Thus, according to Klein, even though "[t]he 'measure' is here, to be sure, applied only to symbolic formations," "this does not change the fact that Descartes basically retains the traditional—Peripatetic—understanding of the ἕν." This understanding, as we have seen,[168] takes the ἕν—in contrast to the Platonic view of it as a "common thing" (κοινόν)—to be a common "measure" (μέτρον). By understanding the "unit" in this manner, Descartes, in Klein's judgment, "*fails* to pursue his own, different, original notion, according to which the unit is a 'res simplex communis' and therefore a *res simplex intellectualis*."[169]

On Klein's view a major consequence of Descartes's understanding of proportion in terms of *ordo* and not *mensura* is that "a plane or linear 'figure' represents no less and *no differently* a 'multitude or number' (multitudinem sive numerum) than a 'continuous magnitude'" (215/204). This means that "Each 'figure' is therefore not a representation of a *determinate amount* of units of measurement (as are the straight lines of Euclid's arithmetical books)" (215/205). Rather, "it is the primary task and the *truly proper function* of such 'figures' 'to image' the 'true idea' (vera idea) of number (numerus)." When we are "[c]onfronted with an indeterminate multitude, i.e., with any algebraic quantity," we are therefore in a position to be "misled by the pure intellect" and to "want to understand" such a quantity "as a 'mere multitude' (sola multidudo), that is, as a formation separated from all 'enumerated things' (res numeratae)." In order, then, "not to fall into error, we must indeed . . . 'represent' to ourselves 'some subject measurable by many units' (subjectum aliquod per multas unitates mensurabile)," which, according to Klein, is precisely what is accomplished by Descartes's "figures." They accomplish this because like the letters in algebra (*notae*), they are visible—and thus something that is "measurable by many units"—even though their status as figures is emphatically *not* representative of the figures belonging to ordinary geometry. In other words, Descartes's figures represent the "indeterminate multitude" of what, although it is "in truth" not separated from enumerated and extended things, is nevertheless *not* conceptually equivalent to them either. This is to say, every figure, as just such a representation, is "a 'symbol'—obtained through 'symbolic abstraction'—of an *indeterminate multitude*." Moreover, Klein holds that this symbol "is exactly the same as the letter sign (together with its 'degree' designation) that occurs in 'algebra,' especially in Vieta's analytic."

Indeed, for Klein it is precisely "[t]his 'symbolic' character of Cartesian 'figures' that first makes possible the mutual correspondence of 'lines'

168. See § 87 above.

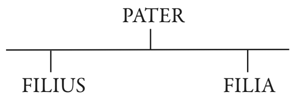
169. On the symbolic understanding of the 'unit' or 'one', see § 118 below and n. 187.

with 'letter signs' or 'ciphers' which obtains in Cartesian mathematics" (215–16/205). Klein reports that "Descartes himself says [this] explicitly: 'It should be noted . . . that we . . . here *abstract no less from numbers themselves than* we did just before *from geometric figures* or from anything else you like.' (. . . Rule XVI, 455 f.)" (216/205). Thus, Klein concludes that this means that "Precisely the same reinterpretation which the traditional concept of ἀριθμός undergoes at the hands of Vieta, Stevin, and the other contemporary algebraists, is effected by Descartes—and this is his original achievement—in the realm of traditional γεωμετρία" (217/206). Hence, on Klein's view, "The symbolic abstraction which leads from the ordinary numbers [*numeri*] to letter signs, to the 'notae' or 'termini generales' . . . , also called 'puri et nudi' . . . , needs the imagination in exactly the same way as when it leads to 'figures'" (216/205). In connection with this, Klein stresses: "The mode of being of these 'figures' is, therefore, to repeat, none other than that of algebraic 'numbers'—of Vieta's 'species'" (216–17/206).

§ 115. Klein's Use of the Scholastic Distinction between "First and Second Intentions" to Fix Conceptually the Status of Descartes's Symbolic Concepts

Klein therefore maintains that "Descartes' '*mathesis universalis*' corresponds completely to Vieta's 'zetetic,' by means of which is realized, with the aid of 'logistice speciosa,' the 'new' and 'pure' algebra, interpreted as a general 'analytical art'" (217/206). This correspondence obtains because "both have in mind a *universal* science." Where they differ concerns the relationship between this science and traditional arithmetic and geometry. As we have seen, "Vieta sees the most important part of analytic in 'rhetic' or 'exegetic' . . . , in which *numerical* computations and the geometric constructions represent two different possibilities of application (so that the traditional conception of geometry as such is here preserved)." In Descartes, however, the traditional conception of geometry is not preserved, since he "*begins* by understanding geometric 'figures' as formations whose 'being' is determined *solely* by their 'symbolic' character." Indeed, Klein reports (215 n. 189/299 n. 319) that

> Descartes, incidentally, from the very beginning distinguishes "two kinds of figures" (duo genera figurarum), namely those which represent a "multitudo," e.g.,
>
> 
>
> i.e., a "numerus triangularis" . . . , or



> i.e., a genealogical "tree which displays someone's family relations" . . . , and on the other hand, those which represent a "magnitudo," e.g.,
>
> □, △, etc.
>
> (Rule XIV, 450 f.).

However, "The symbolic 'figure' which forms the object of the *mathesis universalis*" is *not* to be found among these "completely traditional *nonsymbolic* formations," as it is rather precisely "what is 'common to' such figures, that is, 'figurality' itself."

Klein again uses "the contemporary literature of the schools to fix yet more exactly that conceptual character of algebraic symbols which has already been variously outlined" (218/206),[170] although in this case he does so in conjunction with "Descartes' assertions" about the mediation of the imagination in reconciling the mathematical problem of the relation between determinacy and indeterminacy, and, in addition, to bring greater clarity to "the sense in which we spoke earlier of 'symbolic abstraction'" (222/208). Klein begins by establishing a connection between Descartes's designation of "the 'sola multidudo' which the intellect 'separates' from the 'idea' of number [*numerus*] it finds available in the imagination as an 'abstract being' (ens abstractum)" (220/206) and what is "called an *ens rationis* in the language of the schools." Klein reports, following Étienne Gilson,[171] that "the *Summa* of Eustachius a Sancto Paulo" (220/206) characterizes "'*second intentions*' (secundae intentiones)" (220–21/207) as one of three kinds of *ens rationis* ('being of reason') that "owes its 'being' to the operation of the intellect alone" (221/207). Thus, second intentions, in Eustachius's words quoted by Klein, "'do not belong to things *unless a certain operation of the intellect is presupposed*, wherefore these beings of reason are said to depend on the intellect for their existence and connection.'" For Klein, then, Descartes's "'mere multitude' (sola multitudo), multitudinousness as such, which has its 'being' by grace of the 'pure intellect,' is truly an *ens abstractum* or *ens rationis* in the sense of a '*second intention*.'"

Moreover, the "process of separating" that Descartes claims is responsible for bringing about the "being" in question insofar as "the 'pure intellect' ventures into the 'alien realm' of the 'imagination'" "must," according

170. See § 106 above.
171. Étienne Gilson, *Index scolastico-cartésien* (Paris: Alcan, 1912), 107.

to Klein, "be more precisely described" in a manner that takes its clues from Eustachius's "more exact" determination of the second intention. In this determination, Eustachius, "appealing to prevailing terminology, defines 'second intention' as an *ens rationis* 'that is conceived as belonging to a thing known by virtue of its being known, and which cannot exist except *as something present in the intellect*, since it is conceived {not originally but} *secondarily and through a self-relating action of the mind*' (quod concipitur accidere rei cognitae, ex eo quod cognita est, quodque non aliter potest existere quam objective in intellectu, cum secundario et per reflexam mentis operationem concipitur)." Klein describes as follows the "process of separating" in Descartes in accord with this "more exact" determination of second intention: "The intellect, when directed to the 'idea' of a number [*numerus*] as a 'multitude of units' (multitudo unitatum – πλῆθος μονάδων) offered to it by the imagination, . . . turns, as its nature requires, toward its own 'directedness,' its own knowing" (221/207–8). In so doing, "it sees the multitude of units no longer 'directly,' no longer in the 'performed act' (actus exercitus . . .), but 'indirectly,' 'secondarily' (secundario), or, in the terms of another scholastic expression, in the 'signified act' (actus signatus[172])." With this turn, then, what "is now immediately 'objective'" for the intellect "is its own conceiving of that 'multitude of units,'" a conceiving that having become objective is equivalent to "the 'concept' (conceptus) of number [*numerus*] as such." Notwithstanding the fact that what is being conceived by the intellect in its conceiving is a *multitude* of units, however, "this *multitude itself* nevertheless appears to it as 'something,' namely as *one* and therefore an 'ens,' a '*being*.'" For Klein, "This is precisely what the 'abstraction' [*abstractio*] which the intellect undertakes consists in: It transforms the multitude of the number [*numerus*] into a seemingly 'independent' being, into an '*ens*,' if only an '*ens rationis*.'" Klein emphasizes that what is of "crucial importance" (221–22/208) in order for this abstraction to generate a symbol is that "the '*ens rationis*,' as a 'second intention,'" or, more precisely, "as the intended object of a second intention" (222 n. 194/306 n. 324), "becomes graspable—with the aid of the imagination—in the mode that the intellect, in turn, is able to take up as a 'first intention'" (222/208). This is to say, the imagination assists the intellect in making into *the object of a first intention* something that, in the language and conceptuality of the schools, *is properly the object of a sec-*

172. The translator of *Greek Mathematical Thought and the Origin of Algebra*, Eva Brann, adds the following parenthetical gloss of *actus signatus*: "i.e., an act the object of whose intention is already an expressly signified concept, as opposed to the object of the first intention which is a being on which the *actus exercitus* is immediately exercised" (222/208).

ond intention.[173] When this happens—and for Klein it *does not happen* within the context of the horizon of the conceptual presuppositions of the school philosophy of the scholastics[174]—"we are dealing with a *symbol*, either with an 'algebraic' letter-sign or with a 'geometric' figure as understood by Descartes."

§ 116. Descartes on the Non-metaphorical Reception by the Imagination of the *Extension* of Bodies as Bridging the Gap between Non-determinate and Determinate Magnitudes

According to Klein, the imagination's "'service' function" (222/208) with "respect to the intellect," in which it is defined by the way it "insures the pos-

173. Richard Kennington, in his otherwise excellent synopsis of Klein's account of how the "Cartesian *numerus* is acquired" (Kennington, Review of Klein's *Lectures and Essays*), inexplicably identifies the genesis of its symbolic status with the abstractive transformation of the "multitude" belonging to a "multitude of units" into a "multitude itself" or "'general magnitude.'" Thus, he writes: "An amount is initially abstracted from a multiplicity of things; but this amount is then taken just as amount and not an amount *of* any determinate sort of things, hence becoming a 'general magnitude,' that is, a 'symbol' which because of its indeterminacy can be represented algebraically; the result of computation of such second intentional entities is then imputed to first intentional beings." As can be seen from the account just given, however, Kennington here is at odds with Klein's presentation of the issues on three related points: 1) While the abstractive transformation of a determinate multitude or multiplicity into multitude itself or general magnitude is indeed *part* of Klein's account of the process of symbolic abstraction, Klein clearly identifies the imagination's mediating role as what is "crucial" to this process, since it functions to make the "intended object of a second intention," namely, this "multitude itself" or "general magnitude," *graspable as an object of a first intention*. 2) This, in turn, is accomplished by the algebraic letter signs or the "figures" of Descartes's *mathesis universalis*, which, "with the help of the imagination," transform the object of a second intention (the unimaginable pure concept of "multitude/magnitude itself") into the object of a first intention (the letter sign or Cartesian "figure"). Thus, it is not the case that what is "represented algebraically" is the general magnitude understood as a symbol, because, as we have seen, for Klein it is not the indeterminacy of the general magnitude that is symbolic but rather the *representation* of this indeterminacy by something that is, in a fashion, determinate and thus accessible in a "first intentional" manner: letter signs or Cartesian "figures." Finally, 3) because for Klein these symbols are precisely the objects of first intentions, even though what they represent are objects of second intentions, there is no need to "impute" the results of "computation of such second intentional entities" to "first intentional beings," since, again, it is precisely the grasping of these "second intentional entities" as "first intentional beings" that comprises their "symbolic" status. The question, then, that Kennington raises with respect to Klein's account of symbolic abstraction, of "whether the first and second intentions have been confused" (presumably by Descartes's thought and therefore the modernity it makes possible), seems to be off the mark insofar as Klein uses these scholastic distinctions in the service of his descriptively orientated de-sedimentation of something, namely "symbolic cognition," whose condition of possibility for Klein rests upon something that cannot be got at in terms of a putative confusion of levels of intention.

174. See § 106, n. 147 above.

sibility of symbolic knowledge in general and, in particular, of the *mathesis universalis* as a general theory of proportions and equations," is something that "depends . . . on a 'real,' and not a 'metaphorical,' 'rendering' of the corporeal world" (224/210) by the very same imagination. Thus, Klein concludes that, for Descartes, "*Everything* which we receive from the 'world' with the aid of the 'external senses' (sensus externi) is transferred by way of the *sensus communis* to the imagination by a series of 'seal impressions,' and this occurs in such a way that the parts of the body affected by the 'impressions' (impressiones), *including the imagination*, take on the shape, the 'figure,' which belongs to that part of the world which is making the impression (cf. Rule XII, 412 ff.)" (223/210). This reception by the imagination, via impressions, of "the *actual* [wirkliche] '*extension*' of bodies" (222/209), of "*their corporeality as such*," something that Descartes stresses "'must not be thought to be said metaphorically' (neque hoc per analogiam dici putandum est)" (223/210), means that "*this very same corporeal nature belongs*, *according to Descartes*, *to the imagination* (or *phantasia*) *with all the ideas present* '*in*' *it*" (222/209). For Klein, "This and nothing else explains why the imagination can guarantee the ability of the *mathesis universalis* to grasp the structure of the 'true world' and so to prove itself a 'wonderful science' (scientia mirabilis)" (224/210).

Indeed, Klein notes that notwithstanding the fact that "The doctrine of 'impressions' (τυπώσις – *typosis*), especially in relation to the *imaginatio*, is, to begin with, again unquestionably Stoic" (223 n. 196/306 n. 326), the "precise distinction between the traditional part and what is specifically Descartes' own and therefore 'new'" is located in Descartes's rejection of the metaphorical understanding of the impression's status in the soul. Thus, while "in ancient philosophy τυπώσις is *always* understood in such a way that the 'impression' in the soul is not taken literally but is simply regarded in each case as some sort of counterpart to the '*looks*' of the thing *precisely as it presents itself to our* ('*external*') *senses*" (223 n. 196/307 n. 326), this is a "position which Descartes explicitly opposes." Moreover, "the Cartesian conception of this process is completely original in reducing *everything* perceptible by the (external) senses, that is, besides the 'things' themselves also their colors, warmth, coldness, hardness, roughness, sweetness, etc., to 'figures,' which are supposed to present the *true* 'nature' of the 'things' or 'forces' or 'properties' in question, namely precisely that nature which is inaccessible to the external senses" (224 n. 196/307 n. 326). Hence, what "the intellect 'sees' when it 'turns toward' the 'impressions' 'in' or 'on' the part of the brain which *is* the 'phantasia vel imaginatio'" (224 n. 196/308 n. 326) are precisely "the variety of the infinitely many possible 'figures'" (224 n. 196/307 n. 326) that are responsi-

ble for "[t]he variety of these 'things,' 'forces,' or 'properties.'" Finally, for Klein, even though Descartes initially "introduces only as a *suppositio*" (224 n. 196/308 n. 326) this understanding of perception, "these original assumptions, especially those underlying the understanding of *extensio*, remain not only the basis of Descartes' later writings, but are, in fact—admittedly or unadmittedly—presupposed by all modern physiology and physiological psychology even to this day."

However, the magnitudes that belong to the extension of bodies in the world and therefore to the corporeal nature of the imagination itself are "specific," which is to say, determinate, while "the subject of proportions and equations" (222/208) treated by "the *mathesis universalis* deals only with 'magnitudes *in general*' (magnitudeines in genere)" (222/209), that is, "indeterminate magnitudes," which, as we have seen, "are conceived by the intellect as '*entia abstracta*.'" Regarding either type of magnitude, Klein writes that "Descartes argues in Rule XIV" that "Only that kind of thing" (222/208) "'which accepts the more or less' (quod recipit majus et minus) . . . can be called magnitude" (222/209). The determinacy of "specific" magnitudes and the indeterminacy of "magnitudes in general" give rise to the issue of whether, and if so, how, exactly, they are related. On Klein's view, Descartes's argues for their relationship on the basis of the legitimacy of the transfer of what is said about indeterminate to determinate magnitudes. To justify this transfer, Descartes appeals both to the "'figurality'" of the "*actual* [wirklich] *extension*" of the determinate magnitudes presented in the imagination and to the fact that the pure intellect relies on the "aid" of the imagination in its treatment of general magnitudes. When everything but the figurality of the actual extension of bodies is "disregarded," then it is nothing other than this "figurality itself," as it is depicted by the imagination, that allows it to form "a bridge to the world of 'bodies' for Descartes" (222/208). This is because "figurality" has its basis in the imagination, which "always presents precisely that within the corporeal world which *actually* [wirklich] constitutes its true nature, its 'substance,' its 'corporeality'" (224/210). Klein quotes Descartes at length to substantiate this claim (223/209–10):

> In order to *imagine* something even then {in the treatment of general magnitudes} and so as not to use the intellect purely but *with the aid of specific* [i.e., particular] *forms depicted in the image-making organ*, we must finally note that nothing can be said of magnitudes in general which cannot also be ascribed to some specific form or other. From this we easily conclude that there will be no little profit in transferring that *which the intellect allows us to say about magnitudes in general* to that specific form of magnitude which is depicted most easily and distinctly in our imagination; this is indeed the *real extension of a body* abstracted from everything else, except that it *has figure*, as follows from what was said about it in the twelfth rule, where we conceived

> of the *image-making organ itself*, with the ideas existing in it, as being nothing but *a true body, really extended and having figure*. (. . . Rule XIV, 440 f.).

From this transferability of what is said about magnitude in general to specific or determinate magnitude, Klein draws the conclusion that "Extension has, accordingly, a *twofold* character for Descartes: It is 'symbolic'—as the object of 'general algebra,' and it is 'real'—as the 'substance' of the corporeal world" (225/210–11). Indeed, Klein articulates this more precisely when he writes: "what in Descartes' thinking confers on extension the dignity proper to the substantial '*being*' of the corporeal world is exactly its *symbolic* objectivity in the framework of the *mathesis universalis*" (225/211). With this, "the foundation on which Newton will raise the structure of his mathematical science of nature," namely, "the conceptual basis of 'classical physics,' which has been called 'Euclidean space,' has been created." The fact that "the importance of the *imaginatio* in the Cartesian system declines more and more" (225 n. 198/309 n. 328) in the development of Descartes's thought, "which leads him to find the guarantee for the certainty of 'clear and distinct,' that is, above all, *mathematical* cognition—and consequently also the guarantee of the possibility of a 'true physics'—via a detour into metaphysics whose warrant is ultimately granted by God," leaves "[t]he character of the *extensio* itself," in Klein's judgment, "essentially untouched by this development."

§ 117. Wallis's Completion of the Introduction of the New Number Concept

According to Klein, John Wallis's (1616–1703)[175] book, *Universal Mathematics* [*Mathesis universalis*], *or the Whole arithmetical enterprise as handed down both by the Philosophical and the Mathematical Tradition, embracing both Numerical and Specious or Symbolic Arithmetic as well as the Geometric method of calculation; furthermore the traditional theory of Ratio and Proportion, also the Doctrine of Logarithms; and other matters which will be indicated by the Table of Chapter Headings* (1657),[176] presents "[t]he final act in the introduction of the new 'number' concept" (225/211). Wallis's discussion 1) makes apparent "in what the difference between 'thinking' (cogitare) and 'saying' (dicere), as Bachet formulates it . . . really consists" (227/213),[177] 2)

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175. Wallis contributed substantially to the origins of calculus and is generally regarded as the most influential English mathematician before Newton.

176. Klein states that "he quotes from the complete edition undertaken by Wallis himself (1693–1699)" (226 n. 200/309 n. 330): John Wallis, *Opera mathematica*, 3 vols. (Oxford: Oxford University Press, 1693–1699).

177. See § 98 above.

stresses the "'arithmetic' nature" (229/216) of the "'symbols or species' (symbola seu species)" with which the *logistice speciosa* calculates, such that they are understood to be "both '*general magnitudes*' and precisely also—'*numbers*'" (231/218), and, finally, 3) presents "[t]he object of arithmetic and logistic in their algebraic expansion . . . as 'number,' . . . symbolically conceived as *ratio*" (235/223). For Klein, all of this means not only that the "conception" of the object of arithmetic and logistic is now completely "consonant with that of algebra as a general theory of proportions and ratios," but also that "The 'material' of this universal and fundamental science is no longer furnished by 'pure' units whose mode of being may be subject to dispute, since they can be conceived as formations that are either independent or obtained by 'abstraction' (ἀφαίρεσις)" (235/223–24). Rather, this "material" is now furnished by "'numbers,' whose being no longer presents any problem since, as the products of symbolic abstraction, they are immediately graspable in the notation" (235/224).

§ 118. Wallis's Initial Account of the Unit Both as the Principle of Number and as Itself a Number

Klein reports that the point of departure for Wallis's consideration of the "true 'principle of number'" (226/212) is "the question 'whether the unit is a number' (an Unitas sit Numerus – pp. 24–27)." When Wallis's initial answer to this question—that 'one' is a number while the 'unit' is the principle of number—is compared with "conceptual mode of ancient arithmetic" (227/213), the sedimentation of the distinction between thinking and saying that, for Klein, occurs in the conceptual mode of the modern (symbolic) understanding of number[178] is brought to the fore. Wallis's reason for treating the "unit" as the principle of number has its basis in "the complete parallelization of 'arithmetical' with 'geometrical' procedure" (226/212), which is "by now accepted altogether as a matter of course." Hence, Klein quotes Wallis to this effect: "'As far as principles are concerned, the *point* is that of magnitude, while the *unit* is that of number, a principle *commonly* proposed . . . since number is *commonly* defined as a multitude of units' (. . . p. 20)" (226/212).

This understanding of the principle of number, however, does not prevent Wallis from deviating from the conclusion that is commonly drawn from the unit's status as the principle of number (defined as a *multitude* of units), namely, that because it is lacking in multitude, it is *not* a number. He reasons that "one" is indeed a number, "for 'one' answers the question 'How many

178. Ibid.

are there?'" Owing to the importance of Wallis's elaboration of this issue for highlighting the sedimentation under discussion, Klein quotes at length Wallis's elaboration of the reason why "one" is a number (226–27/212–13):

> When I assert that "four" has the same force as "four units," then "units" {here} are neither number nor a {constituent} part of a number, but either the denomination {i.e., the number-name} of the number or the term giving the denomination, or the thing numbered itself. However, "four" is indeed the *number* of these units. Thus also, when I assert that "one" has the same force as "one unit," "one monad," then "unit" is the denomination of the term giving the denomination of the number, but "one" is a *number*, namely the multitude of units (the word *multitude* being taken in a loose sense . . . : for "one" says "how many" or "how many units" are asserted to be present, namely a single one.

On Klein's view, the distinction Wallis draws here between "the 'unitas' or μονάς" (227/213), which "one might on this account be able to deny . . . is 'number,'" and the "'unum' or ἕν," whose "'numerical' character it is impossible to deny," has its basis in precisely the previously mentioned distinction between 'thinking' and 'saying'. Thus, "By 'four' or 'six,' we do, indeed, mean four or six units, but by way of a *detour* through the '*number*,' which alone is directly expressed and addressed, and this means precisely that it is understood *without the counted things*, and consequently only as 'symbol.'" For thinking, then, "units" are indeed what a number means. However, in saying 'number', what is said "in the context of the symbolic conception" of number is precisely "objectless." Klein here emphasizes once more that "This 'indirection' of the understanding of a 'definite amount of definite objects,' which characterizes modern consciousness, can, to be sure, become visible only if the conceptual mode of ancient arithmetic is kept in mind." As we have seen, within the context of the latter "the concept of the 'four' or 'six,' or even of the 'unit,' refers *immediately* to the particular unit or units, be they presented as sensibly perceived things or pure noetic formations and be these formations, moreover, understood as independent or as merely 'abstracted.'" Furthermore, Klein again stresses that "there is no immediate occasion for questioning the mode of being of the 'symbol' itself," since "The whole ontological problematic of the ancient concept of definite amount," which, as we have seen, is preoccupied with the issue of accounting for the mode of being proper to the "one over many" unity of the ἀριθμός,[179] becomes quite literally "without an object"[180] within the context of the symbolic number conception. Finally, Klein again cautions that "This state of affairs is, of course, completely dis-

179. See n. 79 above.

180. That is, it loses the direct reference of the ἀριθμός to a sensible or noetic multitude.

torted if it is simply asserted that the modern number concept possesses a remarkably high degree of 'abstractness,' without bothering to investigate the real mode of this 'abstractness'" (227/213–14).

§ 119. Wallis's Account of the Nought as Also the Principle of Number

Wallis's initial account of the 'unit' as the "'*principium numeri*,' corresponding to the '*numerus*' understood as '*unitatum multitudo*'" (226/211) is, according to Klein, introduced by him "explicitly only to save the 'usual' definitions" (226/212). Hence, Klein quotes Wallis's view that "'As can be ascertained with certainty by anyone who looks ⟨into the matter⟩ more deeply as soon as the arithmetic operations ⟨i.e., calculations⟩ are compared with the ⟨accompanying⟩ geometric constructions'" (227/214), "the 'nought'"—which "presents for the number domain the sole analogue to the geometrical point"—is the "true 'principle of number.'" Wallis attempts to bring the "traditional opinion . . . into accord with his own" (228/214) on the basis of an appeal to the two senses in which "[s]omething can be a 'principle' of something": when it "is *the* '*first which is such*' (primum quod sic) *as to be of the same nature* as the thing itself," and when it "is *the* '*last which is not*' (ultimum quod non) *such as to be of the same nature* as the thing itself." It is thus "[i]n the first sense [that] the unit *may* indeed be called the 'principle of number,' while the nought is a 'principle' *only* in the second sense."

The status of the nought as the principle of number allows Wallis—as the following passage from his *Mathesis universalis* cited by Klein makes clear—to assert the priority of arithmetic over geometry, "'since, in truth, the objects of arithmetic are of a higher and more abstract nature than those of geometry'" (229/215). This priority leads to the claim that "'Universal *algebra is in truth arithmetic*, *not geometry*, and must therefore be explained rather on arithmetic than geometric principles' (. . . p. 56)" (229/215–16). Wallis maintained, moreover, that "The ancients happened to have overlooked" (228/214) this priority insofar as they failed to realize "the fact that the analogy which exists [between the principles of arithmetic and geometry] is not between the 'point' and the 'unit,' but between the 'point' and the '*nought*.'" As a result, the ancients "were able to develop their algebra only for 'geometric magnitudes' (quantitates Geometricae – p. 53) and, worse yet, only for 'heterogeneous' geometric magnitudes: 'for lines, and planes compared with solids, and also other, imaginary quantities of still more dimensions'" (228/214–15). Wallis asks, "'But why did they resort to geometric (rather than arithmetic), and also to heterogeneous (rather than homogeneous) quanti-

ties?'" (228/215). And he answers, "I see no reason more likely than that they chose the 'one' (and not, as they ought to have, the 'nought') among arithmetical objects to equate with the geometric 'point.'" Klein notes that Wallis is "clearly following here" (228 n. 206/310 n. 336) "the opinion of Stevin," though, as we shall see, his working out of "the crucial point—to understand 'Algebra or Analytic' as an '*Universal Art*' and yet to confine it within the bounds of the realm of arithmetic" (228/215) takes him beyond both Stevin's number concept and his understanding of the "analytic art."

§ 120. Wallis's Emphasis on the Arithmetical Status of the Symbol or Species of the "General Analytic"

The priority of arithmetic over geometry for Wallis means that "'geometry is, as it were, subordinated to arithmetic'" (229/216), which makes it "'to that extent a special application of the *universal assertions of arithmetic*[181] to its [special] objects.'" With this, Klein maintains that the "twofold character" that belongs to the "object" of the "general analytic," that is, to the "species," as on the one hand a 'general magnitude' and on the other—"*at the same time*"—as something whose "determination is essentially 'arithmetic,'"[182] undergoes a shift away from Vieta's "emphasis . . . on the universality of the species." In Wallis, "the same twofold character is displayed by the algebraic magnitudes, the 'symbols or species' (symbola seu species)," with the important difference that "here their 'arithmetic' state is more perspicuous: they are unambiguously '*numeri*'—'numbers.'" Klein thus quotes Wallis, who is here "speaking of algebraic nomenclature . . . : 'Let it suffice at any rate to point out that the various algebraic powers, by whatever name they may be called, *are nothing else* than numbers *or* lines *or also* other *mutually homogeneous* quantities in continuous proportion.' (. . . p. 57)" (231/218). For Klein, this means that the "universality of the 'potestates algebricae' excludes them neither from having a 'numerical' character attributed to them nor from being interpreted 'arithmetically' or 'logistically.'" This is because, in Klein's judgment, Wallis does not see them "as mere (reference) signs but as *symbols*: in themselves they not only 'represent,' but they also 'are' mathematical objects."

Klein supports this judgment on the basis of Wallis's statement that "'Since all numbers {*properly so-called*} are constituted out of units, they are in fact homogeneous quantities' (cum . . . numeri omnes [proprie dicti] ex unitatibus constituantur [. . .] sunt vere Homogeneae [sc. quantitates] . . .)"

181. That is, 'arithmetic' understood as *logistice speciosa*, not *logistice numerosa*.
182. See § 114 above.

(230/217). Klein maintains that even though Wallis's argument here "apparently fails to do justice precisely to the universal character of 'algebraic numbers,' for the argument holds only if the algebraic 'numeri' are conceived also as 'numbers properly so-called' (numeri proprie dicti)," that is, as "'definite amounts of units,'" it is nevertheless actually the case that the "argumentation itself already presupposes a 'symbolic' understanding *even* of the ordinary 'definite amounts of units,' i.e., of the numeri proprie dicti." Klein maintains this on the basis both of an analysis of 1) the bracketed words in Wallis's text that he (Klein) omitted from the quote above and 2) a consideration of how Wallis deals with relations expressed in equations that are made between "algebraic powers" that are not of the same "degree."

Regarding (1), the words in question are "'vel saltem ad unitatem vere rationem habeant' (or at least are really in a ratio with the unit)" (230 n. 210/311 n. 340). According to Klein, "They invalidate the meaning of the main sentence and show directly the ambiguity which the expression 'numeri proprie dicti' has in Wallis." This is the case, because, as we will see below, when number is understood as something that "*indicates* a certain 'ratio'" (233/221), it "no longer means a 'definite amount of. . . .'"

Regarding (2), "the requirement of 'homogeneity,' which is that 'all comparisons [i.e., all relations expressed in equations]of quantity with a view to their equality must be made only among quantities of the same degree'" (229/216), and, which, as we have seen,[183] in Vieta is represented by the explicitly stated "'first and eternal law of equations or proportions'" (181/173), is, on Klein's view of the matter, "fulfilled, as it were, automatically" (229/217) for Wallis. Klein draws this conclusion because, for Wallis, "algebraic equations are in themselves homogeneous," as can be seen when he writes, "'it often happens that equations are made among various powers not of the same *height* [of degree]; *this, although acceptable for arithmetic degrees*, is by no means acceptable for geometric dimensions.'" For Wallis, then, "equations between powers of various 'height,'" of which he gives as examples "$2a^2 = 6a$ and $2a^3 = 6a^2$," are homogeneous, because "The 'arithmetic degrees' (Gradus Arithmetici) are nothing other than '*numbers* in continuous proportions' (numeri continue proportionales)." Indeed, for him, "'The universal art of 'algebra' or 'analytic' rests on this as on a foundation; and if these things which we have said about degrees, ascending or descending in whatever order, are well understood, a great light flashes forth for the understanding and for the handling of so-called 'algebraic powers'" (230 n. 208/310–11 n. 338). Thus, Klein writes that "The homogeneity among all 'ordinary' as well as algebraic 'numbers' is for Wallis, in ef-

183. See § 106 above.

fect, the result of their membership in one and the same 'genus,' namely the 'number' genus as such" (230/217). Whereas for Vieta "homogeneity" was "a requisite *condition* for operating with 'algebraic magnitudes'" (230–31/218), for Wallis it "characterizes their very composition: they are *essentially* homogeneous *and just for this reason* '*universal*.'" Specifically, "the *commonality of having the character of signs*" (230/217) that belongs to "all the members" of the "number" genus "immediately manifests" "the unity of this 'genus,'" such that "this and *only* this makes them appear as 'homogeneous'" (230–31/218).

§ 121. The Homogeneity of Algebraic Numbers as Rooted for Wallis in the Unity of the Sign Character of Their Symbols

Klein takes the "symbolic" character of these signs—manifest in the inseparability of their composition as signs from their status as mathematical objects, specifically from their status as "algebraic numbers"—to mean that, for Wallis, "All possible 'numbers' now belong to one and the same, dimensionless, 'genus'—*their homogeneity is identical with their symbolic character as such*" (231/218). And because of this, "The dimension, more exactly, the 'altitude' or height of an algebraic magnitude no longer changes its 'genus,'"[184] which means that "Wallis no longer needs special 'rung' (i.e., degree) designations for each single algebraic 'number.'" The "genus" of such unknown magnitudes no longer changes because "As symbols they are both '*general magnitudes*' and precisely also—'*numbers*.'"

Klein confirms Wallis's identification of the homogeneity of *numeri* with their status as symbols with a consideration of Wallis's views on the respective ancient and modern understandings of the 'unit' and the different understandings of 'number' that follow from this. Klein shows how the divisibility of the unit for Wallis leads to the view that "a 'ratio,' a 'relation,' underlies every 'number' as such" (232/220). Moreover, he shows that the "real reason for this interpretation of 'number'" (233/221) is rooted in the fact that 'ratio' for Wallis is "symbolically conceived" (235/223), which, in turn, means that for Wallis the "comparisons" constitutive of ratios belong to the "'numerical genus' (genus numerosum)" (234/222) and are therefore numbers.

Regarding the unit, Wallis was well aware of the fact that the ancients "'allowed almost no numbers other than integers; nor did they allow the material of arithmetic to be infinitely divisible like that of geometry, but they required [division]to stop at the unit' (... Chap. XXXV, p. 183)" (231/219). By contrast, "'When arithmetic wishes to imitate in some way the infinite di-

184. Which was the case for Vieta. See § 106 above.

visibility of geometry, it supposes a unit or a one which is something whole, as it were, but divisible into as many parts as you please.' (. . . Chap. XII, p. 60)." Thus, for the "'moderns' it [the 'unit'] is, *as something continuous* (ut quid continuum), divisible into as many (equal) parts as you please" (231/218–19). Consequently, in contrast to the ancients, who speak only of "'*true numbers properly so-called* (meaning *integers*) which are composed of units'" (231/219), "The moderns . . . are entitled, on the basis of the internal 'continuity' of the unit as well as of the whole numerical realm, to speak of fractional numbers (numeri fracti), of irrational numbers (numeri surdi), and also of 'algebraic numbers'" (232/219). Klein emphasizes, however, that because "numeri, and especially the *unit* itself, can, in turn, be characterized as continuous only by reason of a symbolic interpretation, or more exactly, only in symbolic abstraction" (232/220), "Wallis, who makes a continual effort to remain as true as possible to tradition and to retain the ancient terminology, has, on the one hand, great doubts whether fractions, for instance, can be understood as 'numeri.'" Thus, Wallis writes, "But 'fractions,' or 'broken numbers' *are not so much numbers as* 'fragments of the unit' (. . . Chap. XII, p. 60)," and, as such, "in between 'one' and 'nought.'" Yet he also writes, "But I add that we have now passed 'into another genus' . . . so that the 'broken number' is *not so much a number as an index of the ratio numbers have to one another* (. . . Chap. IV, p. 27)." In Klein's judgment, then, "this last remark itself shows that 'fractions' *are*, in fact, nothing but 'numbers,'" since what they "index," a "ratio," Wallis articulates as having as its terms precisely "numbers." Moreover, Wallis states explicitly that "*a 'ratio,' a 'relation,' underlies every 'number' as such*." He states this "In his discussion of the fifth book of Euclid's *Elements* (that is, of the 'general theory of proportions'), whose propositions Wallis undertakes to prove 'arithmetically,' i.e., 'algebraically.'" Klein quotes from this discussion at length (232–33/ 220–21):

> For this fifth book of the Elements is, like the whole theory of proportions, arithmetical rather than geometric. *And so also the whole of arithmetic itself seems, on closer inspection, to be nothing other than a theory of ratios*, and the numbers themselves nothing but the "indices" of all the possible ratios whose common consequent is 1, the unit. For when 1 or the unit is taken as the {identical} *reference quantum, all the rest of the numbers (be they whole, or broken or even irrational) are the "indices" or "exponents" of all the different ratios possible* in relation to the reference quantum. (. . . [Chap. XXXV,] p. 183).

Klein remarks, "Thus here 'number' no longer means a 'definite amount of . . .'" (233/221), since Wallis is quite clear that "a 'number' now *indicates* a certain 'ratio,' a λόγος in the sense of Euclid." As such, it "can be designated as a 'whole' or a 'broken' or an 'irrational' *number*—or, more briefly, as a 'rational' or 'irrational' *number*—only with reference to this ratio." However,

for Klein, despite Wallis's interpretation of the ratio indicated by number as a ratio in Euclid's sense, a consideration of Wallis's account of "'comparatio,' the relating of magnitudes to one another," discloses that the ratio at issue for Wallis is not Euclid's but "a symbolically conceived ratio" (235/223).

Klein holds that Wallis's deviation from Euclid is rooted in the distinction Wallis makes regarding "two possible ways of 'comparing'" the magnitudes at issue in Euclid's "requirement of homogeneity," which holds that "only magnitudes of 'the same kind' can be 'compared' with one another, can have a relation to one another." Thus, Klein finds that Wallis maintains that "We may either ask whether, or by what 'part,' or by 'how much,' one [magnitude] exceeds the other, i.e., what the '*difference*' between them is; or we may ask how many times one magnitude contains the other, or how many parts of one the other forms, i.e., what their '*relation*' (in the narrower sense of *ratio*), is (p. 134)" (232/221). The "'difference' (differentia)" in the first case "is found by 'subtraction' (subductio)," while the "'ratio'" in the second case "is always the result of a division of one magnitude by another" (233/222). Because the difference established by subtraction "is always *of the same kind* as the magnitudes compared," Klein maintains, for Wallis, "it follows that the *differences themselves* cannot be put in relation to one another without further 'comparisons' with one another" (233/221–22). By contrast, because "the 'ratio' is always the result of a division of one magnitude by another" (233/222), it is established directly, *without further comparisons*, as "it can be read off the 'quotient' which is the result of this division."

§ 122. Wallis's Understanding of Algebraic Numbers as Symbolically Conceived Ratios

Klein makes perspicuous the relation of the quotient to the ratio that is crucial for understanding Wallis's concept of symbolic number by unpacking what Wallis means by the "reading off" of the ratio from the quotient. The relation between "dividend and the divisor" yields two possibilities. Either they "are 'of the same kind' (homogeneous)" or they "are 'of a different kind' (heterogeneous)." In the former case, it follows that "they have the same amount of 'dimensions,'[185] so that the quotient indicates *how often or how*

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185. Klein maintains that "'Dimension' is here understood exactly as in Descartes (*Regulae*, X, 447–449), except that Descartes does not stress the dimensionlessness of the *numeri* themselves as emphatically as Wallis does" (233 n. 215/311 n. 345). (Descartes's account of 'dimension' in *Rule* X is, in part, as follows: "By 'dimension' we mean simply a mode or aspect in respect of which some subject is considered to be measurable. Thus, length, breadth, and depth are not the only dimensions of a body: weight too is a dimension. . . . The mode which gives rise to number is strictly speaking a species of dimension" [*Regulae*, 447].) Klein goes on to say,

many times one magnitude is contained in the other" (233–34/222). The "quotient itself" (234/222) therefore, is a number that "has *no* dimension," because "'where any species is divided by another of exactly the same dimensions, *a quantity of no dimension arises*' (. . . cf. p. 103)." In the latter case, where the dividend and divisor are of different kinds, it follows that they "do not have the same amount of dimensions, in which case division proper is not involved but an 'application' (applicatio),[186] although it is carried out *as if* the 'quantities were considered *as good as* numeri.'" The quantities involved in an *applicatio* "likewise produce quotients which are a dimensionless number [*numerus*]." Klein notes that for Wallis the realization of either possibility belonging to the relation of the dividend and divisor is such that "*every* '*quotient*' *of a division is . . . a* '*number*' (numerus)" (233/222), and, indeed, a number without dimensions.

In Klein's judgment, then, the "dimensionlessness" of such numbers for Wallis is thus "identical" (234/223) with their "'indicating' or 'exponential' role . . . as the '*indices* of ratios.'" Accordingly, Wallis concludes from this that "all 'quotients' (and this means *all* '*ratios*,' *all relations*, no matter of what *kind of magnitude*) are—as 'dimensionless' formations—'of the same kind' (homogeneous), such that they *can all be compared with one another*" (234/222). Klein bases his view of this matter on the following passages cited from Wallis (234/222–23):

> And hence it is clear that *all ratios of whatever quantities taken in turn are homogeneous among one another*.
>
> When a comparison in terms of ratio is made, the resultant ratio often {namely with the exception only of the 'numerical genus' itself} leaves the *genus of the quantities compared, and passes into the numerical genus*, whatever the genus of the quantities compared may have been.

And, Klein stresses, "this can only mean that *all numeri are homogeneous*" (234/223), and thus that "*Their homogeneity is identical with their dimensionlessness*."

For Klein "The *dimensionlessness* of numbers is therefore in fact identical with their *symbolic character*," because their role as indices of ratios "is immediately visible in the notation, the '*notatio*.'" Klein writes, "This is why

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"The reason for this is that for Descartes the 'figures' with which the *mathesis universalis* operates are exactly as symbolic as the 'numbers'" (233 n. 215/311 n. 345), whereas in "Wallis . . . as in Vieta, the traditional conception of geometric structures is preserved, as it is, incidentally, also later."

186. Klein notes that *applicatio* corresponds "to the Greek term παραβολή (parabola, that which falls alongside)" (234 n. 216/312 n. 346), and that "Vieta, who refuses to recognize the law of homogeneity as valid precisely in the case of division, also uses the 'adplicare' for 'dividing.'"

Wallis can say: 'And since indeed "double" and "half" and "triple" and "third," etc., are to be taken just as names of ratios, while the symbols of the half and the third, i.e., ½, ⅓, are reckoned among the numbers (namely those that are fractioned), *why not so reckon the symbols of the double and the triple*, *i.e.*, 2⁄1, 3⁄1 *or* 2, 3?'" (234–35/223). Not only is it clear from this passage that "the 'notation' (notatio) leads Wallis to call the 'one' or 'unit' a '*denominator*'" (235/223),[187] but "in this very passage he adds explicitly: '*And this* [i.e., the notation] *is the chief reason*[188] why I assert that the whole theory of ratios belongs more to arithmetical than to geometrical investigations.'" The priority that Wallis places on arithmetic over geometry is also evident in his understanding of the meaning of the word 'magnitude' in Euclid. Klein again quotes Wallis: "'and the whole fifth book of Euclid is arithmetic, however specifically, as if concerned with [geometric] magnitudes, propositions may be presented, propositions which could meanwhile be carried out just as correctly for any quantities desired in general, and this is the sense in which the word "magnitudes" must be understood in Euclid'" (235 n. 218/ 312 n. 348).

Klein thus concludes that it is "only in terms of a symbolic reinterpretation of the ancient 'definite amounts of definite objects,' of the ἀριθμός," that "the universality of arithmetic as a 'general theory of ratios,' which depends on the homogeneity of all 'numbers,' can be understood" (235/223). A major consequence of this reinterpretation is that "The object of arithmetic and logistic in their algebraic expansion is now determined as 'number,' and this means as a symbolically conceived ratio." Not only is this conception of number, as ratio, "consonant with that of algebra as a general theory of proportions and ratios," but also as the "material" of arithmetic and logistic, its "being no longer presents any problem," since, unlike the problematical mode of being of the ancient "material" (ὕλη), its mode of being "is immediately graspable in the notation" (235/224). That is to say, unlike the "pure" units that make up the "material" of ἀριθμός, whose mode of being can be subject to dispute because "they can be conceived formations that are either independent or obtained by 'abstraction' (ἀφαίρεσις)" (235/223–24), Klein finds that in Wallis's *Mathesis Universalis* the symbolic 'being' of "'numbers'" (235/224)—their character as dimensionless—is indisputable.

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187. Wallis's "symbolic" understanding of the 'one' or 'unit' thus realizes, in effect, Descartes's notion of the unit as a *res simplex communis* and therefore a *res simplex intellectualis*, a notion, as we have seen, that Klein maintains Descartes did not "pursue" (see § 114).

188. Klein's emphasis.

Part Four

Husserl and Klein on the Origination of the Logic of Symbolic Mathematics

Chapter Twenty-four

Husserl and Klein on the Fundamental Difference between Symbolic and Non-symbolic Numbers

§ 123. Klein's Critical Appropriation of Husserl's *Crisis* Seen within the Context of the Results of Klein's Investigation of the Origin of Algebra

Having completed our exposition of Klein's historical investigation and desedimentation of the origin of algebra, we are now in a position to take up and critically assess the questions raised in the conclusion to Part One of our study.[1] Klein's deviation from Husserl in his account of the actual development of symbolic mathematics, as well as the questions left open in its wake, can now be reconsidered within the context of Klein's *Greek Mathematical Thought and the Origin of Algebra*. This reconsideration is required inasmuch as this work is undoubtedly the basis for that deviation, despite the fact that Klein did not mention it in his 1940 article, "Phenomenology and the History of Science."[2] The context provided by the former work will allow us to assess critically the systematic relationship of Klein's thought to Husserl's regarding the origination of the logic of symbolic mathematics. And it will also allow us to explore the implications of Klein's thought for Husserl's unfinished phenomenological project of securing, in the transcendental constitution of the perceptual life-world, the ontological foundation of the logic operative in symbolic mathematics.

As we saw in Part One, Klein's articulation of his deviation from Husserl has two foci. Each is related to his overall assessment of the limits of Husserl's recognition of the role of symbolic formulae in the universal formalization of nature accomplished by mathematical physics. Husserl did not

1. See Part I, §§ 21–22.
2. See Part I, § 4.

recognize that this physics' anticipation of an exact nature involves an arithmetical sedimentation that is coincident with the origin of its employment of symbolic mathematics. The first of these foci concerns Husserl's failure to appreciate "the importance of Stevin's algebraic work" (*PHS*, 70) for "the Cartesian idea of a *mathesis universalis*." For Klein, his failure is evident in the fact that Husserl ascribes to Leibniz the origin of "the conception of a universal and symbolic science (*mathesis universalis*, *ars combinatoria*) which is prior to any 'material' mathematical discipline and any 'material' logic." The second focal point concerns Husserl's failure to recognize the importance of the "fusion of the traditional theory of ratios and proportions with the 'algebraic' *art* of equations" (80) for the anticipation of an exact nature and the mathematical formalization that is a necessary condition for bringing it about.

Klein's deviation from Husserl raises three questions whose answers remained open at the conclusion of Part One above. 1) What precisely does the method of symbolic abstraction entail, especially in connection with its difference from the ancient ἀφαίρεσις? 2) Why does Klein formulate the reactivation of this method, and the rediscovery of the "original arithmetical evidence" of Greek mathematics promised by its desedimentation, as a *task* in 1940 by Klein—without any mention of his *Greek Mathematical Thought and the Origin of Algebra*? 3) Does—and if so, how—what Klein characterizes as the "*third* task arising from the attempt to reactivate the 'sedimented history' of the 'exact' nature" (84), the task of "the rediscovery of the prescientific world and its true origins," differ from Husserl's formulation of this problem?

The stage for our reconsideration of Klein's deviation from Husserl and the questions it raises will be set by first analyzing what Klein neither analyzed nor commented upon, Husserl's investigation of the origin of symbolic numbers in *Philosophy of Arithmetic*. Husserl's failure to account for the logical origin of signitively symbolic numbers on the basis of authentic cardinal numbers will then be analyzed from the perspective of Klein's account of the origination of symbolic numbers in symbolic abstraction.

§ 124. Husserl and Klein on the Difference between Non-symbolic and Symbolic Numbers

We had occasion to comment provisionally, in Part Two of our study,[3] on the significance of Klein's statement that Husserl's logical researches "amount to in fact a reproduction and precise understanding of the 'formalization'

3. See Part II, § 27.

which took place in mathematics (and philosophy) ever since Vieta and Descartes" (70). We did so by stressing that Klein's qualification ("amount to") here suggests that Husserl, lacking a detailed knowledge of the *actual* history of this formalization, was not aware of this fact. Klein's research in *Greek Mathematical Thought and the Origin of Algebra*, of course, purports to provide just such a detailed account of the actual history, a history rooted neither in empirical events nor accidental contingencies, but in an aspect of the historicity of the aprioricity of the formal meaning structures[4] that comprises one, albeit an essential one, of the conditions that make modern mathematical physics possible.

Klein's account of this history will provide the context for, first of all, our consideration of his articulation of the conceptuality of the Greek ἀριθμός-concept in relation to Husserl's articulation of the authentic cardinal number concept.[5] Our consideration will substantiate our argument, ad-

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4. Regarding the historicity of a priori meaning structures, see Part I, § 20.

5. As we have seen, both Klein and Husserl use the same term to designate non-symbolic numbers, *Anzahlen*, although (as we have also seen) Klein maintains a strict terminological distinction between *Anzahl* and *Zahl* (see Part II, § 36, n. 8), whereas Husserl (see Part III, § 40, n. 2) does not. Because it is our claim that Husserl's and Klein's accounts of non-symbolic numbers correspond to one another without, however, being identical, we will refer to the number concept at issue here for Husserl as the "authentic cardinal number concept" and the one for Klein as the "Greek ἀριθμός-concept."

Talk of 'concept' in connection with either Husserl's or Klein's account of non-symbolic numbers is, of course, misleading. Both accounts identify and indeed stress that the most salient feature of such numbers is that they are *not* concepts in the sense of *abstracta* whose mode of being is independent from the items to which they refer qua their status as definite amounts *of something*. That is, both stress that the "content" of number concepts is *not* conceptual, but a multitude of items that are discrete and generically arbitrary. Nevertheless, when comparing Husserl's and Klein's characterizations of non-symbolic numbers, it is difficult to avoid formulating and referring to what is involved here as their respective "concepts" of such numbers. By this, it must be emphasized, nothing more is meant than the calling of attention to the similarities and differences that come to light as a consequence of comparisons of their respective accounts of *numbers*. Likewise, Klein's account of the ancient and the modern characterizations of non-symbolic and symbolic numbers, and Husserl's account of symbolic and non-symbolic numbers, encounter this same difficulty.

Moreover, talk about either thinker's account of non-symbolic number concepts must be sharply distinguished from their talk about the "*concept of*" non-symbolic or symbolic numbers. We have seen (Part III, §§ 61–62) that, for Klein, the "concept of number" does not refer directly to the composition characteristic of the specific numbers themselves as "number concepts;" for instance, among the latter it can be said—in the case of both the Greek ἀριθμός-concept and Husserl's authentic cardinal number concept—that neither zero nor fractions are included, while it cannot be said that zero or fractions either are, or are not, "concepts of number." On the contrary, with respect to the "concept of number" it is the *structure* common to the non-symbolic or symbolic numbers themselves that is at stake. For example, both Husserl and Klein characterize as determinate the *concept of* non-symbolic numbers. This means that common to non-symbolic numbers is their reference to a definite amount of definite items. It

umbrated in Part Three above,[6] that a correspondence obtains between these two concepts of number. Establishing this correspondence will provide the basis for substantiating the rest of the argument, namely, that Husserl's failure in *Philosophy of Arithmetic* to establish authentic cardinal numbers as the foundation of signitively symbolic numbers mirrors Klein's account of the fundamental difference between the Greek ἀριθμός-concept and the symbolic number concept that came into existence with the origin of algebra. Finally, notwithstanding this correspondence, a decisive difference between what Klein desediments as the Greek concept of ἀριθμός and what Husserl in *Philosophy of Arithmetic* and later writings characterizes as the concept of cardinal number will emerge. This difference concerns the all-important logical issue of the relation between non-symbolic numbers and formal meaning per se. Its thematization and analysis will provide the point of departure for our assessment of Husserl's and Klein's accounts of the origination of the logic of symbolic mathematics.

Husserl's account of authentic cardinal numbers, like Klein's account of the Greek ἀριθμοί, rests on an acute awareness of what is taken to be a *given*, namely, that there is a fundamental difference between symbolic and non-symbolic number concepts. Both thinkers single out the same two crucial factors in this difference. The first is that non-symbolic numbers refer directly to items that are discrete, multitudinous (i.e., more than one), and precisely so many. As such, non-symbolic numbers manifestly are *not abstracta*, that is, conceptual entities that are capable of existing apart from the items to which they refer. On the contrary, the very status of each authentic cardinal number *as* an authentic cardinal number for Husserl and each ἀριθμός *as* an ἀριθμός for Klein is inseparable from the items to which they refer, items that are determined with respect to their amount by the number in question. The second factor is that symbolic numbers *are* capable of existing as *abstracta*, as conceptual entities that are separable from, which is to say, *independent* of, the

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does not mean, however, that the *concept* of being determinate is inseparable from the composition of these numbers "themselves." Again, we will refer to Husserl's and Klein's respective accounts of non-symbolic numbers as their accounts of "non-symbolic number concepts," in recognition of the fact that even though they both concern what is ostensibly the same thing (namely, non-symbolic numbers), their accounts, when compared, are nevertheless not identical. Finally, we should mention that Klein's distinction between ancient and modern number concepts emerges from his comparison of the different modes of being that he presents as characterizing what for each tradition composes the true being of numbers themselves. Hence, it is this difference that is referred to when their respective "number concepts" are mentioned, and not necessarily the view that for either (or both) of these traditions the numbers in question *are* themselves concepts.

6. See Part II, § 38.

items to which non-symbolic numbers necessarily refer for both Husserl and Klein.

Likewise for these two thinkers, the insight into this difference, which concerns both the ontological and the logical status of symbolic and non-symbolic numbers, occasions the investigation of the relationship between these two different kinds of number concepts. In Husserl's case, the investigation of this relationship presupposes that notwithstanding the fundamental difference between authentic and symbolic number concepts, they are nevertheless *logically* equivalent insofar as each has *the same object*[7] as its referent—albeit one has it directly (the authentic) and the other indirectly (the symbolic). In Klein's case, the investigation of this relationship is guided by the opposite assumption, that the two kinds of number concepts are *not* logically equivalent insofar as one (the ἀριθμός) has as its direct referent the definite amount of a multitude of definite items, while the other (the symbolic) has no referent—direct or indirect—*at all in this sense*. Rather than refer to anything, the symbolic number is itself the direct referent of a perception that is inseparable from the calculational rules that assign to it a numerical status.[8] Thus, for Klein, symbolic numbers are, properly speaking, *not* "concepts" at all but signs whose numerical status is dependent upon (and therefore inseparable from) a set of canonically established syntactical rules, rules that are based in turn on an axiom system that determines the "object" to which they *apply*.

Husserl's attempt to demonstrate the logical equivalence[9] of authentic and symbolic number concepts by clarifying their presentations in psychological experience gradually led him to recognize that symbolic number concepts, or, more precisely, signitively symbolic numbers (which, properly speaking for Husserl, are not *concepts* at all), are logically equivalent to neither authentic nor ideally inauthentic numbers.[10] In the course of Husserl's analyses, signitively symbolic numbers proved to have their basis not in concepts that, properly understood, have a cognitive relationship to individual objects but rather in precisely the demonstrable lack of this type of cognitive relationship. These numbers are based in the manifestation of sensible signs whose numerical meaning is established by the conventionally established rules for their calculation,[11] not by their direct or indirect relationship to multitudes of

7. See Part III, § 47.

8. See Part III, §§ 103, 106, 112, and 121.

9. Concerning Husserl's understanding of the logical equivalence of concepts, see Part III, § 44.

10. Concerning Husserl's distinction between signitively symbolic, inauthentic, and authentic number concepts, see Part III, §§ 46–48.

11. See Part III, § 50.

units. These rules, in turn, acquire for Husserl their numerical warrant on the basis of their establishment as "signitive surrogates" for the systematic number *concepts* and systematically *conceptual* number operations that form the basis of arithmetical *knowledge*.[12] Indeed, establishing the rigorous parallelism between the rules that secure the numerical meaning of the former and the concepts and conceptual operations of the latter is precisely what, at the end of *Philosophy of Arithmetic*, Husserl attributes to the accomplishment of universal arithmetic. It is an accomplishment, moreover, that guarantees that calculation with signitively symbolic numbers will yield results that are logically equivalent to operations on ideally symbolic but nevertheless *conceptual* numbers. And, therefore, it is an accomplishment that establishes "the rules of the game" governing signitively symbolic calculation as a surrogate for systematic arithmetic's conceptual operations and the *knowledge* it produces.

§ 125. Husserl on the Authentic Cardinal Number Concept and Klein on the Greek 'Αριθμός-Concept

The main reason that Husserl's investigation of the origin of symbolic numbers reaches the conclusion that they are not logically equivalent to authentic cardinal numbers is shared by Klein's point of departure for the investigation of modern symbolic numbers in relation to the Greek ἀριθμοί. For both Husserl and Klein, non-symbolic numbers specify a definite amount of items belonging to a discrete multitude, items that are theoretically formulated as a multiplicity of units. Due to this agreement, they both identify the number 'two' as the first non-symbolic number.[13] They both agree also that symbolic numbers manifestly do not specify definite amounts of definite items or units. The similarity of their accounts does not end here, however. Their accounts also agree that non-symbolic numbers are initially accessed through the activity of enumeration or counting. And they both contend that counting, in turn, presupposes the availability of an indeterminate multitude of items and an interest in answering, with respect to this multitude, the question 'How many?' Hence, for Husserl and Klein, accounting for non-symbolic numbers is closely connected to accounting for what it is that makes a multitude available for enumeration.[14] And, in what is perhaps the most important correlation between Husserl's account of au-

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12. See Part III, § 49.

13. Concerning Husserl's position on this point, see Part III, § 41; concerning Klein's, see § 61.

14. For Husserl's assessment of this issue, see Part III, § 45; for Klein's, see §§ 61, 84, and 87.

thentic cardinal numbers and Klein's account of ἀριθμοί, they agree that the unity of the whole proper to the numbers in question manifestly *does not have its basis in their parts*.

This last point deserves closer scrutiny. Both Husserl and Klein consider the parts of non-symbolic numbers, the items or units of a multitude determined with respect to its definite amount, *not* to be parts of that which composes the unity of this amount. For Husserl, this is the case because, either in themselves or in terms of their relations, these parts are incapable of providing any basis sufficient to account for the collective unity into which they are combined as a definite authentic cardinal number.[15] For Klein, it is the case because what can be truly said of each of the parts proper to an ἀριθμός is something that *cannot* be truly said of the unity of the whole that composes it and vice versa. Specifically, what can be truly said of the parts is that each is one, while what can be truly said of the ἀριθμός as a whole is that it is precisely not one. Furthermore, what can be truly said of all the parts taken together, that they are just this ἀριθμός, cannot be said of each part.[16] For example, in the ἀριθμός 'two', what is true of each item or unit that is a part of this ἀριθμός is that it is 'one', which is something that is not true of the ἀριθμός itself, which is precisely not 'one' but 'two'.

Husserl's and Klein's recognition of this non-identity of the parts and the whole proper to non-symbolic numbers yields further important points of agreement in their accounts of such numbers. One is that, strictly speaking, there is no authentic cardinal number or ἀριθμός "in general." That is because each thinker regards the commonality at issue in non-symbolic numbers as precisely *not* a characteristic that pertains to each of them *as a different number*. Each number is a whole whose unity determines a *definite* amount of parts and thus determines them such that the whole of each number determines a *different* definite amount of parts. For instance, the non-symbolic numbers 'two' and 'three'—*qua* their delimiting just *two* and just *three* of something—have no commonality because, considered precisely as *amounts*, what is *numerically decisive* is their *difference* as amounts.[17] For Husserl as for Klein, then, it makes no sense (for non-symbolic numbers) to talk about a 'number concept in general' since no such thing exists.[18] What

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15. See Part III, §§ 41–42.

16. See Part III, § 70.

17. Consequently, to speak of the difference between each number as a difference in the "property" of the multitudes characteristic of each (different) number is misleading, for it suggests what is precisely *not* the case, namely, that the *differentia* between numbers is the result of their sharing the *common* quality of a property.

18. For Husserl on this issue, see Part III, §§ 41 and 49; for Klein, see § 61.

is common to non-symbolic numbers *as numbers* is therefore something that is not, properly speaking, *numerical*. For Husserl, the commonality in the authentic meaning of the 'generic concept of cardinal number' concerns the fact that each cardinal number is a *species* of the concept of multiplicity.[19] However, each species is *different* because a different amount of *content* falls under each one, for example, 'two items or units', 'three items or units', and so on—and not a different amount of the *concept* of multiplicity. For Klein, it is the same: each ἀριθμός presents a delimitation of the *contents* of a multiplicity that is different, and each different delimitation, as precisely "so many" items or units, is manifestly not directed to the *concept* of multiplicity.[20]

19. See Part III, §§ 41 and 45.
20. See Part III, §§ 61–62.

Chapter Twenty-five

Husserl and Klein on the Origin and Structure of Non-symbolic Numbers

§ 126. Husserl's Appeal to Acts of Collective Combination to Account for the Unity of the Whole of Each Authentic Cardinal Number

Notwithstanding the agreement on these points between Husserl and Klein regarding non-symbolic number concepts, important differences emerge when their respective accounts of the origin of these concepts are considered. These differences are not confined to the origin of non-symbolic numbers, but (as we shall see) also concern the very structure of non-symbolic numbers. Nevertheless, even here there is an important point of coincidence insofar as the question of origin for each thinker stems from the recognition of the same problematic. Husserl's philosophical motivation for turning (in *Philosophy of Arithmetic*) to psychology to assist in a matter of *conceptual* clarification is rooted in his recognition that the peculiar collective unity of the whole of each authentic cardinal number *cannot* be accounted for by any relation among or within the items or units that are inseparable from it.[21] In other words, the whole that each authentic cardinal number presents is a whole of parts that presents a unity that is manifestly irreducible to the unity belonging to the parts of which it is the whole. It is precisely in the service of attempting to account for this *numerical* unity that Husserl—not only in *Philosophy of Arithmetic* but, as we shall see, in all his subsequent analyses—turned to the cognitive *act* in which the unity at issue is presented so as to clarify what conceptual analysis per se proves to be incapable of clarifying.

Klein's account—or, more precisely, Klein's account of the Greek philosophical accounts of the mode of being of ἀριθμοί-concepts—likewise is rooted in the recognition of the problem that the peculiar whole mani-

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21. See Part III, § 45.

fested by numerical unity presents for conceptual analysis.[22] However, on his telling, neither of the two paradigmatic Greek analyses of this problem, Plato's and Aristotle's, attempts to resolve it in a manner even remotely similar to Husserl's attempt, though, as we shall see, elements of each of these are sedimented in Husserl's analyses.

Husserl's initial[23] approach to the peculiar unity of the numerical whole in question appealed to acts of "collective combination," which he claimed are locatable in the experiences that generate both authentic multitudes and authentic cardinal numbers.[24] He sought to account for the origin of the unity proper to the wholes of each by appealing to what he interpreted as the *psychological* (inner) experience of "grasping-as-one" either the indeterminate assemblage of items (or units) in a multitude (or group) or their delimitation in a determinate aggregate, namely, in an authentic cardinal number. The unity of a multiplicity or cardinal number was supposed to arise in the *reflexion*[25] on the experiences of collective combination, a reflexion that grasps the respective collections at issue *as* collections and thereby yields what the conceptual analysis of the unity of the assemblages cannot: the peculiar whole responsible for the unity that exceeds the relationship and composition of the parts belonging to the collection.

§ 127. Husserl's Appeal to Psychological Experience to Account for the Origin of the Categorial Unity Belonging to the Concept of 'Anything' Characteristic of the Units Proper to Multiplicities and Cardinal Numbers

Husserl also appealed to psychological experience to account for the origin of the categorial unity proper to the whole of the *concept* of multiplicity and thereby to the unity of the delimitation of this concept that the species of cardinal numbers manifest. For Husserl, the unity of this concept, together with the unity of each of the concepts by which it is delimited, is such that there is no restriction with respect to the genus of the content it takes in, so long as each constituent is something, a certain one. Consequently, both the concept of multiplicity and the concept of authentic cardinal number belong

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22. See Part III, §§ 69, 88, and 90.

23. That is, Husserl's approach in *Philosophy of Arithmetic*, which unless otherwise noted, is the source of the views being attributed to him here.

24. See Part III, § 45.

25. Dallas Willard correctly notes in his "Translator's Introduction" to *Philosophy of Arithmetic* that for Husserl "'Reflexion' with an 'x' indicates reflexivity, self-reference" (lxi), and should therefore be distinguished from 'reflection', which indicates "a process of consideration, meditation, thoughtfulness."

to the categories that are most universal and emptiest of content insofar as each also contains the concept of 'anything' (*Etwas*).[26] The psychological experience wherein the concept of multiplicity originates is rooted in an abstractive process based on individual contents as they are given in collective combination. The items manifest in the latter, even the most heterogeneous, are passed over in the abstraction in which the most universal and emptiest concept of multiplicity originates. The slightest attention to them as contents with any kind of determination is therefore lacking in this abstraction, such that the main interest is focused on their collective combination. Each individual content is therefore "considered and attended to only as some content or other, each one as a *certain something*, a *certain one*" (*PA*, 79).

Regarding the psychological origin proper to the concept of 'anything', which Husserl explicitly states "the concept of the multiplicity also contains" (80), he writes that "Obviously the concept *anything* owes its origination to reflexion upon the psychical act of presenting, for which precisely any determinate object may be given as the content." This means that this concept "itself must be designated as a relative determination" because it "belongs to the content of any concrete object only in that external and non-literal fashion common to any sort of relative or negative attribute." However, despite its status as a relative determination, Husserl contends that the "reflexion mentioned" is always "carried out" when "some sort of content is present" and thus that this concept of 'anything' plays a "crucial role . . . in the origination of the general concept of multiplicity." It does so insofar as "each particular one from among the determinate contents encompassed by the concrete presentation of a multiplicity is thought under the mediation of the concept of *anything*, and is attended to only insofar as it falls under that concept" (80–81). It is in this way, and this way alone, then, that, according to Husserl's account in *Philosophy of Arithmetic*, "there comes about that utter depletion of content which confers upon the concept of multiplicity its universality" (81).

Husserl observes that the collective combination is indicated in language by the conjunction 'and' and maintains that when this observation is combined with the result of the abstraction process that originates the concept of multiplicity, the content of the concept of multiplicity in general is expressed. This concept means nothing other than "a certain something and a certain something and a certain something, etc.; or, some one and some one and some one thing, etc.; or, more briefly, *one and one and one*, etc." (80). Authentic cardinal numbers originate when the indeterminateness of what

26. See Part III, § 41.

is expressed by the concept of multiplicity is done away with, in enumeration or counting, such that 'one and one' are delimited, 'one, one, and one', and so on.

§ 128. The Decisive Contrast between Husserl's and Klein's Accounts of the Being of the Units in Non-symbolic Numbers

Before considering Husserl's own critique of this account of the *origin* of the concept proper to multiplicity and its delimitation as the authentic cardinal numbers,[27] we shall first consider Klein's articulation of Plato's and Aristotle's contrasting accounts of the ἀριθμός-concept and its origin. This is necessary because, on our view, these accounts, when compared to Husserl's account of the concept of cardinal number and its origin, disclose an important difference in Husserl's and Klein's respective articulation of the structure of non-symbolic numbers: a difference, moreover, that is common to Plato and Aristotle despite their otherwise quite different accounts of the origin of ἀριθμοί. Because (as we shall see) Husserl's accounts, subsequent to *Philosophy of Arithmetic*, of the origin of the concept of multiplicity and that of authentic cardinal number do *not* modify his view of their structures but only of the "constitution" of their origins, the results of this comparison will assume a systematic significance for our study.

According to Klein, Aristotle's dispute with Plato over mathematical objects was not about their "being" but their "mode of being."[28] In the case of arithmetic, this means that they were in agreement that arithmetically mathematical inquiry has as its object a discrete field of "pure," which is to say *noetic*, monads, each of which is "one" and therefore all of which are identical. Moreover, it means that they were in agreement that each ἀριθμός delimits a definite amount of such monads, that the amount delimited by each is different, and that therefore each ἀριθμός is demarcated from another. Finally, they agreed that 'one' is the ἀρχή of ἀριθμός.

Prior to our consideration of Klein's articulation of Plato's and Aristotle's different accounts of the mode of being of ἀριθμοί, both in relation to each other and to Husserl's account of the origin of non-symbolic cardinal numbers, it is necessary to point out a decisive contrast between the structure of the ἀριθμός-concept in Plato and Aristotle and the authentic cardinal number concept in Husserl. While both exhibit identical structures insofar as each ἀριθμός and each authentic cardinal number is understood

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27. See § 158 below.
28. See Part III, § 84.

to delimit a definite amount of definite units, the structure of these definite units differs significantly in Husserl's and Klein's accounts. In Klein's exposition of the Greek account, the structure of what the definite amount delimits is *ontologically* determined as a multitude. That is, *the noetic monads that are the units of an ἀριθμός are themselves multitudinous*. As such, they are *not* "concepts" but beings—noetic beings, to be precise. The non-conceptual status of the monads in a multitude is evident from the fact that they are not characterized as εἴδη, which is how they would have to be characterized to exhibit a conceptual structure within the context of Plato's and Aristotle's accounts of ἐπιστήμη.[29] Moreover, neither do they "fall under" any concepts (including, of course, the concept of multiplicity) nor are they determined by concepts that are not identical to them but which nevertheless "express" them. Indeed, the controversy between Aristotle and Plato over *noetic* monads concerns neither their discreteness nor their multitudinousness as *beings*, but rather whether their mode of being multiple or discrete is dependent upon, or independent of, the multiplicity and discreteness of *somatic* beings.

For Husserl the units of an authentic cardinal number are manifestly *not* beings in the sense that they are for Plato and Aristotle; rather each is an extension of the universal and materially empty *concept* of anything, or of one for short. Each is an extension of this concept in the precise sense that as a relative determination of anything at all that is presented in experience, the units that belong to each multitude delimited in counting by authentic cardinal numbers do not have the *ontological* status that Klein finds attributed to the monads of an ἀριθμός. The units at issue in Husserl are not already available to the mind prior to the activity that yields both a multitude and its delimitation; they are not *available* prior to the inception of collective combination but become so only subsequent to it. This contrasts sharply with Klein's account of Plato's and Aristotle's *noetic* monads, since these are available to the mind prior to its initiation of the process that yields ἀριθμοί, that is, prior to counting.

The significance of the ontological difference noted here between the characterization of the units belonging to non-symbolic numbers in Husserl and Klein will be pursued in detail below when their respective accounts of the origin of such units is compared in view of the problem of the origination of the formalization that makes possible the logic of symbolic mathematics.

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29. See Part III, § 63.

§ 129. Klein on Plato's Account of the Purity of Mathematical Ἀριθμοί

Plato's recognition of two factors led him[30] to posit as "pure" the mode of being belonging to the mathematical ἀριθμοί that are the objects of both the *praxis* of counting and theoretical arithmetic. One factor is the supposition of the unchanging and exact nature of arithmetical knowledge (ἐπιστήμη), while the other is the availability of ἀριθμοί to the soul *before* it begins counting sensible beings (αἰσθητά), the definite amounts of which are delimited in each ἀριθμός. This factor is what is responsible for the *dianoetic* supposition (ὑπόθεσις) that the true referents of counting and arithmetical knowledge are not the *monads* or units proper to sensible beings, which, as unequal and divisible (thus, subject to change), are therefore incompatible with knowledge. Rather, the true referents are the identical and indivisible intelligible (νοητόν) units that thought (διάνοια) supposes to underlie its counting of sensible beings. These units are intelligible because their identity, along with their indivisible and unchanging nature, is recognizable only by *thought*—not by the senses.[31] The second factor that led Plato to posit the "pure" mode of being of mathematical ἀριθμοί is what is responsible for his so-called *chorismos* thesis. The availability of ἀριθμοί to the soul before its διάνοια counts αἰσθητά points to the ontological independence of the ἀριθμοί whose units are intelligible from the ἀριθμοί whose units are sensible. More precisely, this availability points to the intelligible units' separateness from sensible beings that must be supposed in order to account for the unchanging and exact status of arithmetical knowledge.[32]

Thus the availability of ἀριθμοί to the soul *prior* to its having rendered definite the amount of this or that multitude of sensible beings, combined with the supposition of the exact and unchanging nature of knowledge, leads to the Platonic thesis that the mode of being of the pure ἀριθμοί investigated by arithmetically mathematical knowledge is *separate* from sensible beings.

§ 130. Klein on Plato's Account of the "Being One" of Each Mathematical Ἀριθμός as Different from the "Being One" of Mathematical Monads

The peculiar state of affairs indicated by each ἀριθμός, whereby it permits a

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30. Unless otherwise noted, the following summary and analyses of Plato and Aristotle represent Klein's interpretation of both.

31. See Part III, § 62.

32. See Part III, § 66.

multitude of either sensible or intelligible beings to be grasped as *one* definite amount (e.g., *as* 'two', 'three', and so on), gives rise to another Platonic supposition: the supposition that the peculiar "one over many" unity yielded by each ἀριθμός has its basis in a common thing (κοινόν) whose unity is rooted, in turn, in an ontological kind (γένος) that is *fundamentally* different from the mathematical kind that is responsible for the unity of each one of the multitudinous units unified by each ἀριθμός.[33] That is, it gives rise to the supposition that the unity proper to each 'two', each 'three', and so on, is something that has its basis in a unity that is different in kind from the unity of the monads that mathematicians suppose to be unified in each ἀριθμός in order to account for counting and arithmetical knowledge. That is because the kind of unity belonging to the units that compose either the sensible beings counted in *praxis* or the pure beings investigated by theoretical arithmetic is incapable of accounting for the difference in kind that characterizes the unities belonging to each different ἀριθμός. For instance, the kind of unity belonging to the *being one* of 'two', 'three', 'four', and so on, is *different* in the case of each ἀριθμός and thus cannot be accounted for by the kind of unity characteristic of the "being one" of each monad, which is the *same* for each monad. More explicitly, because each ἀριθμός *is* precisely the delimitation of a *different* amount of multitudinous items, the *being one* of the "one over many" unity belonging to each ἀριθμός likewise *has to be* different for each different number. According to Plato's reasoning, this is something that the "being one" of the units in a multitude is not able to account for, because the being one of each unit in a multitude is identical.

Hence the "being one" of the units (and therefore their kind of unity) that compose a multitude is unable to account for the "being one" that belongs to the unity of each different ἀριθμός. For the unity proper to the beings in a multitude does not exclude its opposite. So the unity of a multitude is unlike the arithmetical unity that comprises the integral whole of each definite amount of units. The "being one" of the latter whole, that is, the whole of an ἀριθμός, composes the exactness proper to its delimitation of the units in a multitude, an exactness that excludes its opposite—the "unlimited"—from its unity. On the contrary, the unity of each being in a multitude is intelligible only as a unit among other units. That is to say, as a result of its being a part of a multitude, each unit is intelligible only insofar as it is thought of as being inseparable from the other units that compose the multitude. The unity of the unit's intelligibility therefore includes (in precisely this sense) its opposite, the "unlimited." Whether the beings that compose a multitude are unequal and di-

33. See Part III, § 75.

visible (as in a sensible multitude) or exact and indivisible (as in an intelligible multitude), the *being* "one" of each such being is a being one among *other* ones. As such, the "unity" of each one is homogeneous with each of the unlimitedly many other members of the multitude to which each, as a unit in a multitude, necessarily belongs. Thus, the "being one" of each sensible being is homogeneous with the other ones insofar as they are all alike in being unequal and divisible (and therefore changing). Likewise, the being one of each intelligible being is homogeneous to the other ones, insofar as they are all identical and indivisible (and therefore unchanging). In either case, the kind of "unity" that belongs to the "being one" of each monad *must* be different from that of the kind of "being one" of each ἀριθμός because the "being one" of each ἀριθμός—or, more precisely, of each *different* ἀριθμός—is *not* homogeneous with and therefore is different from every other.[34]

§ 131. Klein on Plato's Account of the Non-mathematical Unity Responsible for the "Being One" of the Whole Belonging to Each Ἀριθμός

Precisely the recognition of the last-mentioned point, the heterogeneity of 1) the "being one" of the units belonging to the multitude delimited by each different ἀριθμός and 2) the "being one" of each ἀριθμός as the whole that brings about this delimitation, is what occasions the Platonic supposition that another kind of unity (one other than that which characterizes the units in a multitude) is responsible for the "being one" proper to the whole of each ἀριθμός. This other kind of unity is therefore supposed to be different from what determines the unity belonging to the beings in the multitudes dealt with either by the *praxis* of counting or arithmetical ἐπιστήμη. To account for the unity presupposed by the "being one" of the whole composing each (different) ἀριθμός,[35] the following is supposed: that the unity of the arithmetical "being one" composing each of these different wholes is provided by a *different* γένος. As a function of the difference between each of the γένη that provides the basis of the integrity belonging to each (different) ἀριθμός, the character of each γένος as a κοινόν (common thing) is, paradoxically, not common to any other γένος; that is, it is not common among

34. See Part III, §§ 68 and 75.

35. The "being one" of this unity, we want to stress, is something that neither the sensible nor the intelligible ones that belong to the multitudes unified by the integrity of the whole of each number can account for, because the unity of each ἀριθμός, as a *different* ἀριθμός, must needs be different, whereas the unity of each unit in a multitude of units must be homogeneous.

any of the other γένη that give integrity to the different ἀριθμοί. The non-identity or incomparability of the γένη in question, which must be supposed in order to account for what need *not* be supposed because it is manifestly the case—namely, that there *are* different ἀριθμοί and therefore *differences* among the "being one" composing each different ἀριθμός—thus points to intelligible structures whose "being one" unifies each different delimitation of a multitude *differently*. Furthermore, it points to a resultant unity that is nevertheless fundamentally different from the unity of the beings that compose a mathematical multitude.[36]

Insofar as the intelligible structures pointed to in the manner under consideration are responsible for the unification of the integral wholes that are responsible, in turn, for differently delimited multitudes (ἀριθμοί), they are posited as having something in common with the structure of such integral wholes, that is, with the integral wholes that compose ἀριθμοί. Specifically, they share the latter's "one over many" mode of being. However, because precisely the unity belonging to this "one over many" mode of being remains unaccounted for by the kind of unity that mathematicians suppose accounts for the "being one" of the units that compose the multitudes that each ἀριθμός delimits, the (one over many) *kind* of unity under consideration here points to a unity that is *different* in kind from that which belongs to ἀριθμοί; more exactly, it points to a unity that is different from the unity that characterizes *mathematical* ἀριθμοί.[37] Mathematical ἀριθμοί delimit units that are homogeneous in relation to one another and that maintain their homogeneity in relation to the integrity of the whole composing each different ἀριθμός. The composition of these ones, together with their relationship to the wholes (ἀριθμοί) that delimit them, contrasts sharply with the delimitation of the integrity of the different wholes themselves that belong to each ἀριθμός. Each ἀριθμός, as a *different* ἀριθμός (i.e., 'two', 'three', etc.), points to a delimitation of units—namely, to each ἀριθμός itself considered as a unity—that must be heterogeneous from the integrity of the wholes proper to all the other ἀριθμοί that differ from it. Each ἀριθμός must be heterogeneous, for inseparable from the delimitation of one mathematical ἀριθμός from another is its differentiation as the integral whole responsible for the "being one" proper to each different ἀριθμός. Moreover, the fact that there are arbitrarily *many* such integral wholes[38] (i.e., many 'twos', many 'threes', and so on) in the realm of sensible and intelligible units means that the mathematical ἀριθμοί that delimit these

36. See Part III, § 75.
37. See Part III, § 76.
38. Ibid.

units are, like these units themselves, also inseparable from *their* opposite: in being many, the mathematical ἀριθμοί that *delimit* the sensible and intelligible units supposed by mathematicians therefore are also *unlimited*.

§ 132. Klein on Plato's Account of the Solution Provided by Ἀριθμοὶ Εἰδητικοί to the Aporias Raised by Ἀριθμοὶ Μαθηματικοί

Plato's solution to the aporias that are generated by the inseparability from their opposite (from the unlimited) of both kinds of mathematical unity—the unity of the units comprising multitudes and the unity of the ἀριθμοί that delimit them—is provided by the supposition of ἀριθμοὶ εἰδητικοί. By positing an *ontological*—and therefore higher—mode of being as belonging to the νοητόν (intelligible being) than the mathematical one, the ἀριθμοὶ εἰδητικοί address the paradoxical "mixing" of the one and the many (and therefore, the mixing of opposites) that characterizes the mathematical νοητόν. Because their mode of being is ontological, the ἀριθμοὶ εἰδητικοί provide the foundation for the mathematical mode of being of the νοητόν. Insofar as they are ontological, their mode of being is "higher": the ἀριθμοὶ εἰδητικοί render intelligible the mathematical mode of being of ἀριθμοί, which—being aporetic—would otherwise remain incomprehensible. The γένος of each ἀριθμὸς εἰδητικός is posited as the condition responsible for the unity of the κοινόν that is, in turn, responsible for the mathematical unity of each different ἀριθμὸς μαθηματικός. Being responsible for the unity belonging to the latter's integrity, the unity of the different γένη of ἀριθμοὶ εἰδητικοί must reside in a commonality that is inseparable from the structural elements of the ἀριθμοὶ μαθηματικοί. This, of course, contrasts with the "being one" of the monad's mathematical unity, which is completely independent of the "being one" proper to the mathematical structure of the ἀριθμός and therefore has nothing in common with it.[39] The ontological unity, which is inseparable from the commonality definitive of the κοινόν that provides the foundation for the aporetic mode of being that characterizes mathematical unity, must be unique to the ἀριθμὸς εἰδητικός in question; it cannot exist beside or outside its structural parts. It cannot do so because otherwise the unity in question would be common to other ἀριθμοὶ εἰδητικοί and therefore common to other ἀριθμοὶ μαθηματικοί inasmuch as it is precisely the supposed function of the ontological mode of being of the former ἀριθμοί to

39. The unity of the monad is not numerical because each is precisely one and not many, while the unity of the mathematical ἀριθμός is *both* one and many.

provide the foundation for the mathematical mode of being of the latter. A common unity, of course, is precisely what *cannot be* if something like ἀριθμοὶ μαθηματικοί are to be at all.[40] To insure the intelligibility of ἀριθμοὶ μαθηματικοί, then, the parts of ἀριθμοὶ εἰδητικοί, namely their εἴδη, must—in explicit contrast to the parts of ἀριθμοὶ μαθηματικοί—be incomparable.[41]

The mode of being belonging to the unity of each of the ἀριθμοὶ εἰδητικοί, which must be supposed as incomparable in order to account for the "being one" of each of the different ἀριθμοὶ μαθηματικοί, must also be supposed as what is responsible for the sequence (two, three, four, and so on) "natural" to the latter ἀριθμοί. That is because the supposition of the homogeneity of the multitude of mathematical monads cannot account for the sequence that their arithmetical delimitation unquestionably yields. It cannot do so for the simple reason that nothing is ordered within the identity of what is homogeneous. The order (τάξις) of ἀριθμοὶ μαθηματικοί, then, must be supposed to originate in the order proper to ἀριθμοὶ εἰδητικοί. It must be supposed to originate in the γένος of each one of them, the κοινόν of which, because it does not exist beside or outside the elements it unifies, determines its relation to its neighboring γένη in terms of prior and posterior. And, again, it must be supposed to do this because the natural sequence of ἀριθμοὶ μαθηματικοί would otherwise remain unintelligible.[42]

Finally, at the head of the τάξις of ἀριθμοὶ εἰδητικοί is the "One itself," whose absolute priority—as complete, whole—must be supposed in order to render intelligible the unity inseparable from the "being one" of both mathematical and eidetic ἀριθμοί. What must be supposed is a one (ἕν) that is unlike the ones supposed by mathematics, that is, unlike the ones whose unity is delimited by its opposite, namely, the multitude from which each mathematical one is inseparable. Only the supposition of a ἕν whose unity needs nothing other than itself, and therefore needs nothing other, no opposition, in order to be delimited as what it is, can make intelligible something like the "being one" of wholes that delimit mathematical and eidetic multitudes.[43]

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40. See Part III, § 76.

41. Mathematical monads, being identical, are therefore incapable of providing a basis for the *difference* between, e.g., two monads and three monads, that is, the difference between the mode of being one (the integrity) of the "dyad" and the mode of being one of the "triad." Eidetic monads, being incomparable with one another, include within themselves the articulation of the condition of *difference* that is exhibited by the in fact different mathematical ἀριθμοί. See Part III, § 75.

42. See Part III, § 77.

43. See Part III, § 81.

§ 133. Klein on Aristotle's Account of the Inseparable Mode of the Being of Ἀριθμοί from Sensible Beings

Aristotle's account of the origin of ἀριθμοὶ μαθηματικοί represents a fundamental critique of the Platonic account. Moreover, it is an account that is comprehensible only in the context of the Platonic one, since it explicitly takes issue with the *chorismos* thesis, the attribution of a *generic* unity to ἀριθμοί, and the necessity of the supposition of a non-mathematical one (ἕν) in order to ground arithmetically mathematical unity. It bears emphasizing, however, that what is disputed is *not* the *being* of ἀριθμοί, characterized as the discrete delimitations of the field of noetic units into definite amounts, the ἀρχή of which is one but rather their *mode* of being.

The Platonic determination of the mode of being of "pure" ἀριθμοί on the basis of the attempt to account for the possibility of counting sensible beings misses precisely the ontological dependence that is characteristic of each ἀριθμός. From the fact that it is possible to articulate the parts of something in declarative speech (τῷ λόγοῳ) before denominating the whole, it does not follow that the "being" (τῇ ουσίᾳ) of these parts has priority over the being of the whole. Likewise, it does not follow from the assertion that there is an ἀριθμός of something that this ἀριθμός exists outside of that which it delimits with respect to its definite amount. For example, in calling a human being 'white', no other being is meant than precisely this white human being. Likewise, in the assertion 'three trees', 'three' has the same status as the 'white'; the definite amount of trees (e.g., 'three') therefore has no proper φύσις. The "being so many" of trees, like their being green, is dependent on there being *trees*.[44]

For Aristotle, then, the ontological status of ἀριθμοί is determined by their natural meaning: the assertion that certain things are present in a specific ἀριθμός means only that *such a thing* is present in just this definite multitude. This characterization of the mode of being of number, however, presents the problem of how one is to account for the purely noetical quality of ἀριθμοὶ μαθηματικοί. This is a problem for Aristotle because, unlike Plato, who posits an ontological independence of the νοητόν from sensible beings, Aristotle's reliance on the natural meaning (revealed in the analysis of ordinary speech) of ἀριθμοί precludes the supposition behind the Platonic position. That is, it precludes the ὑπόθεσις that the homogeneous, indivisible (and therefore unchanging) characteristics proper to the ἀρχή of ἀριθμοί in mathematical ἐπιστήμη are grounded in a mode of being separate from αἰσθητά. Aristotle instead articulates the being belonging to these characteristics as one of ἐξ ἀφαιρέσεως, of being "lifted off," "drawn off," or, in other

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44. See Part III, § 85.

words, being "abstracted" from sensible beings. The mathematical objects (τὰ μαθηματικά) studied by mathematical ἐπιστήμη, which in their being are not detached from sensible beings, are therefore nevertheless studied *as if* they were detached or separated.

§ 134. Klein on Aristotle's Account of the Abstracted Mode of Being of Mathematical Objects

How is it that someone who thinks mathematical objects is able to do so as separate from sensible beings, even though they are not separate? The answer to this question arises by considering how the "single parts" (μέρη) of sensible beings are gotten hold of in the λόγος. When the aspects of a sensible thing are distinguished in speech, one after the other, from the concrete context of their being, a context without which they would not exist—for example, 'this' 'round' 'white' 'column'—it is apparent that the nexus of being that links all the parts together is disregarded in a manner that allows each part to be singled out and apprehended separately. This "disregard" establishes a new mode of seeing that allows something *in* sensible beings to come before its regard in a manner that, for all their variety and transitoriness, is unchanging. As such, it remains always in the same condition and therefore satisfies the demand that for Aristotle, as for Plato, must be satisfied for a being to be an object of ἐπιστήμη. Thus, Aristotle writes: "Each thing may be viewed best in this way—if one posits that which is *not* separate *as separate*, just as the arithmetician and the geometer do" (*Metaph.* M, 1078 a 21–23).[45]

The "lifting off" characteristic of abstraction expresses nothing other than the "disregard" that makes possible the articulation in the λόγος of the single parts of a sensible thing, a disregard in which sensible beings are deprived of their sensible qualities and individual differences. In a manner of speaking, they wither away, becoming mere pieces of bodies or mere bodies themselves, such that a demonstrative discipline becomes possible, one that, as it were, "reads off" such pieces or bodies their arithmetical and geometrical aspects, namely, how many or how extensive they are. Moreover, the theoretical mathematician, in making the subject matter of study that which comes into view in abstraction, no longer views what has been abstractedly lifted off as having its basis in mere bodies. Rather, disregarding all that is sensible, the mathematician regards what has been abstracted in this manner merely as "pieces," pieces whose content, being indifferent to all that is sensible, leaves only that which is asked about in the question 'how many' together

45. See Part III, § 86.

with continuous magnitude. In the case of what is investigated with respect to its ἀριθμός, these abstract pieces are transformed into neutral monads, into merely countable pieces of things whose sensible qualities have withered away. Thus, it is not an *original* separation but a *subsequent* indifference that characterizes the mode of being of pure ἀριθμοί. The task of determining how this mode of being itself is to be understood, however, belongs not to mathematics but alone to πρώτη φιλοσοφία. That is because mathematics simply has to accept the mode of being of the various *original* abstract beings that comprise the *pregiven* contents of arithmetic and geometry (e.g., the 'one', the 'line', the 'plane', and so on) and deal with them only insofar as their noncontradictory connections are demonstrable.

§ 135. Klein on Aristotle's Critique of the Platonic Solution to the Problem of the Unity of an Ἀριθμός-Assemblage

It follows for Aristotle from the monad's abstract mode of being that the Platonic solution to the problem of the unity of an ἀριθμός-assemblage, that is, to the question how the "many" can be understood as "one" at all, is untenable. In the first place, it is untenable because the positing of a "common thing" (κοινόν) *above* and *alongside* the multitude of units supposedly unified by the integrity of its γένος attributes unity to something that, properly speaking, cannot be one at all. It cannot be one because what is meant in speaking of an ἀριθμός is precisely something that is *more* than one thing. Things are one by immediate contact, mingling, or the disposition of their parts, none of which are possible when it comes to the monads in the dyad, triad, and so on. Rather, just as two men are *not* one thing over and above *both* of them, so too in the case of two pure monads.[46] In the second place, on account of *what* ἀριθμοί are one, "no one says anything" (*Metaph.* Λ, 1075 b 34). The 'no one' here being Plato and the Platonists, all of whom (on Aristotle's view) are silent about what causes something that is intrinsically more than one nevertheless to be one, which on Aristotle's understanding of ἀριθμός is patently impossible.

The Platonic view of the generic unity of ἀριθμοί is the consequence of the supposition of the detachment and therefore independence of noetic monads from sensible beings. This supposition removes the basis for appealing to the natural articulation of ever-different and divisible sensible beings to account for the *origin* of the delimitation and unification of single ἀριθμοί. Having eliminated this ultimate foundation of all possible unity, the *chorismos*

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46. See Part III, § 87.

thesis seduces the one who posits it into embracing the view that the possibility of collecting together *two* monads in *one* ἀριθμός-assemblage has to be the effect of an original and therefore independent γένος or εἶδος. However, being in truth nothing other than sensible beings that have been reduced by abstraction to mere countable pieces of such beings, monads are, like ἀισθητά, *divisible*. This means that when "one" monad is divided into "two," there is nothing but their *being two* that may be termed their 'twoness'. That is, there is no "one thing"—the whole of which is beyond or beside the monads in question—that provides the integrity of their delimitation *as two*. Thus, for Aristotle, an ἀριθμός is precisely not *one* thing but a "heap" (σωρός) of sensible beings or abstract monads. An ἀριθμός, therefore, is precisely *nothing more* than these parts, for it is only what has been or can be counted.

This last point is crucial to Aristotle for understanding properly the soul's preknowledge of all possible ἀριθμοί, which, following Plato, should be called a "stored possession" (κτῆσις)—in contrast to a "possession in use" (ἕξις). Because an ἀριθμός is something that coincides with what is counted, the "pure" (that is, "indifferent" to the determinate qualities of sensible beings) noetic structures available to the soul prior to counting must not be spoken of as *one thing* that, in turn, points to a κοινόν that should be understood as a whole above and outside the multitude of counted objects. On the contrary, because the availability of such structures originally becomes known *in* counting, it is likewise rooted in the practice of counting sensible multitudes and extracting from them, ἐξ ἀφαιρέσεως, "pure" monads. As a consequence, ἀριθμοί of "pure" monads involve, no less than ἀριθμοί of sensible beings, "heaps"—in this case, heaps of "pure" monads. They are therefore "one" only in the sense that something can be said to extend "over the whole" (καθόλου), which rules out their being "one thing" any more than ἀριθμοί of sensible beings.[47]

§ 136. Klein on Aristotle's Answer to the Question of the Unity Belonging to Ἀριθμός

Aristotle's answer to the question of *what* it is that is responsible for the unity proper to ἀριθμός, a question that he maintains is unanswered in the generic Platonic account of ἀριθμός, begins by posing it only for *actually* counted multitudes. Such multitudes, as multitudes of homogeneous ones, comprise a unity insofar as each multitude is measured by its own one. Counting presupposes the homogeneity of that which is counted, which means that in

47. See Part III, § 88.

counting one and the same thing is fixed upon, such that its definite amount is arrived at only after *one and the same thing* has been counted over. The 'one', then, has priority *not* in counting as the superiority of a genus over a species but rather in its character as the "measure" (μέτρον) by which the definite amount of a multitude is determined. The "being one" of sensible beings marks both the possibility of their being counted and the *indivisibility* of the "one" that, insofar as it supplies the measure of what is counted, is "one sensible thing" and therefore undivided. For example, the "being *one*" of each apple in an ἀριθμός of apples is not divided and therefore does not have a division, even though each apple as a sensible being can be divided, as can any other sensible being. Indivisibility therefore belongs to what is counted only insofar as it is the origin of the measure of the count, because "whatever does not have a division, insofar as it does not have it, is in that respect called one" (*Metaph.* Δ 6, 1016 b 4–6). Any specific ἀριθμός is therefore "a multitude measured by the one" (I 6, 1057 a 3–4). As such, its "being" (οὐσία) is the multitude of units as such, in the precise sense of the "how many" it indicates. Thus, οὐσία is understood here to be derived insofar as *what* each ἀριθμός is is not something that is separate or detached from the definite amount of homogeneous units it delimits. Thus, for example, "six" units are not "two times three" or "three time two" units but precisely "once six." For Aristotle, then, there is no such thing as *the six*, with a noetic being that would be distinct from the many hexads that delimit this or that multitude of "once six" units.

§ 137. Klein on Aristotle's Account of the Origination of the Μονάς as Measure

The "totally indivisible" (πάντῃ ἀδιαίρετον) and "completely exact" (ἀκριβέ - στατον) status that the arithmetician understands the μονάς to possess arises for Aristotle on the basis of the elevation of a habitual procedure to the rank of ἐπιστήμη. The habitual expression of the sensible beings in every count in terms of their "being one"—for example, instead of saying 'one apple, two apples, three apples', what is said is rather 'one, two, three'—points already to the purely arithmetical status of sensible beings as countable material. When this status is abstractedly "lifted off" sensible beings, the mathematical μονάς originates. And it originates as nothing more than the character of being a measure as such, a character expressed through its indivisibility and exactness. The character of the one as measure is what is responsible for the universal applicability of "pure" ἀριθμοί, namely, the applicability of the μονάς to any arbitrarily countable being whatsoever. The μονάς is applicable in this way because its mode of being is *not* one of *being* separate from the sensible beings that are

the source of its abstracted *origin*. Hence, it is only because sensible beings (*as* the *kind* of beings that they are) are one and indivisible that the arithmetician (having already abstractedly posited the μονάς as totally indivisible) is then able to see what always follows from any given sensible being insofar as it is subject to being counted or calculated with as a "unit." Thus, for example, a human being as the kind of being it is, namely *as human being*, is one and indivisible and as such the *abstract* μονάς is applicable to it.[48]

48. See Part III, § 89.

Chapter Twenty-six

Structural Differences in Husserl's and Klein's Accounts of the Mode of Being of Non-symbolic Numbers

§ 138. The Different Accounts of the Mode of Being of the "One" in Husserl, Plato, and Aristotle

The differences in the mode of being proper to non-symbolic numbers presented in Husserl's account of authentic cardinal numbers and Klein's account of the ἀριθμός-concept in Plato and Aristotle concern three issues, all of which are crucial for understanding and assessing Husserl's and Klein's presentations of the origination of the logic of symbolic mathematics. The first issue is the status of the unit or the one, which all accounts characterize as discrete and multitudinous and as the element whose delimitation in counting yields non-symbolic numbers. The second issue is the status of the "being one" of the whole composing each non-symbolic number, an issue that includes both the peculiar "one over many" unity belonging to the whole of each non-symbolic number, as well as each number's ordered differentiation—qua precisely this unity—from other non-symbolic numbers. The third and final issue is the scope of the intelligibility of non-symbolic numbers, their ability to render definite the amounts of any arbitrary thing at all that happen within the ambit of counting.

The articulation of these differences will permit us to consider their significance for Husserl's and Klein's accounts of the origination of the logic of symbolic mathematics, a consideration that will enable us to assess the philosophical significance of Klein's deviation from Husserl on the nature of the origin of the mathematization of nature.

The "one" for Husserl is a formal category that is nearly identical to that of the formal category of *anything*, the correlation with the concept of multiplicity being that which "alone marks out the concept of *one* over and

against that of *anything*" (*PA*, 84). This means that in Husserl's account, as in Plato's account of the "being one" proper to mathematical monads, the concept 'one' is inseparable from its opposite, the concept of multiplicity. Husserl maintains, however, that for "number abstraction" this correlation "is not a point that in any way comes into consideration." His reason for maintaining this is that when it is counted, "each object of the multiplicity is thought merely as a 'something'," which means "the 'something' is already 'one'" as far as the delimitation—qua cardinal numbers—of the items in a multiplicity is concerned.

The one for Plato is differentiated in accordance with the distinction and foundational relationship between the "being one" of mathematical beings and the incomparable unity of the "One itself." As the ultimate foundation of the unity belonging to mathematical and ontological beings, the "One itself" transcends them both. The noetic one is a mathematical being whose unity is independent of somatic beings but dependent on the unity of the One itself, because, as multitudinous, mathematical being is incapable of accounting for the singularity composing the unity of the "one" that the mathematical mode of being a multitude nevertheless presupposes.

For Aristotle the one is that aspect of somatic beings that functions as their measure when they are counted, and it is therefore a being—albeit a secondary being. The mathematical being of the noetic one is nothing other than the isolation of this measure aspect of somatic beings, an isolation that at once disregards all the sensible characteristics of these beings that are unrelated to their being a measure and "lifts off" from any sensible context what has been so isolated. What remains as a result of this process is a "pure" piece of somatic being, the *νοητόν* of the one.

§ 139. The Different Accounts of the "Being One" and Ordered Sequence Characteristic of the Wholes Composing Non-symbolic Numbers in Husserl, Plato, and Aristotle

The "being one" of the whole composing each non-symbolic number for Husserl is rooted in a psychological unity, namely, in the reflexion on the act of combining together the ones in a multiplicity. The ordered sequence of non-symbolic numbers is likewise rooted in a psychological unity, again in the reflexion on the collective combination that unfolds with the successive addition of 'ones' to delimited multitudes, beginning with 'one and one'.

The "being one" composing the whole of each non-symbolic number for Plato is rooted in a trans-mathematical, which is to say, ontological generic unity, albeit one that is different for each different number. Therefore, when

the mode of "being one" of more than one number, the mode of "being one" of *numbers*, is considered, it can be said—in precisely *this sense*—that the unity in question here is "heterogeneous." It is a unity that is the condition responsible for the structural integrity of the somatically independent and generically unique assemblages of ontological beings (γένη or εἴδη) that make possible the "one over many" unity of each mathematical number. Moreover, the unity of the structural integrity of each such assemblage is dependent upon the unity of the trans-mathematical and trans-ontological "One itself," because the mode of being belonging to both mathematical and ontological multitudes presupposes (rather than accounts for) the unity of a "being one" whose completeness excludes any possible relation with what is *not* one and therefore *other* than its unity. The ordered sequence of non-symbolic numbers is determined by the priority and posterity that characterizes the relationship to one another of the generic wholes composing each of the heterogeneous assemblages of ontological beings that are responsible for the integrity of the natural succession of mathematical numbers.

For Aristotle the "being one" of the whole composing each non-symbolic number is nothing other than the measure by which the definite amount of ones that are delimited by each non-symbolic number is determined. The ordered sequence of non-symbolic numbers is determined by the natural articulation of the "heaps" of ones that make up each such number.

§ 140. The Different Accounts of the Conditions Responsible for the Scope of the Intelligibility of Non-symbolic Numbers in Husserl, Plato, and Aristotle

The scope of the intelligibility of non-symbolic numbers, such that it includes the enumerative delimitation of any arbitrary object whatever, is accomplished for Husserl on the basis of the generic emptiness, which is to say, the formal generality of the category *anything*. The abstractive lack of attentiveness to the content of what is counted, which brings about the utter depletion of its specificity that allows it to fall under the materially indeterminate concept of *anything*, is what confers universality on multiplicities and their numerical delimitation.

In Plato, the scope of the intelligibility of non-symbolic numbers is also unlimited, though the basis for this is different than in Husserl. Non-somatic noetic beings, the indivisibility and unchanging nature of which comprise the purity of the true referent proper to counting and calculation, are what permit any arbitrary being whatever to be enumerated. They permit this, because what are really numbered when more than one of any spe-

cific kind of being is counted are just these pure noetic beings, and not beings determined in any other way.

For Aristotle noetic beings likewise are what are responsible for the numeration of any arbitrary being, but (unlike in Plato) not because of their independence from somatic beings. On the contrary, it is precisely their dependence on such beings that is responsible for this, as abstracted pieces of the being one and being indivisible of the generic *what* that belongs to any somatic being. Because any somatic being, *as* the kind of being it is, is both one and indivisible, the abstracted being of the noetic one is applicable to any arbitrary kind of somatic being.

§ 141. The Structural Differences between Husserl's and Klein's Accounts of the Mode of Being of Non-symbolic Numbers

Husserl's and Klein's accounts of the mode of being proper to non-symbolic numbers diverge in ways that are significant not only in their own right but even more so for their respective accounts of the origination of the logic of symbolic mathematics. Among these divergences, the most important concern their radically different accounts of the origin and character of the "ones" that are the elements of the multitude delimited by non-symbolic numbers. Notwithstanding Husserl's appeal to "abstraction" in his account of their origin, specifically in his subsumption of abstracted individual contents under the formal category of *anything* that yields the ones that are a part of the concept of multiplicity, the abstraction and therefore their origin is radically different from Klein's presentation of what is involved in Aristotle's appeal to ἐξ ἀφαιρέσεως.

The intelligibility of noetic ones for Aristotle is inseparable from their origin in somatic beings, an origin whose involvement in abstractive "disregard" is admittedly similar to Husserl's account of the abstractive depletion of individual contents. Nevertheless, Husserl's abstractive disregard prepares the way for thinking these contents via the mediation of a concept whose origin *is not abstractive*, namely, under the concept of *anything* that originates in the *reflexion* on the act of presenting, while for Aristotle the disregarding at issue in abstraction is not preparatory to anything. Disregarding the sensible characteristics of somatic beings, save for their "being one," and thereby isolating what makes them countable in a manner that allows this to be treated *as if* it were separate from what it—in truth—is inseparable from, has *nothing* to do with *concepts* for Aristotle: what is abstracted in this manner is not a concept but still a being, a somatic being shorn of all its indi-

vidual differences and sensible characteristics. The purely noetic quality of the abstracted being that follows from this mode of origination is therefore inseparable from that *in* which it originates. As such, not only is the abstracted noetic one still a part of the somatic being from which it is abstracted, but its sensible indeterminacy is also limited because of this. This last point becomes quite evident when that which is responsible for the abstracted noetic one's applicability to any arbitrary sensible being whatever is considered, the isomorphism of *each* being one and undivided in their kind.

In a word, then, the indeterminacy of the ones' content in Husserl's account of the mode of being of the elements in the multitude delimited by non-symbolic numbers is the result not only of a concept but of a concept that is totally foreign to both Aristotle's and Plato's accounts of the noetic one, namely, of the 'object in general'. We have just seen why the intelligibility of the noetic ones in Aristotle not only supposes nothing of the kind, but also is incapable of doing so. And again, the reason is that the noetic ones are beings whose mode of being is inseparable from their origin in determinate beings, which means that their applicability to any beings whatever is rooted in an isomorphism between their being one and indivisible and the being one and indivisible of the *kind* that characterizes the *what* of any determinate being.

For Plato the non-conceptual mode of being of the noetic ones is even more striking, for the *chorismos* thesis that posits them supposes that their mode of being *as beings* is completely independent of somatic beings: not only are such beings therefore *not* abstract in Aristotle's sense, but neither are they general in the sense of Husserl's concept of 'anything'. They are not "abstract" because their ontological independence precludes their being "lifted off" anything; they are not general because as beings they are determined by both multitudinousness and homogeneity.

§ 142. The Divergence in Husserl's and Klein's Accounts of Non-symbolic Numbers

Husserl's account of the diversity of the collectively combined and delimited ones—that is, the different cardinal numbers—is radically different from Plato's and, while in some ways similar to Aristotle's, ultimately radically different from his as well. Like Aristotle and therefore unlike Plato, Husserl does not appeal to heterogeneous unities in order to account for the differences in integrity proper to the "one over many" unity belonging to the (different) aggregates of 'one and one', and of 'one, one, and one', and so on. For Husserl, the difference between the unity of each, in the former case

'two' and the latter 'three', is nothing more than the difference in the form of the combination of delimited ones. Likewise for Aristotle—or, more properly, Husserl's account is Aristotelian in just this regard—the unity of each non-symbolic number is just the counted heaps of ones, and nothing more. However, whereas for Aristotle the unity—as an ἀριθμός—of the heap originates in the measure character of the one that is counted repeatedly in arriving at the amount of the *same thing*, for Husserl the unity belonging to the whole of each non-symbolic number originates in the reflexion upon the collective combination that combines and delimits the multitude of ones (that answer the question 'how many?'). The unity that results from this reflexion is neither ontologically generic in the Platonic sense nor formally conceptual in the sense in which Husserl himself characterizes the unity that belongs to the category of *anything*, but rather psychological. It is psychological in the sense that the origin of the relation between the ones that compose the delimited multitudes of non-symbolic numbers is established by the psychological *act* that combines them. (Precisely the problem of the logical status of this relation, which is obscured in *Philosophy of Arithmetic*'s account of its genesis in psychological acts of combining, is what Husserl addresses in his subsequent self-critique of this work, considered below. Out of this criticism will come his attempt to account for the relation of arithmetic and mathematics generally to logic by understanding mathematics as part of logic.)

Neither Husserl's nor Aristotle's account of the diversity proper to non-symbolic numbers, however, addresses the main issue raised in Klein's presentation of the Platonic position: the inability of the kind of unity that belongs to the homogeneity of the ones proper to unlimited and limited multitudes alike to account for the differences between the exact amounts that characterize each limited multitude as a different non-symbolic number. The exact delimitation of 'one, one, and one' cannot account for their being 'three' according to the Platonic position, because each 'one' is one and the same;[49] nowhere in and nothing about these ones in any way specifies or otherwise addresses why, for instance, 'one and one' should be both *identified* as 'two' and *differentiated* from 'one, one, and one' (the *identification* of which as 'three' is equally problematic), etc. Moreover, the appeal to the unity of the "one over many" relation of the whole that is generated in the reflexion on the psychological act of collective combination cannot ac-

49. J. N. Findlay essentially repeats the Platonic argument against Husserl when he notes that "Husserl has not considered what may be involved in the necessary *diversity* of the abstract somethings collected, since something and something and something is not three if the somethings are one and the same" ("Translators Introduction," *LI*, 14).

count for the identity and differentiation at stake here either, because (on Husserl's own account) what differentiates one such whole from another—which is to say, one non-symbolic number from another—is precisely the combination of ones: 'one and one' is therefore differentiated from 'one, one, and one'. The "one over many" structure of the collective connection (expressed by the 'and') of a) 'one and one' is therefore the same as b) 'one, one, and one', and so on.

Aristotle's account of the being of non-symbolic numbers as heaps faces the same problem, since there is nothing in different heaps of homogeneous ones to specify or address the issue of why any heap should be *identified* as a dyad or triad. There is also the problem of why heaps identified as either of the latter should be *differentiated* from the other in accordance with the relation of prior and posterior[50]—that is to say, why heaps of identical ones should be related sequentially.

The Platonic solution to the problems connected both with the "being one" of the differentiated "one over many" kind of unity characteristic of mathematical numbers and with their sequential order, the ontological supposition of ἀριθμοὶ εἰδητικοί, is at best only a limited one, however. While this solution does indeed make clear the aporetic nature of both of these problems when one attempts to solve them using the kind of unity that determines a mathematical multitude, its supposition of a generic unity belonging to non-mathematical numbers has the disadvantage that it exceeds the sphere of intelligibility provided by so-called natural predication.[51] Specifically, the natural meaning of ἀριθμός, as a multitude of countable items, is lost because the monads that compose an ἀριθμὸς εἰδητικός are incomparable and therefore uncountable.

The final point of divergence between Husserl's and Klein's accounts of the mode of being of non-symbolic numbers concerns how they characterize the condition or conditions responsible for the unrestricted scope of their intelligibility. At issue here is *not* whether such numbers have the capacity to delimit amounts of any arbitrary beings or objects whatever: that non-symbolic numbers are able to accomplish this is something that neither Husserl nor Klein contests. However, the comparison of their accounts of *how* these numbers are able to accomplish this, of what in their structural modes of being makes *possible* the unrestricted scope of the intelligibility involved in this accomplishment, discloses that their respective accounts are indeed at odds with one another. For Husserl, non-symbolic numbers can

50. See Part III, § 90, n. 87.
51. See Part III, § 83.

delimit any arbitrary beings or objects whatever because part of their very structure *includes* the formal concept of—in effect—'anything whatever'.[52] Any object or thing can be counted because, once its determinative qualities are abstracted, it is able to fall under the formal concept 'anything whatever'.[53]

Klein's account of the mode of being characteristic of non-symbolic numbers (the ancient Greek ἀριθμοί) that is responsible for the scope of their intelligibility with regard to any arbitrary kind of being whatever is *non-conceptual*. Neither the formal concept of 'anything whatever' (or 'object in general') nor the concept of 'number in general' is involved for either Plato or Aristotle in the structure of ἀριθμοί that is responsible for their ability to delimit unrestrictedly any kind of being. Beings—not concepts—are what allow this, beings that, notwithstanding their different origins for Plato and Aristotle, can only be thought. Thought as discrete, multitudinous, and indivisible, noetic beings are neither undetermined nor "general" in Husserl's —distinctly modern—sense of an 'object in general'. Nevertheless, as determinate intelligible beings, "pure" monads can account for the unrestricted scope of the intelligibility of non-symbolic numbers. For Plato, they accomplish this by being the true objects of both practical and theoretical arithmetic. For Aristotle, their abstractive relation to the mode of being one and indivisible belonging to the "kinds" of sensible beings allows them to enter into an enumerative relation with the "kind" that belongs to any sensible being.

52. In *Philosophy of Arithmetic* this formal concept is referred to as the *anything* (*Etwas*), although he will eventually articulate it as the *Etwas-überhaupt* (anything whatever).

53. Using Husserl's terminology in *Philosophy of Arithmetic*, the formal concept of the *Etwas* can never be the *content* of a presentation, because its conceptual status originates in the *reflexion* on the act of presentation itself. This is why he there considers it a "negative" determination of the content of presentational acts, content that, to be sure, on this account must be present in order for the concept of the *anything* to be thought. It should be also noted here that even after Husserl abandons this overtly psychological account of the origin of the formal concept of the *anything*, neither his account of its materially empty character nor its inability to be experienced in (using Husserl's later terminology) straightforward perceptual experience will be altered.

Chapter Twenty-seven

Digression: The Development of Husserl's Thought, after *Philosophy of Arithmetic*, on the "Logical" Status of the Symbolic Calculus, the Constitution of Collective Unity, and the Phenomenological Foundation of the *Mathesis Universalis*

§ 143. The Need to Revisit the "Standard View" of the Development of Husserl's Thought

The widespread view that Husserl both acknowledged and adequately addressed the shortcomings of *Philosophy of Arithmetic*'s psychologism, at least in principle, in his *Logical Investigations* has resulted in what we shall refer to here as the "standard view" of the development of his thought subsequent to his critique of his first work. According to this view, from the Sixth Logical Investigation onward, Husserl's phenomenology, in both method and substance, "overcomes" his first work's illegitimate appeal to psychological acts and structures to account for the objectivity of logical structures. The key to this overcoming is located in Husserl's introduction of the distinction between cognitive "acts" and the "objects" of acts, and his recognition that the "ideality" of logical objectivity can, in principle, be established only by acts that apprehend "categories" as objectivities *given to* cognitive acts rather than (as in psychologism) by acts that apprehend the structure and content of the acts that present these objectivities.

From the standpoint of our guiding concern with Husserl's account of the origination of the logic of symbolic mathematics, the standard view of the development of Husserl's thought is too general. Under the categories that are

putatively given to cognitive acts, it includes the collective unity of cardinal numbers and multitudes or sets (*Mengen*)[54] and assumes that the basic intentional paradigm of acts that present "empty cognitive intentions" and acts that present their "fulfillment" in the givenness of the logical objectivities themselves, which Husserl developed with respect to the "categorial intuition" of logical "states of affairs," holds also with respect to the apprehension of the ideality of collective unity. On our view, this assumption cannot withstand a critical scrutiny of the content of Husserl's attempts to resolve the "logical" problem posed by multiplicities, because these attempts (from the *Logical Investigations* on) clearly distinguish the kinds of logical unity characteristic of the whole and parts of a collection from the whole and the parts of an individual object and, correspondingly, the nature and "constitution" of their respective unities.

Related to the standard view on Husserl's "overcoming" of psychologism is the likewise standard view that Husserl's mature logical investigations in *Formal and Transcendental Logic* present a phenomenological theory of judgment that adequately accounts for the origination of the distinction as well as unity of the formal logic and formal mathematics that compose the "pure" (completely formalized) *mathesis universalis.* Husserl's account of the origination of the logic of formal, if not symbolic, mathematics, is therefore to be found in this text. On our view, his analyses in *Formal and Transcendental Logic* of the structure and composition of the "pure" *mathesis universalis* not only are insufficiently complete to warrant the view in question, but also Husserl himself is acutely aware of the programmatic nature of his analyses that is responsible for their incompleteness and says as much in their presentation. To this it has to be added that, on our view, the shortcomings of those later analyses are rendered especially perspicuous if they are approached from within the context of the development (subsequent to *Philosophy of Arithmetic*) of his thought related to the logic of symbolic mathematics found in both his major works and other key texts.

To expose the prejudices that inform the standard view on the matters that are of central concern in our study, a significant digression is required before we take up our final discussion of Husserl's and Klein's accounts of the logic of symbolic mathematics. The task thereby will be to present the development of Husserl's thought on these matters in a manner that brings into bold relief his mature views on them. To this end, we shall trace this development as it relates to his views on the logic of the symbolic calculus, his critique of psychologism, and the phenomenological foundation of the "pure" *mathesis universalis*.

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54. Regarding the transformation in the general meaning of the term *Menge* subsequent to *Philosophy of Arithmetic*, see n. 60 below.

Chapter Twenty-eight

Husserl's Accounts of the Symbolic Calculus, the Critique of Psychologism, and the Phenomenological Foundation of the *Mathesis Universalis* after *Philosophy of Arithmetic*

§ 144. Husserl's Account of the Symbolic Calculus after *Philosophy of Arithmetic*

Subsequent to *Philosophy of Arithmetic*, Husserl's position on the origination of the logic of symbolic mathematics is no longer ambiguous in his writings inasmuch as they clearly express his abandonment of the thesis that the technique of symbolic calculation in universal arithmetic is logically equivalent to arithmetical calculation that employs arithmetical concepts. Husserl characterized symbolic calculation as the "surrogate" for genuine arithmetical thinking and concepts, but he now characterizes the logic that permits it to accomplish this as "external," in the precise sense of a technical procedure that is grounded in syntactical rules—the "rules of the game"—rather than in genuine arithmetical thinking and concepts. Because the external logic of the symbolic calculus operative in universal arithmetic (and, for that matter, in formal logic) produces valid results when measured by the standard of non-algebraic and therefore "genuine" mathematical (or logical) thinking, Husserl still maintains that it functions as a "surrogate" for the latter. However, he does not take this surrogate function to originate in the identity of the logical content of the symbolic calculus and genuine thinking. In the case of universal arithmetic, Husserl understands its symbolic algorithm to function independently of the various concepts of its possible objects. In the case of formal logic, the algebraic (and therefore symbolic) treatment of the rules for correct judgment *presupposes* rather than establishes the *logical* criteria for the truth and falsity of the concepts (formal categories) of genuine logical judgment.

The clarity with which Husserl establishes that the symbolic calculus is neither deduction nor its logic is not matched by a similar clarity regarding the relation of its external logic to genuine logical thought. That is, his thesis regarding the relation of the symbolic calculus to genuine thought is negative: the symbolic calculus is not an artificial language—indeed, it is not a language at all because it is *not* expressive of any kind of thought. The operational rules and signs that compose the symbolic calculus are conceptually blind surrogates for genuine deduction, and its algorithm is a mechanical procedure that saves us from having to engage in complicated and laborious deductions and enables us to produce its results with greater speed, certainty, and ease than conceptually deductive thinking. But the "rules of the game" that compose the "algebra of logic" are themselves not logic, which gives rise to what Husserl himself characterizes as the difficult questions about the essence and logical justification of its calculative method. Yet it is precisely these questions, despite Husserl's recognition of their importance, that nevertheless remain unaddressed and therefore unanswered in his mature works.

We shall show that when Husserl discusses the symbolic calculus in his mature works, he always does so in terms of the negative thesis that its "rules of the game" represent neither genuine logic nor the theory of such logic. To this end, we shall first consider in detail the text that initially presents this thesis, Husserl's "Review of Ernst Schröder's *Vorlesungen über die Algebra der Logik*."[55] Published in 1891, Husserl's account therein of the symbolic calculus as "mere" calculational technique clearly goes beyond the view of it expressed in his 1890 letter to Stumpf, where Husserl characterizes the symbolic calculus of universal arithmetic as part of formal logic and describes formal logic itself as a symbolic technique. In the Schröder review, Husserl distinguishes the calculus from "pure" deduction because he now understands the technical device of a *calculus* of pure deduction to be external to the logic proper to the pure implications belonging to any judgments whatever. Rather than being capable of providing a logic of pure deduction, Husserl understands the symbolic calculus to be a technique for making deduction "superfluous." In 1903, Husserl credits his Schröder review with having "laid bare the follies of extensional logic,"[56] that is, the logic of classes that Schröder and

55. Edmund Husserl, "Ernst Schröder's *Vorlesungen über die Algebra der Logik*," *Göttingische gelehrte Anzeigen* (1891), 243–78; reprinted in *Hua* XXII, 3–43; English translation: "Review of Ernst Schröder's *Vorlesungen über die Algebra der Logik*," in *Early Writings*, trans. Dallas Willard, 52–91. Henceforth cited as 'Schröder Review', with page references to the Husserliana edition and the English translation, respectively.

56. Edmund Husserl, "Melchior Palágyi, *Der Streit der Psychologisten und Formalisten in der modernen Logik*," *Zeitschrift für Psychologie und Physiologie der Sinnesorgane* 31 (1903),

others use as the basis for the symbolic calculus. That precisely the view of the essential distinction between pure logic and the "logical" calculus expressed in his Schröder review informs all of Husserl's subsequent thought on the symbolic algorithm will be shown next, with the consideration of his accounts of the "rules of the game" status of the symbolic calculus in his last work (*Crisis*), as well as in his two major logical works (*Logical Investigations* and *Formal and Transcendental Logic*).

In a 1913 discussion of the development of his thought, Husserl articulates a more or less straight line leading from his preoccupation with symbolic thinking in mathematics[57] to his "breakthrough"[58] to phenomenology. He characterizes how, "[a]bove all" (*ILI*, 126), this breakthrough was connected with his investigations of "the cognitive accomplishment of arithmetic and pure analytical mathematics in general," and their focus on "its purely symbolical procedural techniques, in which the authentic, originally understandable meaning is violated under the heading of the 'imaginary' and appears to be turned into countersense." Husserl attributes the appearance of this countersense as what "led his thinking to the signitive and purely linguistic aspects of thought and cognitive processes, and from that point on to necessary universal investigations, investigations that concerned a universal clarification of the meaning, exact delimitation, and characteristic accomplishment of formal logic." He relates that "after many troubles," he was able to "understand theoretically" what he characterizes as "the immense importance for consciousness of the 'merely symbolic thinking' belonging to the so-called external logic operative in mathematics." However, he also writes: "But how symbolic thinking is 'possible,' how objective mathematical and logical connections are themselves constituted in subjectivity—and how the evidence for this is to be understood, how, in other words, the mathematical can be given in the medium of what is psychic and nevertheless be objectively valid, all of this remained mysterious" (127–28). Husserl's breakthrough to phenomenology, then, is, in his words, "tied to these investigations," which means for him that they are two-sided. There is, "on the one side, the so to

152–61, here 155; "Review of Melchior Palágyi's *Der Streit der Psychologisten und Formalisten in der modernen Logik*," in *Early Writings*, 197–206, here 199.

57. In 1906, Husserl wrote, "Extension of my efforts to the whole domain of the purely logical was no doubt occasioned, more than anything else, by engagement with the logical calculus during the Winter of 1890" (*Personal Notes*, 491).

58. Edmund Husserl, "Entwurf einer 'Vorrede' zu den *Logischen Untersuchungen* (1913)," ed. Eugen Fink, *Tijdschrift voor Philosophie* 1 (1939), 106–33, here 124; English translation: *Introduction to the Logical Investigations*, trans. Philip J. Bossert and Curtis H. Peters (The Hague: Nijhoff, 1975). Henceforth cited as *ILI* with German page references, which are included in the margins of the English translation.

say ontological delimitation of the pure mathesis universalis" (128) and "on the other side, the separation [*Ablösung*] [of mathematics and logic] from psychologism."

We shall trace below the path that leads from Husserl's consideration of the possibility of symbolic thinking in mathematics and logic to both sides of the breakthrough to phenomenology, with the goal of assessing the extent to which his phenomenological investigations succeed in accounting for this possibility.

§ 145. Husserl's Critique of *Philosophy of Arithmetic*'s Psychologism

On the side of the "separation" from psychologism, we shall consider Husserl's critique of *Philosophy of Arithmetic*'s psychologism. This critique locates a species of "logical psychologism" in that work's attempt to account for the logical unity of the collection in an abstraction directed toward the inner perception of the mental act of collecting. Husserl now recognizes such an abstraction to be capable only of accounting for the concept of collecting, but not for the logical unity of the collection itself characteristic of the concept of cardinal number. He credits his "fully conscious and radical turn" (128) to Platonism with enabling him to realize that the logical unity of the concept of number and every other logical concept is "pure," in the precise sense that logical meaning "in itself" excludes all psychological content. Husserl initially ties the phenomenological overcoming of logical psychologism to the recognition that the object of judgment is arrived at by abstracting not from the mental act of judging but from this act's object. The symbolic presentation that is part of the act of judgment is therefore understood to relate signitively to the presentation of a categorial object, and not to the presentation of the act of judging. Husserl terms the non-signitive presentation of a categorial object 'categorial intuition', which occurs in an abstraction from the sensuous perception of objects that renders present the categorial object itself in a non-symbolic manner.

Our critical assessment of Husserl's initial account of categorial intuition in the *Logical Investigations* will show (in §§ 163–69 below) that he distinguishes the acts and objective contents of the judgment from the acts and contents of the collection and that his analyses of categorial intuition focus on the judgment's predicative states of affairs. This will have the significant consequence of leaving incomplete the phenomenological account of the collectively objective correlate of the symbolically significative presentation of the 'and' that Husserl's analyses maintain refers to the collection as the logical

object proper to the act of collecting. We shall therefore conclude that the account of categorial intuition in the *Investigations* does not resolve the problem of *Philosophy of Arithmetic*'s psychologistic account of the logical unity of the concept of cardinal number and that of collectiva generally.

Husserl's "mature" treatment of the constitution of the logical unity of the collection and cardinal number in *Experience and Judgment*[59] follows the *Logical Investigations* in distinguishing judgments that posit predications in relation to substrate objects from judgments that posit the collection as an objective unity. The analyses in *Experience and Judgment* go beyond those in the *Investigations*, however, insofar as they locate a decisive difference in these two types of positing. In predicative positing the substrate object is pregiven to the judicative spontaneity that posits the predicate in relation to the substrate, whereas in collective positing the collection is not pregiven as an objective substrate. Husserl attributes the emergence of the collective unity as a substrate object—that is, as a "set"[60]—to the reflective thematization of the presentation of the collection that results from the collective combination of objects. Our critical assessment of these analyses shows their proximity to *Philosophy of Arithmetic*'s discredited psychologism and concludes that they do not satisfy the "truly" Platonistic standard that Husserl himself set for overcoming psychologism, that pure logical meaning "in itself" must exclude in principle all psychic content.

Husserl's mature statement of how phenomenology is able to overcome psychologism no longer appeals to abstractive categorial intuition but to the capacity of temporally individuated multiplicities of acts to refer to numerically identical objects that, as such, are not temporally individuated. The "supratemporality" of the reference of multiple acts to the "same" ob-

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59. Edmund Husserl, *Erfahrung und Urteil* [1939], ed. Ludwig Landgrebe (Hamburg: Meiner, 1985), 254; English translation: *Experience and Judgment*, trans. James S. Churchill and Karl Ameriks (Evanston, Ill.: Northwestern University Press, 1973), 215.

Landgrebe, under Husserl's direction and supervision, prepared this volume on the basis of the texts of Husserl's four-hour lecture course entitled "Genetic Logic," first given in winter semester 1919–20, together with supplementary manuscripts from 1910–14 and other lectures from the 1920s. However, the resultant text includes no critical apparatus that would enable the reader to identify sources or dates of the material. See, however, Dieter Lohmar, "Zu der Entstehung und den Ausgangsmaterialien von Edmund Husserls Werk *Erfahrung und Urteil*," *Husserl Studies* 13 (1996), 31–71, which provides a detailed reconstruction of Landgrebe's sources and establishes their dates—or most likely dates—of composition.

60. Because Husserl published *Philosophy of Arithmetic* four years prior Cantor's *Beiträge zur Begründung der transfiniten Mengenlehre* (1895), we have avoided translating *Menge* as 'set' in Husserl's first work on the grounds that because set theory proper is inaugurated with Cantor's work, such a rendering would be anachronistic. In Husserl's works subsequent to the publication of Cantor's work, however, we render *Menge* as 'set' where the context requires it, as we have done here.

ject implies, according to Husserl, the "omnitemporality" of their numerically identical referent. Husserl therefore maintains that the basis for the principled distinction between the real and the ideal is secured by the latter's omnitemporality and that with it psychologism, understood as the reduction of the ideal to the reality of the psyche, is overcome once and for all. Our critical assessment of these analyses has demonstrated that they do not address the kind of psychologism that Husserl himself identifies in *Philosophy of Arithmetic* and, moreover, that their appeal to the numerical identity of the referent of a multiplicity of acts presupposes rather than establishes the *logical* unity of this numerical identity.

§ 146. Husserl's Account of the Phenomenological Foundation of the *Mathesis Universalis*

On the side of the ontological delimitation of the *mathesis universalis*, we shall consider Husserl's assignment of the task of investigating the pure logical "in itself" to a new science (pure logic), his formulation of this science in accordance with (what he takes to be) Leibniz's idea of a *mathesis universalis*, and his account of its phenomenological foundation. Husserl's embrace of Platonism in order to overcome the logical "embarrassment" of psychologism created the problem of how to establish the relationship between the signitive thinking characteristic of logical judgment and the logical meaning "in itself" characteristic of categorial form. Husserl's phenomenological approach to this problem is informed by his pre-phenomenological formulation of the distinction between the signitive thinking that defines the "merely symbolic" technique of the symbolic calculus and the signitive thinking that defines thinking in the pure logical judgment. The latter thinking is also "symbolic" due to its status as a presentation that refers to its categorial object in a non-intuitive manner, though it is not symbolic in the non-conceptual sense of the algorithmic manipulation of sense-perceptible signs according to the syntax provided by the "rules of the game." Because he is convinced of the logically derivative status of the "merely symbolic" technique of algorithmic calculation, Husserl's phenomenological investigations of the relationship between the signification categories of pure logic and the formal categories of formal ontology do not raise what he himself characterized as the difficult questions about the logical justification of the symbolic calculus. That is, Husserl begins with the conviction that the elemental and operational signs of the latter are *not* logical objects in their own right but that they derive their "logical" significance *entirely* from the combination o forms proper to materially empty *objects* ("empty anythings")—of which their letter signs represent the lawful

forms. We shall show that Husserl does not investigate how these letter signs are able to represent the lawful forms of formal logical combination.

What Husserl does investigate is Leibniz's idea of the *mathesis universalis* as a single systematic science embracing both formal logic and formal mathematics. Husserl formulates this investigation as the phenomenological project of establishing the unitary object domain of logical and mathematical analytics. This project entails accounting for both the unity and distinction between formalized logic and formalized mathematics, which Husserl comes to term (respectively) apophantic logic and formal ontology. Due to the traditional difference in the objects investigated by logic and mathematics, Husserl is acutely aware that establishing the idea of the *mathesis universalis* as the unity of the distinct disciplines of apophantic logic and formal ontology hinges on establishing the unity of the object domain investigated by each discipline. That is, because of the singular nature of the object investigated by predicative logic and the plural (manifold) nature of the "object" investigated by mathematical analysis, the unity of the object domain investigated by formalized logic and formalized mathematics (formal ontology) is not something Husserl thinks can be justifiably assumed at the outset of the project of establishing a phenomenological foundation for the Leibnizian idea of a *mathesis universalis*. Husserl acknowledges in his final work on logic (*Formal and Transcendental Logic*) that his first logical work (*Logical Investigations*) did not establish the unity of these sciences and that therefore the unity of their object was likewise not established. That is, he recognized that the correlation between the signitive judgment characteristic of formal logic and the categorial judgment characteristic of formal ontology is not established phenomenologically in the *Investigations*.

Husserl's investigations in *Formal and Transcendental Logic* present three considerations indicative of the phenomenological foundation of the correlation between apophantic logic and formal ontology: 1) that the mathematical theory of sets and the theory of cardinal numbers relate to the same object domain as apophantic logic, namely, the empty universe of 'any object whatever' or 'anything whatever'; 2) that all formally ontological categorial forms of objects ultimately make their appearance in apophantic judgments; and 3) that the "ontological" meaning that is inseparable from the formalized categories 'any object whatever' and 'anything whatever' has its genesis in the perceptual experience of individual objects. Our critical assessment of these considerations will show that each falls short of establishing the foundation for the unity of apophantic logic and formal ontology, that is to say, each falls short of establishing their unitary formalized object domain. And because of this, we shall show that Husserl's mature logical in-

vestigations provide neither a justification for the logic of symbolic mathematics nor a satisfactory account of the origin of the formalization that makes symbolic mathematics possible.

In the case of (1), we shall demonstrate that Husserl himself recognizes that the mathematical and logical reference to the object domain of the 'anything whatever' does not establish the unity of formal logic and formal mathematics because of the singular modality of its logical treatment and the plural modality of its mathematical treatment. In the case of (2), we shall demonstrate that the formalizing logical operation of "nominalization" to which Husserl appeals to account for the transformation of the plural judgment into an apophantic judgment directed toward a singular logical substrate *presupposes* that the pre-formalized unity of the plurality has already been accounted for. That is, nominalization does not resolve the problem of the logical unity of the collection that Husserl acknowledges gave rise to *Philosophy of Arithmetic*'s psychologism. In the case of (3), we shall demonstrate that Husserl's account of the relation to the experience of individual objects of the ontological meaning proper to the formal categories belonging to formal ontology is inconsistent with the formalized logical meaning of these categories. By appealing to an abstraction that takes its point of departure from the individual object, Husserl's analyses in *Formal and Transcendental Logic* and elsewhere are incapable of accounting for the formalization in which the materially empty concept of 'any object whatever' or 'anything whatever' originates.

Chapter Twenty-nine

Husserl's Critique of Symbolic Calculation in His Schröder Review

§ 147. The Applicability to Mathematics of Husserl's Critique of the Symbolic Calculational Technique in Logic

Husserl's discussion of symbolic calculation in the Schröder review is guided by his critique of the author's presentation of an algebraic calculus *as* logic, and in fact as an "'exact' logic" (*Schröder*, 20/68). Nevertheless, it is clear that Husserl considers his critique of Schröder's mistaken, and indeed self-deceptive[61] approach to the relationship between logical *thinking* and calculational technique to be applicable, mutatis mutandis, to the relationship between mathematical thinking and calculational technique in mathematics. Husserl buttresses his main critical point here as follows: "The logical calculus is, thus, a calculus of pure deduction; but it is not *its* logic. In it we have its logic as little as the *arithmetica universalis*, which spans the whole domain of numbers, is a *logic* of that domain" (8/57). When considering the cognitive value of the logical calculus itself, Husserl argues that the failure of its practitioners to properly understand its scope and limits is no more reason to reject it than there is reason to reject universal arithmetic, for "the most gifted of its [universal arithmetic's] representatives are, and always have been, far removed from a deeper grasp of its fundamental principles" (22/70). Indeed, he then points to a parallel between logical and mathematical calculation:

> those who have bravely worked on through the [logical] calculus and familiarized themselves to some degree with its technique, will find it hard to deny that it performs a function similar to that of the arithmetical calculus in the domain

61. Husserl writes that "one can hardly deceive himself more about his true goal than the author does here" (*Schröder*, 5/54), and explains that he "will go into the details of this deception" because "it is characteristic of the whole of extensional logic."

> of number—even though one not so grand in scope. One can hardly deny that the calculus puts at our disposal methods which, in the restricted domain accessible to it, spare us extremely complicated and laborious deductions, and achieve their results more quickly, more certainly, and above all more easily—presupposing, of course, a full mastery of the calculus! (22/70–71)

The conclusion Husserl draws from one of the two main points of his review—that the symbolic calculus is not a true logic but rather a time-saving technique—is also applicable, on his view, to the symbolic calculus operative in mathematics. His point that the symbolic calculation belonging to the logical calculus substitutes "a rule-governed process of transposing and replacing signs with signs—for actual inferring" (21/69), and is therefore "an external surrogate for deduction" (8/56) that functions as a time-saving technique rather than as a true *logic*, is also applicable to the external logic operative in symbolic mathematics. Therefore, in both cases Husserl understands calculation as "a blind procedure with symbols, [which operates] according to mechanically reiterated rules for the transformation and transposition of the signs in the respective algorithm" (7/55–56).

§ 148. The Two Foci of Husserl's Schröder Review: Calculus Is Not Deduction and Extensional Logic Is Part of Intensional Logic

Turning to the details of Husserl's review, we note its two foci. The first is that "calculation is not deduction" (8/56). The second is that "insofar as extensional logic [*Logik des Umfangs*] has any sense and meaning at all—and thus not as a new *logic*, but rather as a special logical technique—it belongs totally within intensional logic [*Inhaltslogik*]" (20/68). Husserl bases his argument for the first point on his claim that there is an "essential difference between a *language* and an *algorithm*" (21/69)—the latter being understood as a step-by-step procedure for solving a problem in a finite number of steps. This difference means that the major presupposition of Schröder's argument for presenting logic as an algebra and, as such, as an exact logic in contrast to the old logic, that it "invents an *artificial* language" (20/68) free of the ambiguities that plague *natural* language, is utterly unfounded. Husserl bases his argument for the second point on his claim that "every judgment about extension is truly a judgment about content" (19/68) and therefore "that when doing extensional logic we yet stand within intensional logic, or subordinate to it" (20/68).

Husserl's argument that calculation is not deduction is embedded in a deeper and farther-reaching set of claims about the nature and scope of

logic itself. Although these claims are neither explored nor substantiated in the review, they are nevertheless significant because they touch on the problematic nature of the relationship between logic and mathematics, a relationship that Husserl's mature self-understanding considered all but resolved, save for the details, in the ontological delimitation of the pure *mathesis universalis* in the *Logical Investigations*.[62] Husserl's claims regarding the nature and scope of logic are situated within the context of his critique of Schröder's characterization of the "essential elements of deductive logic" (5/54). This critique takes Schröder to task for: 1) confusing "a mere partial domain" within deductive logic, the domain of "pure deduction" (6/55), "with deductive logic itself"; 2) failing to realize that "logic, *as* the universal theory of deduction," includes more than the activity of deduction; specifically, it includes "the theory of all those mental activities which, though not themselves deductions, yet serve in the derivation of scientific truths" (7/56), mental activities that are therefore part of the "logical activities which are used in all deductive disciplines and which constitute the essence of their methods" (6/55); and, finally, 3) falling short of his goal of "presenting a *logic* of pure deduction alone" (7/56), but instead presenting "no more and no less than a *calculus* of pure deduction."

§ 149. The Calculus of Pure Deduction Is Neither Deduction per se Nor Its Logic

The cumulative result of these three shortcomings, according to Husserl, is that "the 'laws' of the calculus" (8/57) "are also nothing less than they are the norms of all 'valid thinking,' or, more precisely, of inference conforming to pure implications." As norms, these "laws are not rules to which everyone does and must conform, so far as they infer correctly." In marked contrast to this, "they are only rules which one *can* follow on any occasion, with full confidence of a correct result." Deciding the issue of "whether the deduction was a correct one," however, "is something which they cannot decide," and they cannot do so because "the logical *theory* of deduction" (9/57) is "not even touched upon" by the "algorithmic formulae" (8/57) that present these rules. In short, "the canon for the deductive activities involved in knowing" is "in nowise mirrored" in the logical calculus of formal deduction.

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62. Illustrative of this is Husserl's quoting at length in *Formal and Transcendental Logic* his characterization in the *Logical Investigations* of "the strict characterization of the idea of a formal theory of theory forms—correlatively, a formal theory of multiplicities" (*FTL*, 79), the latter being the idea of what he subsequent to the *Investigations* calls a formal ontology, because "I cannot improve upon it."

Husserl characterizes the mere partial domain in Schröder's misunderstanding of the scope of deductive logic as "none other than that of *purely formal deduction*" (6/55). Husserl writes:

> More precisely stated, it is the domain of pure implications between any judgments whatsoever—in the case of which, *as* pure, there is no consideration of the peculiar characteristics of the contents of the judged terms. Because of this we have, in the concrete case, the possibility of replacing the terms with general signs, of inferring according to the general schema, and only then replacing the signs by their specific meanings. Whatever can be deduced from any system of given premisses on the basis of their mere "form" falls into this domain—but nothing more.

Husserl's critical concern is clearly not with the formalization of deduction described here but with what he presents as Schröder's restriction of the entire domain of logic to the "pure" deduction that follows from this formalization. Moreover, Husserl takes Schröder to task for failing to provide "a *logic* of pure deduction" (7/56) to which he reduces logic's entire domain. What he provides instead, on Husserl's view, is "a *calculus* of pure deduction," which Husserl characterizes as "a device for making deduction *superfluous*." That is because the *calculus*

> is nothing other than a technique for manipulating signs: one which, by its system of rules, makes it possible, given a fitting symbolization of the premises (their number and complexity may, moreover, be ever so great), to arrive at the totality of pure deductions invested in those premises, or at their symbolic correlates. But calculation is no deduction. Rather, it is an external surrogate for deduction. (8/56)

Being an *external* surrogate for deduction, "the 'algebra of logic'" (8/57) does not address "the hard questions about the essence and logical justification of the calculative method" (8/56–57). It does not do so because

> the *logic* of this algebraic calculus does not fall within the mental horizon of the investigators who take the calculus to just *be* deductive logic—especially since, in fact, the mental operations upon which the calculus is based do not themselves belong within that domain of pure deductions which it exclusively governs. (8/57)

According to Husserl, Schröder's mistake—and that of the formalists who, like him, reduce logic to formal or pure deduction and then reduce, in turn, pure deduction to an algebraic algorithm (in the guise of a symbolic calculus)—is rooted in the mistaken view that the symbolic calculus is a *language*, albeit an artificial one. Invented by the "new logic" (20/68), Schröder and others present the symbolic calculus as an exact logic, because the signs it employs do not suffer from the ambiguities of *natural* language, of which the old logic availed itself and which therefore was responsible for its inex-

actitude. However, for Husserl the signs at issue in the symbolic calculus are manifestly *not* linguistic: "the correlation of symbols and thoughts set up at the outset" (21/69) of the process of substituting a calculus for actual logical thinking does not *express* these thoughts, "For the function of the sign here absolutely is not to accompany the *thought as its expression*" (21/69–70). Not being expressive, the signs that function as the symbols belonging to the calculus do not fulfill the "peculiar function of *language*" (21/69), which "consists in the symbolic expression of mental phenomena." Thus, "A language is not a symbolic method for the systematic derivation of conclusions, and a calculus is not a symbolic method for the systematic expression of mental phenomena." Indeed, Husserl argues, "The art corresponding to linguistic designation is grammar" (21/69), which "does not teach us how we should judge, and it also does not give rules concerning how we can derive correct judgments indirectly, through symbolic devices. Rather it only teaches us how we are to express judgments correctly in language." The implication here is clearly that the art that teaches us how we should judge—namely, logic—is not grammar and therefore *not* linguistic. What exactly logic is, however, Husserl does not say explicitly in the Schröder review; what he does say is what it is *not*: it is identical with neither deduction nor pure deduction; it does not coincide with the symbolic calculus; it is neither a language nor its grammar and therefore it is the art of neither natural nor artificial language. What Husserl does express explicitly about logic is that extensional logic is either part of or subordinate to intensional logic. We shall now take up his arguments in support of this claim.

§ 150. Extensional Logic's Subordination to Intensional Logic

Husserl severely criticizes Schröder's interpretation of the algebra belonging to the new logic as a calculus of *classes*, or as an extensional logic, on the grounds that "every judgment about extensions is truly a judgment of content" (19/68). This means that such judgments cannot be separated—as Schröder claims—from intensional logic and its function of providing "the specification" (16/64) of a concept's "content." Husserl elaborates his argument for this as follows: "If we judge that class *A* is contained in class *B*, then it is affirmed that the object of the concept *class A* is an object of the concept *contained in class B*. Now we could also replace these concepts by their extensions, and form the relevant judgments. But the same point is to be made in turn about these class judgments. And so on *in infinitum*" (19/68). Husserl's point here is that the judgments about a class itself can refer only to its object, the class, insofar as it is the specification of the content of the *con-*

cept 'class'. As a consequence, Husserl concludes, "so little is it true that the logic of extension is to be treated independently of the logic of intension, and so little is the former 'fixed untroubled by the latter' [as Schröder maintains], that when doing extensional logic we yet stand within intensional logic, or subordinate to it." Moreover, "what is meant, and alone can be meant" (16/64), by "the definition of a concept by means of its extension" is "an indirect definition of the conceptual content to be defined by means of another conceptual content corresponding equivalently to the first in virtue of having the same extension." Husserl maintains that this suffices "to make it known that the ideal of an 'extensional logic,' i.e., a logic which in principle considers *only* extensions of concepts, is futile, because it is objectless."

It should also be noted that because Husserl considers "classes by themselves" (15/63) to be "nothing other than collectivities," a close relationship between extensional logic and mathematics obtains, insofar as one understands mathematics—as Husserl does—to have as its subject matter collectivities (multitudes, cardinal numbers, manifolds, and so on). Indeed, because "only insofar as they are collectivities does the calculus consider them [i.e., classes]," it follows "that the calculus in its *universal* conception is to be designated as a universal calculus of collectivities, from which the class calculus proceeds only by means of the special interpretation of the collectivities as extensions of concepts." And, as we shall see, just as Husserl understands the extensional logic yielded by the special interpretation of the universal calculus "as a special logical technique" (20/68) that, as such, "belongs totally within intensional logic," so too will he understand (in his subsequent works) the universal calculus of collectivities (i.e., symbolic mathematics) as something that belongs totally *within* the pure *mathesis universalis*. Finally, we shall also see that by characterizing the *mathesis universalis* precisely as the *correlation* of formal logic and formal ontology, Husserl consciously brings together, under the heading of the general object belonging to the formal category 'anything whatever', the subject matters of mathematics and formal logic. Or, more precisely, we shall see that he characterizes the formal disciplines of mathematics and logic as investigating and therefore referring to the same universal object.

Chapter Thirty

The Separation of Logic from Symbolic Calculation in Husserl's Later Works

§ 151. The Contrast of the Schröder Review's Separation of Logic from Symbolic Calculation with Their Status in *Philosophy of Arithmetic*

Husserl's characterization in the Schröder review of the symbolic thinking operative in mathematics goes beyond his account in *Philosophy of Arithmetic*, and does so in a manner that is consistent with what he says about these matters in the "Introduction to the *Logical Investigations*" of 1913. The concept of an external logic that is applicable to "merely symbolic" thinking is totally absent from *Philosophy of Arithmetic*. The question of the logical foundation of signitively symbolic calculation is left unresolved at the conclusion of the latter work.[63] More precisely, the question left unresolved there concerns which of the two actual number concepts operative in Husserl's investigations—the cardinal number concept or the normative systematic number concept—the signitively symbolic number concepts[64] "are the logically qualified stand-ins for" (*PA*, 272). The assumption behind the very way in which this question is posed, which presupposes that there is a *logical* relationship between signitively symbolic numbers and the number concepts for which they function as surrogates, is precisely what Husserl rules out in the Schröder review. His argument that the symbolic calculus, as an "external surrogate for deduction" (*Schröder*, 8/56), "is not *its* logic" (8/57) is decisive on this point. It is not its logic because it does not touch the "logical *theory* of deduction." Strictly speaking, then, Husserl concludes in the Schröder review that the relationship between the symbolic calcu-

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63. See Part III, §§ 51–52.

64. Strictly speaking, the rule-governed numeral signs that compose signitively symbolic numbers are *not* concepts for Husserl.

lus—logical or mathematical, on this point it makes no difference—and the actual thinking and concepts for which it is the surrogate *is not logical*. Therefore, in contrast to *Philosophy of Arithmetic*, "merely symbolic thinking" in the Schröder review is *not* the *logically* qualified stand-in for the thinking and concepts for which its rule-governed technique substitutes a calculation process, and it is not such because its "logic" is in effect *external* to the thinking and concepts for which it acts as a surrogate.

This understanding of the logical status of the symbolic calculus, however, does not lead Husserl to reject as logically irrelevant what, in the "Introduction to the *Logical Investigations*," he refers to as its "so-called external logic." On the contrary, as we have seen, he applauds—in so many words—the "immense importance for consciousness" (*ILI*, 127) of the calculus insofar as it "puts at our disposal methods which, in the restricted domain accessible to it, spare us extremely complicated and laborious deductions, and achieve their results more quickly, more certainly, and above all more easily" (*Schröder*, 23/71). This last point, of course, is something that Husserl's arguments in the Schröder review arrive at on the basis of what he characterizes in the 1913 "Introduction" as the ability to "understand theoretically" (*ILI*, 127) the immense importance for consciousness of the external logic of *merely* symbolic thinking in mathematics. It should be noted, however, that this theoretical understanding is something that (in the "Introduction to the *Logical Investigations*") he situates within the context of the "mystery" belonging to the question how symbolic thinking in mathematics is possible, the question of how mathematical and logical connections can be constituted in subjectivity and yet, through its psychic medium, give mathematical objects that are objectively valid.

§ 152. The Consistency of the *Crisis* and the Schröder Review regarding Symbolic Calculation

Husserl's major discussions of symbolic mathematics subsequent to the Schröder review are found in the *Logical Investigations*, *Formal and Transcendental Logic*, and the *Crisis*. The strongest evidence that his mature thought on the logic of symbolic mathematics remains consistent with that in the Schröder review is the fact that Husserl's last work characterizes the symbolic calculus operative in algebraic arithmetic as a calculating technique that excludes original thinking and genuine meaning. Thus, in the *Crisis*, Husserl characterizes "the already formal but limited algebraic arithmetic" (*Crisis*, 121/46) as "a sort of *technique*." By this he means nothing other than what he meant in the Schröder review when he characterized signitively symbolic calculation as a technique, as the following makes obvious:

> Like arithmetic itself, in technically developing its methodology it [i.e., algebraic arithmetic] is drawn into a process of transformation, through which it becomes a sort of *technique*; that is, it becomes a mere art of achieving, through a calculating technique according to technical rules, results the genuine sense of whose truth can be secured only in a thinking that is both directed to the subject matter itself and actually skilled in achieving insight into it. Merely those modes of thought and evidence are now in action, which are indispensable for a technique as such. One operates with letters and signs for connections and relations (+, ×, =, etc.), combining and ordering them together according to *the rules of the game*, in a manner that, in fact, is essentially no different from a game of cards or chess. Here the *original* thinking, which gives this technical procedure its authentic meaning and the rule-governed results their truth . . . is excluded.

This passage contains the two hallmarks of his discussion of merely symbolic calculation in the Schröder review, namely, 1) understanding its procedure in terms of a rule-governed process that, ipso facto, 2) excludes the original thinking that establishes both the rules' and the game's cognitive legitimacy. Moreover, Husserl also considers both the formalization and technization involved here to be epistemically legitimate, just as he does in the Schröder review. He thus grants that "the process whereby material mathematics is put into formal-logical form, and formal logic, in turn, expanded and made self-sufficient as pure analysis or the theory of manifolds, is perfectly *legitimate*, indeed necessary, as is the technization with which from time to time the entire procedure completely loses itself in a merely technical thinking" (122/47).

§ 153. The Consistency of *Formal and Transcendental Logic* and the Schröder Review regarding Symbolic Calculation

The Schröder review's approach to symbolic calculation is likewise evident in *Formal and Transcendental Logic*, where, under the heading of § 33, "Actual Formal Mathematics and the Rules of the Game," Husserl discusses both "[t]he danger of becoming lost in an excessive symbolism" (*FTL*, 86) and the "only" way to avoid it. He says that the only way to avoid becoming lost thereby is to realize that if "one builds only a *discipline comprising deductive games with symbols*" (87), then it "does not become an actual theory of multiplicities until one regards the game-symbols as signs for actual objects of thinking—units, sets, multiplicities—and bestows on the rules of the game the significance of *law-forms* applying to these multiplicities." In evidence here is clearly the framework of the Schröder review, whereby the rules of the game belonging to the calculus characteristic of merely symbolic thinking acquire their cognitive legitimacy solely on the basis of a thinking that is "actual," actual in the sense that it is directed to and governed by insight into the genuine subject matter of mathematics and not "its symbolic analog" (88).

§ 154. The Consistency of the *Logical Investigations* with the Schröder Review's Framework for Addressing Symbolic Calculation

Husserl's most sustained discussion in the *Logical Investigations* of "symbolic-arithmetical thought and calculation" (*LI*, 75/305) and "'merely symbolic' mathematical thought" generally occurs in § 20 of the First Investigation, where he radically distinguishes the symbolic character of non-intuitive mathematical thought from the symbolic character of non-intuitive, non-mathematical thought. He writes in connection with this distinction that he is "only concerned to remove confusions readily caused by misunderstanding" two different kinds of symbolic thinking. There is 1) the "[n]on-intuitive symbolic thought" belonging to "most of the range both of ordinary, relaxed thought and the strict thought of science," which is "'merely symbolic'" (73/304) in the sense that it is not accompanied by "illustrative imagery" of its objects. And then there is 2) the non-intuitive thought of mathematics, which is symbolic in the sense that it "employs surrogative operational concepts." Husserl is very clear here that he thinks the symbolic status of each of these types of non-intuitive thought "are two quite different things" (75/305). He is likewise very clear that his removal of the confusion that results from not distinguishing these two types of thought in no way amounts to the "logical justification" and determination of the limits of the merely symbolic mathematical procedure. He explicitly raises the question of whether the account of the symbolic character of non-intuitive, non-mathematical thought, which leads to the distinction between mathematical and non-mathematical kinds of symbolic thinking, "conflicts with quite certain facts involved in the analysis of *arithmetical symbolic thought*, facts I myself have stressed elsewhere (in my *Philosophy of Arithmetic*)" (74/210). Regarding non-mathematical symbolic thought, he says that it "is quite inadequately described if one talks of a '*surrogative function of signs*,' as if the signs themselves did duty for something, and as if our interest in symbolic thinking were directed at the signs themselves" (73/210). This, of course, might appear to conflict with Husserl's contention in *Philosophy of Arithmetic* and the Schröder review that the signs belonging to signitively symbolic calculation function precisely as "surrogates" for systematically conceptual arithmetical thinking. However, Husserl stresses that there is no conflict because, "Looked at more closely, . . . it is not signs, in the mere sense of *physical* objects, whose theory, combination, etc., would be of the slightest use" (74/210) in arithmetical calculation. Rather, the "true meaning of the signs in question" is characterized here exactly as it is in the Schröder review, in terms of "their, so to speak, 'games-meaning,' a meaning orientated towards the game of calculation and its well-known rules"

(74/210–11). These "arithmetical signs" (74/211), when treated "as mere counters in the rule-sense," are what lead to "numerical signs or formulae whose interpretation in their original, truly arithmetical senses also represents the solution of corresponding arithmetical problems" (74–75/211).

§ 155. Husserl's Pre-phenomenological Appraisal of Merely Symbolic Thinking Precludes His Phenomenology from Pursuing Questions regarding the Essence and Logical Justification of the Calculative Method

Husserl's characterization of the "rules of the game" status of the merely symbolical thinking belonging to the algebraic calculus and his clear understanding, in the Schröder review and subsequently, of these rules as bereft of genuine logical meaning and actual logical thinking will prove to be extremely significant when considered in the context of our comparative assessment of his and Klein's thought on the origination of the logic of symbolic mathematics. What is at stake here is no minor philological matter, but rather the fact that Husserl regarded his early, pre-phenomenological appraisal of the external logic operative in the merely symbolic algebraic calculi of so-called symbolic logic and mathematics to have settled the matter. As Willard notes in his introduction to Husserl's *Early Writings* (xxv), Husserl "continued to regard the results of his early critiques of them [i.e., the 'formalists'] as so definitive that 'formalistic' issues are hardly mentioned in the *Logical Investigations* or elsewhere subsequently." To this we would add only that when he does mention them, it is clearly in the terms we have just considered.

Husserl's reliance on the results of his early critique has major implications for his *phenomenological* account of the origination of the logic of symbolic mathematics. Once he fixes the locus of *logic* as *outside* the rules of the game that govern the symbolic calculus, he no longer asks what he characterized in the Schröder review as "the hard questions about the essence and logical justification of the calculative method" (*Schröder*, 8/56–57). We have seen that these questions were precisely what guided his investigations and analyses in *Philosophy of Arithmetic*, and that in the Schröder review he still characterized them as "unavoidable" (8/57)—even as they remained unasked in it. Apparently, however, he no longer saw any need to raise these questions once he had satisfied himself that the true source of the logical legitimacy belonging to the calculational results of merely symbolic thinking stems from *neither* the signs themselves *nor* the operational rules for their combination and manipulation, but solely from the actual thinking and concepts that yield the *laws* for the combination forms proper to the formal *cat-*

egories to which these signs and rules ultimately lead back.[65] This means that the problem governing *Philosophy of Arithmetic*, namely, how symbolic number concepts are able to function as the "logically qualified stand-ins for" (*PA*, 272) the actual number concepts and therefore how the "parallel" between 1) the signs and rules for operating on these signs and 2) the systematic concepts and laws that establish the signs' and rules' cognitive legitimacy is *not* addressed in any of Husserl's subsequent works. In a word, *how* the "fitting symbolization" between signs, rules, and the categorial laws is established in the algebraic algorithms operative in the calculational procedures proper to mathematics and formal logic is not investigated in Husserl's conception of the phenomenology of the foundation of logic.[66] He is convinced that the logical status of these algorithms is *derivative* of and therefore founded in a mode of thinking that is not symbolic in the sense that the algebraic calculus is *merely* symbolic. It is therefore to this *more original* mode of symbolic thinking, a mode that he eventually no longer characterizes as symbolic but as "intentional," that Husserl devotes his phenomenological investigations into the foundation of pure logic.

§ 156. Husserl's Account of the Doubly Derivative Meaning of the Formal Nature of the Calculational Procedure Determinative of the Symbolic Calculus

Husserl's critique of the logical status of the symbolic calculus thus may be said to make an essential distinction between formal logic and the calculational procedure operative in merely symbolic thinking. According to this distinction, not only is the latter *not* the true locus of formal logic, but, strictly speaking, it is not *logical* at all—and therefore a fortiori not a *formal* logic. As a calculational procedure, the symbolic calculus is therefore "formal" in a doubly derivative sense. First, its procedure is limited to "the do-

65. That is, they lead back to these laws and categories insofar as they are their "fitting symbolization" (*Schröder*, 8/56).

66. In this connection, Carl Friedrich Gethmann notes that "Husserl's conception of foundation remains conspicuously unspecific with respect to formal logic"; see his "Hermeneutical Phenomenology and Logical Intuitionism: On Oskar Becker's *Mathematical Existence*," trans. Marcus Brainard, *New Yearbook for Phenomenology and Phenomenological Philosophy* III (2003), 143–60, here 150. In particular, "he does not apply the question of foundation to the instruments that precisely enable combination according to a calculus, namely propositional connectors and quantifiers." To this we would add that Husserl does not apply the foundational question to these instruments for the reasons we have just sketched, namely, because he considers a calculus *not* to *be* formal logic but rather a technique that substitutes a blind calculational procedure for the solving of formal deductions.

main of pure implications between any judgments whatsoever" (*Schröder*, 6/55), namely, to judgments that are formal in the sense that "there is no interest in the distinctive characteristics of the contents of the judged terms." Second, its application is restricted to the formal judgments about classes (i.e., extensional logic), which means that it is but a part, and a technical part at that, of the formal judgments about contents (i.e., intensional logic).

We shall see that Husserl's account of the logically derivative status of the symbolic calculus means that, in significant contrast to Klein, he understands formal logic, and therefore the formally derivative, external logic operative in *merely* symbolic mathematics, to have an origin apart from the symbolic calculus operative in either symbolic mathematics or so-called formal logic. Klein will be shown to hold the opposite view, that the origin of the formalization that makes both formal mathematics and formal logic possible is inseparable from the origin of the symbolic calculus. Thus, while both thinkers are in agreement regarding the materially indeterminate character proper to the "object" of formalized thinking, specifically its status as a "general object," their accounts of its origination differ sharply. In preparation for our assessment of their respective accounts of the origination of the logic of symbolic calculus, we shall now turn our attention to the development of Husserl's understanding, subsequent to *Philosophy of Arithmetic*, of the origin of this general object, his terms for which are the formal categories of 'anything whatever' and 'any object whatever'.

§ 157. Characterization of the Reference to the Empty Universe 'Any Object Whatever' of the Elements Belonging to Multitudes and the Units Belonging to Number

Husserl's mature logical investigations (in *Formal and Transcendental Logic*) trace "the idea of a universal science, a *formal mathematics in the fully comprehensive sense*, whose universal province is unshakably delimited as the domain of the highest form-concept, *any object whatever*—or the domain of anything whatever, conceived with the emptiest universality," to "the naturally broadest universality of the concepts of set and number." More precisely, he relates that it was his consideration of "the concepts of element and unit which respectively determine their meaning" which led to his recognition that "the theory of sets and the theory of cardinal numbers refer to the empty universe, *any object* or *anything whatever*."[67] Husserl characterizes the "for-

67. Husserl's text includes a footnote indicating that this "was already done in my *Philosophy of Arithmetic*" (*FTL*, 68 n. 1).

mal universality" of this domain as one that, "in principle, disregards every material determination of objects," and maintains that "if this mathematics is built up—after the fashion of the expositions in the *Logical Investigations*—within the total complex of the idea of a *logic*" (86), then "it appears as the *highest level of logical analytics*, founded on the essentially preceding lower level" of logic. Moreover, he regards it as "natural to view this whole mathematics as an *ontology* (an a priori theory of objects), though a formal one, relating to the pure modes of 'anything whatever'" (68). Indeed, Husserl maintains that "The formalness of these disciplines lies . . . in this relation to 'any objectivity whatever', 'anything whatever'" (76), and that it is precisely this "universal concept that should delimit the unitary province of these disciplines (which obviously belong together)" (68).

Husserl's articulation of the "idea" of "a full and entire *mathesis universalis*" (87) in *Formal and Transcendental Logic* is tied, then, to this account of "a great problem," namely, that of the "systematic order" involved in "building" what he characterizes as "a formal mathematics that does not float in the air but stands on its foundations and is inseparably one with them." This problem, which "is none other than the problem of a full and entire *logical analytics*," and the progress Husserl makes toward resolving it in the *Logic* will be discussed in greater detail below. Before doing so, however, it is necessary to consider precisely how Husserl understood the logical shortcomings of *Philosophy of Arithmetic*'s appeal to the "reflexion" on acts to account for the origin of logical relations to be overcome in his later works leading up to *Formal and Transcendental Logic*. Because our study is focused on his and Klein's accounts of the origination of the formal category 'general or universal object', we shall focus on Husserl's accounts of the origination of the cardinal number concepts and the domain of "the empty universe *anything* or *anything whatever*" (*FTL*, 68) subsequent to *Philosophy of Arithmetic*.

Chapter Thirty-one

Husserl on the Shortcomings of the Appeal to the "Reflexion" on Acts to Account for the Origin of Logical Relations in the Works Leading up to the *Logical Investigations*

§ 158. The Concept of Cardinal Number Is Essentially Different from the Concept of Collecting

Husserl's account of the development of his thought subsequent to *Philosophy of Arithmetic* zeroes in on what he presents as the shortcomings of the latter's appeal to "psychological reflexion in Brentano's sense" (*ILI*, 127) to account for "the unity of a collection," the "concept of unity" (127), and, finally, "the concept of cardinal number." Significantly, Husserl acknowledges that "The unity of a collection is no material unity, grounded in the collected items" (127) and that therefore there was "certainly something correct" in his view that "the presentation of a 'set' was supposed to arise out of the collective combination." However, he explicitly calls into question the ability of the "'reflexion' on the act of collecting" (127) to yield the *unity* of the set, because "from the reflexion on acts" of collecting "the concept of collecting . . . is all that can result"—not the concept of the unity of the collection. Indeed, Husserl reports that "doubts unsettled, even tormented" him "already in the very beginnings" with respect to the question of whether "the concept of cardinal number [is] not something essentially different from the concept of collecting," doubts that "then extended to all categorial concepts as I later called them and finally in another form to all concepts of objectivities of any sort whatever." It is important to note, for our purposes, that included in what Husserl reports that he was now calling into question is the account of the concept of unity as something that "arises from reflexion on the act of positing-as-anything."

Husserl confirms in the "Introduction to the *Logical Investigations*" what we established in our discussion of his Schröder review's abandonment of *Philosophy of Arithmetic*'s initial thesis of the logical equivalence of authentic and inauthentic (symbolic) presentations.[68] Thus, he writes: "The customary appeal in the Brentano school to inauthentic [symbolic] presentation, presentation through relations, could not help" to dispel his doubts because "that was only an expression in the place of a solution." He likewise calls attention to his view that the concept of the unity belonging to the collective combination is something that "takes its place among the fundamental forms of 'categorial' consciousness in the sense of the *Logical Investigations*" (126), albeit it does so in terms of "its contrast to these forms of unity."

§ 159. The Loss of the Contrast between Mathematically Collective Unity and Logically Categorial Unity in Husserl's Critical Response to Psychologism

The importance of this last point needs to be stressed because Husserl is here differentiating the categorial unity belonging to the concept of multitude, which he is now characterizing as "a basic form of the synthetically multi-rayed consciousness," from the basic forms of unity belonging to the pure logical categories of the *Logical Investigations*. This differentiation has significant implications for our critical assessment of his account of "the inner unity of formal logic with the pure theory of numbers" (130). Husserl's account of the unitary province of formal logic and formal mathematics is based on his premise that each, as a formal discipline, is related to the empty universality of any objectivity or any thing whatever. In our critical assessment, however, we shall argue that Husserl's account of the *difference* between 1) the formal unities belonging to multitudes and cardinal number concepts and 2) the formal unities belonging to the formal categories constituted in pure logical judgments fundamentally calls into question the possibility of realizing his idea of unifying pure mathematics and pure logic on the basis of their both being related to the formal objectivity proper to any objectivity or any thing whatever.[69] We

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68. See Part III, §§ 47–48.

69. See below, §§ 186–87, where we show that notwithstanding the standard interpretation of *Formal and Transcendental Logic*, which maintains that Husserl established in that work the unitary province proper to formal logic and formal mathematics, Husserl is well aware that his investigations in *Formal and Transcendental Logic* are provisional, such that, as he points out at one point, "the problem of the unity or difference of logical analytics [i.e., formal logic] and formal mathematics can by no means be regarded as already resolved" (*FTL*, 70).

mention this now because we are about to articulate the main lines of Husserl's self-critical response to *Philosophy of Arithmetic*'s attempt to account for the unities of multitudes, cardinal number concepts, and the positing-as-anything, in terms of reflexions on psychological acts. We want to point out that his primary concern in *Philosophy of Arithmetic* to account for the unities proper to multitudes and cardinal number concepts gets lost in Husserl's critical response to the inadequacy of such reflexions to account for any kind of logical unity. The radical distinction he draws between what is purely logical and what is psychological, a distinction made in order to overcome the psychologism inherent in *Philosophy of Arithmetic*'s psychological attempt to account for precisely these unities, is addressed only with respect to the independence of pure logical (categorial) unity from psychological acts—and not the independence of pure mathematical (collective) unity from such acts.

§ 160. Husserl's "Separation" of Logic and Mathematics from Psychologism

Husserl formulates the basis of his "separation" of logic and mathematics from psychologism in terms of the following distinction: "everything 'purely' logical is an 'in itself,' an 'ideal'; that which belongs to the proper essential content of this in itself contains nothing 'psychic,' no acts, nothing of the actuality of subjects or empirically factual persons at all" (113/20). This distinction itself is rooted in Husserl's claim that "it is evident that truths are what they are, and that, in particular, laws, grounds, principles are what they are, whether we have insight into them or not" (*LI*, 240/233). More precisely, his point here is that no part of the meaning and validity belonging to the *content* of what he calls the "[*i*]*deal* conditions for the possibility of knowledge" (239/232) is dependent on "the empirical peculiarity of human knowledge as psychologically conditioned" (240/233). This means that such conditions "cannot be valid insofar as we have insight into them," but rather "that we can only have insight into them insofar as they are valid."

However, Husserl does not understand the radical distinction made here, between "the thought-act on the one side and the thought-significance [*Denkbedeutung*] on the other" (*ILI*, 113), to rule out *any* relation between thinking and the purely logical in-itself. Not only is he convinced that "[s]omehow they necessarily belong together," but also the articulation and investigation of this belonging together becomes a major preoccupation of his phenomenological philosophy. Nevertheless, he is also convinced that before this relationship can be properly formulated, one has to feel "the embarrassment of the matter [i.e., psychologism] deeply and in the most in-

tense form possible” (114) and to see oneself “compelled by the critical dissolution of the blinding prejudices of psychologism to recognize the purely logical idealities.” Once the latter are recognized, the claim that “There corresponds to this unique field of existing objectivities a science, a ‘pure logic,’ which seeks knowledge exclusively related to these ideal objectivities, a knowledge that therefore forms its judgments on the basis of pure significations and signified objectivities as such (in completely pure and unconditioned universality)” (113), should likewise be recognized and, indeed, recognized as compelling.

§ 161. Husserl and the Systematic Development of the *Mathesis Universalis*

Husserl’s discussion of the separation of what is logical from what is psychological, both here and elsewhere in his works, oddly enough does not make explicit reference to the implications that the Platonism involved in this separation must have for the specific types of unity he had sought to account for psychologically in *Philosophy of Arithmetic*. Subsequent to Husserl’s “fully conscious and radical turn [from psychologism] and the ‘Platonism’[70] that is given with it” (128), the question of the *origination*—or, in his later terminology, the *constitution*—of the unity of a collection, the cardinal number concepts, and the concept of formal unity itself (the formal category of the *any object or objectivity whatever* or *anything whatever*), if it is addressed at all by him, is answered in a manner that elides the role attributed here to Platonism in his break with psychologism.[71] Husserl’s account of his development in the “Introduction to the *Logical Investigations*” provides an important clue as to why this became the case. He credits his study of Lotze’s logic, specifically “his inspired interpretation of Plato’s doctrine of Ideas” (129), as what “gave me my first big insight,” insofar as “Lotze spoke already of truths in themselves.” From this, “the thought suggested itself” to Husserl “to transfer all of the mathematical and a major part of traditional logic into the realm of the ideal.” Husserl then goes on to relate the following: “With regard to the logic that before I had interpreted psychologistically and which had perplexed me as a mathematical logician, I, thanks to a fortunate circumstance [i.e., his study of Lotze], no longer needed lengthy and detailed deliberations regard-

70. Throughout the “Introduction to the *Logical Investigations*,” Husserl embraces Paul Natorp’s account of pure logic as “‘the ideal in this truly Platonic sense’” (*ILI*, 113).

71. This claim is substantiated below with reference to Husserl’s discussion of these types of unity in the *Logical Investigations*, *Formal and Transcendental Logic*, and *Experience and Judgment*.

ing its separation from that which is psychological." Indeed, armed with the "recognition of this ideal sphere of being and knowledge" (113/20), Husserl turned his attention to "a task that lies in the same direction, namely, to determine the natural boundary of the logical-ideal sphere, and thus to grasp the idea of pure logic in its full scope." As we have already mentioned, Husserl identifies the subject matter of this pure logic as the universal object, the domain of the form-concept 'any object whatever', to which, among other things, the elements of the multitudes and the units of cardinal number concepts investigated in *Philosophy of Arithmetic* refer. Husserl's subsequent investigations of the scope of the idea of pure logic, as well as his investigations of "the 'being-in-itself' of the ideal sphere in its relation to consciousness" (114–15/22), are devoted to working out the *idea* that concludes the first volume of his *Logical Investigations*, the *Prolegomena to Pure Logic*. Specifically, that *because* formal mathematics and formal logic are both defined by their relation to the materially empty universe of the domain of the universal object, they belong together "as the systematic development of the *mathesis universalis*, which . . . was already anticipated by Leibniz" (113/20). In line with this, because formal logic, like formal mathematics, "refers to the formal idea of object, to the 'anything whatever'" (123/30–31), it too, like such mathematics, "can be characterized as formal ontology" (123/31).

§ 162. Husserl's Idea of Pure Logic and the Task of Working out the Theoretical Relationship between Formal Mathematics and Formal Logic

As a consequence of his characterization of the idea of pure logic, Husserl's concern with formal mathematics subsequent to his critique of psychologism in the *Prolegomena* was to work out the *theoretical* relationship between such mathematics and formal logic. By 'formal mathematics' he understands "all of the pure 'analytical' doctrines of mathematics (arithmetic, number theory, algebra, etc.) and the entire area of formal theories, or rather, the theory of manifolds[72] in the broadest sense" (121/28). And by 'formal logic' he understands "that particular concept of formal logic which remains as a residue of ideal doctrines dealing with 'propositions' and validity after the removal from traditional logic of all the psychological misunderstandings and the normative-practical goal positings." For our purposes, three developments in

72. By 'manifold' (*Mannigfalitigkeit* or *Mannigfalt*) Husserl understands a mathematical "object" composed entirely of purely formal relations governed by a finite number of axioms.

Husserl's thought concerning this theoretical relationship are of interest. First is the distinction made in the *Logical Investigations* between 'significance categories' (*Bedeutungskategorien*) and 'formal objective categories', the latter being understood as the correlates of the former. Second is the refinement of this distinction made in *Formal and Transcendental Logic* as a distinction that, strictly speaking, is no longer categorial, namely, the distinction between 'apophantic logic' and 'formal ontology'. (This distinction is likewise made in Husserl's understanding of their essential correlation.) And third is the inconclusive articulation in the *Logic* of the unitary province of 1) traditional formal mathematical analysis and 2) judgments in the traditional sense. By 'traditional mathematics' Husserl understands "the mathematics of multitudes and sets, of combinations and permutations, of cardinal numbers (the modes of how-many), of ordinal numbers belonging to various levels, of manifolds with their well-known forms" (*FTL*, 67), while he understands 'predicative propositions' as judgments in the traditional sense.

In what follows, we shall trace these three developments and show three things: a) that, beyond what we find in *Philosophy of Arithmetic*, Husserl does not significantly modify his account of the origin of the *unity* of multitudes and cardinal number concepts; b) that the form-concept of the universal object is said to "arise by virtue of formalization" (106), a process about which Husserl says very little, other than to characterize it as an "abstraction" from the material content of *the experience of something individual;* and c) that while (a) is rooted in the effects of Husserl's enduring Platonism, specifically his "transfer of all of the mathematical . . . to the realm of the ideal," (b) is rooted in Husserl's equally enduring Aristotelianism, namely, the characterization of the mode of being of mathematical unity as inseparable from individual beings or objects.

Chapter Thirty-two

Husserl's Attempt in the *Logical Investigations* to Establish a Relationship between "Mere" Thought and the "In Itself" of Pure Logical Validity by Appealing to Concrete, Universal, and Formalizing Modes of Abstraction and Categorial Intuition

§ 163. Husserl's Account in the *Logical Investigations* of the Fulfillment of Signification Categories in Formal Objective Categories: Categorial Intuition

Husserl's distinction in the *Logical Investigations* between 'significance categories' and 'formal objective categories' addresses the relation between thinking and the pure "in itself" of ideality that emerges as a result of his break with psychologism. This distinction represents Husserl's initial attempt to account for how it is that the "in itself" of ideality enters into a relationship with thinking by distinguishing between the "mere" thinking of such ideality and the state of affairs wherein the "in itself" of ideality is intuitively rendered present as a logically pure objectivity.

Mere thinking, which occurs "in symbolic acts" (*LI*, 566/694), acts whose symbolic quality Husserl maintains is the "significational essence of expressive acts," manifests what he calls an "intentional relation" (381/555). As intentional, these acts are "directed toward" something either sensibly or categorially objective in a manner that has "a priori precedence over empirical, psychological facticity." This precedence for Husserl is exhibited by what he calls the "unity of the descriptive genus 'intention' ('act-character')," where 'descriptive' points to the heart of his conception of 'pure phenomenology' in the *Investigations*. Husserl explains that pure phenomenology, in

contrast to empirical, psychological facticity, does not apprehend the "contingency, temporality and transience of our [psychic] acts" (175/181), but rather grasps, "in a purely descriptive understanding" (382/55), the "essential determination of 'psychical phenomena' or 'acts.'" Descriptive understanding occurs in what Husserl calls "ideation,"[73] which is a methodical procedure that compares examples of the immediate experience of something (in the case at hand, symbolic acts of thinking) in order to apprehend "the pure, phenomenologically generic idea" of these experiences as such. Such apprehension prescinds from the empirical content of the experiential examples in a manner that allows it to grasp that which, subsequent to this prescission, is inseparable from the compared experiences as such.

Thinking—or more precisely, the "categorial forms in predicating" (667/781), forms that Husserl characterizes as "the categorial forms of significations" (670/784) and that compose "propositional elements" (667/782)—belongs together with the "in itself" of ideality, what he refers to as "the object itself *in its categorial formation*" (671/785). Husserl characterizes the "belonging together" in the following manner. The "state of affairs supposed" (668/782) by a judgment, "[t]he object with these categorial forms" (671/785), is something that "is not merely referred to, as is the case where significations function merely symbolically, but it is set before our very eyes in just these forms." When unpacked, the metaphor 'to set before our very eyes' means that what is signitively referred to by the signification's categorial forms is "*self-given*, or at least putatively given, in the *fulfillment* which at times invests the judgment, the *becoming conscious* of the state of affairs supposed" (668/782). Thus, in cases of fulfillment, what is referred to by formal words, by propositions about forms of quantity and logical categories, is intuited in an intuition that terminates in *what* is referred to, namely, in the formal category *intended* by the signifying intention of the "mere" thinking that characterizes the formal words and propositions. Husserl characterizes judging as involving "not only signification intentions belonging to actual predications, but the fulfillments that in the end fit them completely" (668/783). Thus, "formal words such as 'the', 'a', 'some', 'many', 'few', 'two', 'is', 'not', 'which', 'and', 'or', etc." (657/774), along with propositions about "the forms of quantity and the determinations of cardinal numbers, etc." (667/782), and "the *logical categories* such as being and non-being, unity, plurality, totality, cardinal number, ground, consequence, etc."[74] (668/782), are

73. See § 165 below.

74. Husserl's initial inclusion of the word 'and', together with propositions about quantity, and logical categories such as 'plurality' and 'cardinal number', as examples, respectively,

all capable of being fulfilled. The function of their fulfillment "obliges us to give the name 'perception' to each fulfilling act of confirmatory self-presentation, to each fulfilling act whatever the name of an 'intuition,' and to its intentional correlate the name of '*object*'" (671/785). Husserl terms such intuition, which establishes "*the relation between the signification-intention and the signification-fulfillment*" (538/668), 'categorial intuition' in recognition of the nature of the object given in its fulfilling act.

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of formal words, signification intentions, and categorial intentions that are all capable of being intuitively fulfilled, is very misleading. Most commentators have interpreted this to mean that the structure of the intuitive fulfillment of these examples is identical (or, at the very least, structurally similar) to that of the examples of the logical words and logical categories with which they are included. Thus, in the case most relevant to our concerns, the intuitive fulfillment of the formal word "and" is interpreted to obtain along the lines of the fulfillment of a (logically) categorial signification intention in a (logically) categorial objectivity, which, as we have just seen (see also § 166 below), is how Husserl characterizes the complete structure of logical judgments in the *Logical Investigations*.

For examples of this interpretation, see Barry Smith, who treats "'the conjunctive connection *A and B and C and*...'" on a par with "those categorial acts in which we move from some sensible, material object to the corresponding material species or universal" ("Logic and Formal Ontology," in J. N. Mohanty and William R. McKenna, eds., *Husserl's Phenomenology: A Textbook* [Washington, D.C.: Center for Advanced Research in Phenomenology and University Press of America, 1989], 29–67, here 50), and Robert Sokolowski, who with respect to the conjunction 'A and B' distinguishes between its being "only emptily, signitively, verbally meant" and its being "intuitively meant," in a manner that treats 'A and B' as a "categorial object" (*Husserlian Meditations*, 38). For a notable exception to this interpretative tendency, see Dieter Lohmar, who, in connection with precisely the issue we are raising, notes that for Husserl "collectiva are not 'states of affairs' (*nicht selbst Sachverhalte*)" ("Husserl's Concept of Categorial Intuition," in Dan Zahavi and Frederik Stjernfelt, eds., *One Hundred Years of Phenomenology* [Dordrecht: Kluwer, 2002], 125–45, here 145). See also n. 86 below for a discussion of his account of the peculiar—when compared with that of "states of affairs"—character of the relationship between intention and fulfillment in the case of the "synthetic categorial intention 'and' itself" (144).

What is overlooked in this line of interpretation, however, is that later in the *Logical Investigations* (and, subsequently, in *Experience and Judgment* and *Formal and Transcendental Logic*) Husserl makes a sharp distinction between the categorial status of the collection as such and that of the "state of affairs" (see § 168 below). A consequence of overlooking this is that one fails to recognize the inapplicability of the "signification act intention–signification act fulfillment" judgment structure to the manner in which the act of collection means the unity of the collection as such. This distinction itself is indicated by Husserl's account of the different logical functions proper to the word 'is' (the copula) (see § 169 below) in the state of affairs and the word 'and' in the collection. After his initial, "only superficially indicated and very roughly characterized" (673/786) account of categorial intuition in the *Logical Investigations*, which lumps *all* formal categories under the rubric of the fulfillment structure of judgments, Husserl begins to address the implications of the difference between the intentional unity of the collection as such and that of the "state of affairs" on the respective modes of their intentional fulfillment. Indeed, while Husserl is quite clear that the latter arises as the fulfillment of a significative intention or intentions, we shall endeavor to show that it is not at all clear in the *Investigations* that for him the intentional unity of the collection as such arises similarly (see § 169) and that it is quite clear in his later logical works (see below §§ 173–74) that the unities in question do *not* arise in a similar manner at all.

§ 164. Formal Categories Comprising the "State of Affairs" Are *Not* in Acts of Judgment as Objects of Reflexion But Are the Objects of These Acts

Husserl introduces "the concept of state of affairs" (669/783) to characterize "the objective correlate of the complete judgment," or, more precisely, the categorially objective correlate. In marked contrast to his account of the origin of formal categories in *Philosophy of Arithmetic*, in the *Logical Investigations* Husserl maintains that "the concept of state of affairs cannot arise out of reflexion on judgments, since this could only yield us concepts of judgment or of real constituents of judgments." The "abstractive foundation which allows us to realize the concepts in question" (670/784), namely, the formal categories that compose the "state of affairs," is therefore "[n]ot in these *acts* [of judgment] *as objects* [of reflexion], but in the *objects of these acts*" (670/783).

The precise status and character of the "objects" of the acts of judgment, however, is not exactly easy to discern on the basis of Husserl's presentation of categorial intuition in the *Investigations*, in part because he characterizes it as sharing the "essential homogeneity of the function of fulfillment" (671/785) that is exhibited by "*the sensuous concept of perception*" (672/785). Regarding the act of judgment, Husserl maintains that "It falls within the great class of acts whose peculiarity it is that in them something appears as 'actual,' as 'self-given.'" But when Husserl's investigation moves beyond what he himself characterizes as so far an "only superficially indicated and very roughly characterized" (673/786) account of the "distinction between 'sensuous' and 'supersensuous' perceptions," the strict analogy between the two types of perception breaks down. Thus, while "in *sense* perception, the 'external' thing appears in 'one blow,' as soon as our regard falls upon it" (676/788), in categorial acts the categorial element is either included in or presupposes an act or acts of straightforward perception. Categorial acts are therefore "founded acts" (681/792), and as such they manifest "quite different acts in which concretely determinate states of affairs, collections[75] and disjunctions are given as complex thought-objects, or as objects of higher order, *which include their foundational objects as really intrinsic* [reell] *parts in themselves*" (676/788). More exactly, only in one set of categorial acts is this the case, because for Husserl there is "*another set of categorial acts*, in which the objects of the founding acts do not *enter into* the intention of the founded one, and would only reveal their close relation to it in relational acts" (690/799). These latter acts present what Husserl calls "the region of *universal intuition*" (690/800).

75. But see § 168 below, where we consider Husserl's sharp distinction between the categorial character of collections and states of affairs.

§ 165. Different Abstractive Modes of Categorial Intuition: Concrete and Universal Intuition of Categories

The categorial acts involved in universal intuition differ from those involved in the concrete intuition of categories with respect to their mode of abstraction. Concrete acts are abstract "merely in the sense of a setting-in-relief of some non-independent moment in a sensible object," which is a process that takes place when the same object apprehended in a "straightforward" manner is "grasped by us in explicating fashion" (681/792). By contrast, Husserl characterizes universal intuition in terms of categorial acts whose mode of abstraction is "ideational" (690/800), which, in distinction to the concrete intuition of categories, yields "no such non-independent moment," but instead "brings to consciousness, to *actual givenness*" this moment's "idea, its universal." Thus, whereas the "acts of articulation" (681/792) belonging to the abstractive explication and grasping of the sensible object are "so founded upon straightforward perception that the *synthetic intention was subsidiarily directed to the objects of the founding perceptions*, inasmuch as it held them together in ideal 'contents' or brought them to a relational unity" (690/799), in ideational abstraction "a new categorial act-character emerges, in which a new style of objectivity becomes apparent" (690/800).

The categorial acts of ideational abstraction, however, are also "founded," albeit on a different foundation than that which founds the abstractive acts of concrete categorial intuition. Instead of being directly founded on straightforwardly apprehended sensible objects, ideational abstraction is directly founded on the abstractive acts of concrete categorial intuition, which are founded, in turn, on the straightforward apprehension of these objects. Hence, the acts of ideational abstraction are mediately founded on the perception of sensible objects and immediately founded on the acts of concrete categorial intuition. The abstractive "setting in relief" of "the manifold single moments of 'one and the same species'" (691/800) or formal category that characterizes concrete categorial intuition becomes the basis for the "new act-forms" of ideational abstraction. Husserl characterizes these new acts as "acts of universal determination, acts, that is, which determine objects *universally* as subsumed under a certain species *A*, or acts in which *undetermined* single objects *of a species A* become present to us." He maintains, "We must presuppose such an act in order that this species itself" (690/800), as something that is "able to stand before our eyes *as one and the same*," can be intuited in contrast to the manifold moments of it intuited by the "single acts of abstraction" characteristic of concrete categorial intuition.

Ideational abstraction, then, "operates on the ground of primary intuitions" of the universality characteristic of categorial forms and "becomes conscious of the identity of the universal" (691/800), an identity that is manifest "in an overarching act of identification which brings all such single acts of abstraction into one synthesis." Therefore, "In an act of abstraction, which need not involve the use of an abstract name, the universal *itself is given to us*." It is so given not "in a merely significative manner, as in cases of merely understanding universal names, but we grasp *it*, *it appears* to us."

§ 166. Husserl's Admission That Neither Concrete Nor Universal Categorial Intuition Is Independent of All Sensible Relations

Husserl's account of ideational abstraction, and not simply his account of concrete categorial intuition, represents his response both to the logical shortcomings of his earlier attempt to establish the origin of categorial concepts by reflexion on psychic acts and also to the problem of establishing the relation of thinking to the "in itself" of pure logical objectivity. The mere abstractive setting-in-relief of non-independent moments of the sensible object, in which the categorial object is self-given as the fulfillment of the significational intentions of propositional judgments, does *not* yield the pure logical category at all but only a single moment of it—what Husserl elsewhere calls its "instance" (174/180). Indeed, in whatever manner these moments or instances are understood, this much at least is clear about them: their apprehension, being inseparable from the straightforward sensuously perceptual acts and perceptions on which they are founded, does *not* yield what Husserl's initial exposition of them claimed they yielded, namely, the formal categories *themselves* independent of all sensible relations—*and this by Husserl's own admission*.[76] Ideational abstraction, to which Husserl, fol-

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76. In this connection, it is interesting to note that in Husserl's account of the development of his thought in his "Introduction to the *Logical Investigations*" he mentions that "the radical conquest of 'psychologism' in its most basic and universal form" (*ILI*, 338) was "achieved" (by himself) only "around 1908," when "the important insight was gained that *a distinction between transcendental phenomenology and rational psychology* has to be made."

See John Drummond's excellent discussion of this issue in "The *Logical Investigations*: Paving the Way to a Transcendental Logic," in Zahavi and Stjernfelt, eds., *One Hundred Years of Phenomenology*, 31–40. As Drummond puts it, Husserl's attempt in the *Logical Investigations* to avoid psychologism, by making certain contents of the concrete act "the instantiation of an essence, a meaning-species" (35), involves "some sleight of hand" (36) that precludes it from solving the "problem of accounting *phenomenologically* for the origin of ideal meanings." Drummond articulates the sleight of hand in terms of Husserl's distinction—made in the first edition of the *Investigations* but abandoned in the second—between 1) the "really intrinsic"

(*reell*) contents of concrete acts, namely, "the fact of being directed to a particular object [i.e., the 'matter' of the act] in a determinate manner [i.e., the 'quality' of the act] with a particular 'how' of presentation [i.e., the 'presentational content' of the act, its difference between 'empty' signification and 'full' intuition]" and 2) the "*intentional* contents." When this distinction is in force, only the former, which Husserl characterizes as the really intrinsic, he identifies "psychological contents" (35) as "the proper object of phenomenological description," and because of this Drummond shows that Husserl's appeal to the species content of the latter "to avoid psychologism—carries us beyond the phenomenological contents of the act and cannot be included in a phenomenological description" (36). Drummond clearly lays out what makes the manner of Husserl's appeal here one of "instantiation," namely, Husserl's understanding of "the identity of a species" as "[t]he identity of meaning among many acts intending the same object in the same way." What establishes this appeal as an appeal that extends beyond the phenomenological contents of an act is Husserl's having restricted phenomenological description to the act's "really intrinsic" contents, which "by hypothesis" are *not* intentional. Hence, if this restriction is adhered to, the object and ideal status proper to "[t]he meaning-species appears as a *deus ex machina*." This is because the psychological restriction Husserl places on phenomenological description prevents him from using such description to account for the very *identity* that needs to be established—if the psychological relativity of the really intrinsic contents of concrete acts is to be overcome in a manner that would insure their *intentional* relation to the ideal meaning of the species. The identity in question here, as that of certain moments belonging to the really intrinsic contents of concrete acts, an identity that, as the identical content of more than one *psychological* act, is supposed to establish (via this very identity) *the identity* proper to the ideal intentional content of the species-meaning, therefore *presupposes* "the notion of intentional essence." In other words, "To avoid psychologism, the so-called *phenomenological* description must appeal to the (not descriptively contained) intentional essence precisely because the essence has priority over its instantiation, but this appeal is, by hypothesis, barred."

Drummond identifies Husserl's inclusion (in the second edition of the *Investigations*) of the intentional contents *within* the act's phenomenological content as the manner in which the *Investigations* pave the way to a transcendental logic. Drummond sees in this Husserl's implicit recognition that the "intentional relation of consciousness to the world" (38) is "a whole comprising non-independent parts, some of which are really intrinsic and some of which are intentional," and maintains that this recognition "leads to a fundamental change in Husserl's account of intentionality and meaning." Drummond's articulation of this change as something that "Husserl explicitly acknowledges" (38) in *Formal and Transcendental Logic*, namely, that there is a distinction between "the ir-reality of meaning" (39) and that of "the ideality of a species," such that "Husserl can distinguish the identity of meaning from the identity of a species," will be considered below (see § 188) in our discussion of that text.

For our present purpose, it needs to be pointed out that we support our claim that Husserl recognized the dependence of concrete and universal categorial intuition on sensible relations by appealing to his account of the objective self-givennesses of the concrete and universal categories to which these modes of categorial intuition refer. As we have seen (§ 162), Husserl appeals to different modes of abstraction in order to account for the appearance to us, as actual, of the single moments of the species, as well as the species themselves, in a manner that is not merely significative. In his account of both of these modes of abstraction, he is quite keen to emphasize that the objective status of either the single moment of the species or the species itself is *not* based in an identity of the really intrinsic contents of acts but in the objects of these acts. Thus, in the case of concretely determinate categorial objects, as well as categorial objects of higher order, Husserl stresses that it is "*their foundational objects*" (*LI*, 676/788)—and therefore (presumably) *not* their foundational *acts*—that are included "*as really intrinsic parts in themselves*." Husserl's recognition that categorial objects include sensible

lowing this admission, appeals as being able to yield such "pure" logical concepts, nevertheless also appears to fall short of such an accomplishment, because, as he presents it, even this kind of abstraction is still founded—albeit *mediately*—on sense perceptions.

§ 167. Formalizing Abstraction as Distinct from Both Abstraction as Emphasis on Non-independent Moments and Ideating Abstraction

There is another form of abstraction, however, that Husserl discusses in the *Logical Investigations*, namely "*formalizing abstraction*" (291/482), which he maintains "is something quite different from what is usually aimed at under the title of 'abstraction.'" It differs from abstraction characterized as "the emphasis on a non-independent 'moment' of a content, or the corresponding ideation under the title of 'ideating abstraction'" (292 n. */482 n. 1), which is to say, it differs from the two kinds of abstraction in the *Investigations* that we have considered so far. Formalizing abstraction differs from these other types of abstraction in that, rather than set into relief a universal in a concrete perceptual content—for instance, "the universal redness in a concrete visual datum" (291/482)—or apprehend directly this universal itself, it involves "abstracting from the specificity of the species contents in question" belonging to these other forms of abstraction.

Precisely how one abstracts from such contents in their specificity is something about which Husserl is not very clear in the *Logical Investigations* and—as we have already suggested—in his other works. He writes in the *Investigations*: "In formalization we replace the names standing for the species of content in question by indefinite expressions such as *a certain species of content*, *a certain other species of content etc.* At the same time, on the significational side, corresponding substitutions of purely categorial for material thinking takes place" (291–92/482). In the footnote to this passage, Husserl explicitly calls attention to "the role of formalization for constituting the idea of pure logic as *mathesis universalis*" (292 n. */482 n. 1) and refers to his discussion of both in §§ 67–72 of the *Prolegomena*. In light of this footnote, however, it is surprising to find that while Husserl does discuss the tasks of pure logic in these sections, he makes no explicit mention of formalization. In connection with pure logic, he does mention that "we are dealing with nothing but concepts, which already in their function make it clear that they are *independent of the particularity of any material contents of cognition*"

objects is precisely what is behind our claim here that he "admits" the non-independence of both types of abstraction from sensible relations in the *Investigations*.

(245/237). He includes among such concepts "state of affairs, unity, multiplicity, cardinal number" and comments that "they arise solely in relation to our various 'functions of thinking,' namely, in possible thought acts as such or in the correlates that can be grasped on the concrete basis of these acts" (246/237). But when Husserl addresses here the question of the origin of these concepts, a question he explicitly formulates in terms of "a *phenomenological origin* or—if we prefer to rule out unsuitable talk of [psychological] origins, only bred in confusion—the concern with *insight into the essence* of the concepts involved" (246/238), he speaks not of formalizing abstraction but of ideational intuition. Husserl writes, "We can achieve such an end [i.e., insight into the essence] only by *intuitive presentation* of the essence in adequate ideation, or, in the case of complicated concepts, through knowledge of the essentiality of the elementary concepts present in them, and of the concepts of their forms of combination."

Husserl's apparent inconsistency here, if indeed the ideational intuition appealed to in this context as the source of insight into the essence of the formal categories of pure logic is the same as the ideating abstraction that he elsewhere *distinguishes* from the formalizing abstraction maintained to be responsible for these very same categories, is not our major concern. Rather, our concern lies with Husserl's accounts of the origin—or, in light of his remarks directly above, the phenomenological essence—of the formal arithmetical concepts that he accounted for in *Philosophy of Arithmetic* on the basis of reflexions upon psychological acts. In other words, how, subsequent to his critique of psychologism, does he account for the unity of the collections of units at issue in multiplicities and cardinal number concepts, a unity signified by the 'and', as well as for the unity of the materially empty formal concept 'any object' or 'anything whatever'?

§ 168. The Categorial Distinction between Synthetic Forms of Collectiva and States of Affairs

So far we have seen that in the *Logical Investigations* Husserl rejects any account of logical categories arising through reflexion on certain psychic acts (668/782), categories that include those of interest to us, namely, 'plurality', 'cardinal number', and 'unity'. On the basis of the general account of the concept of judgment that Husserl provides immediately following this rejection, which expands it to include both the act of signification intention and the act of signification fulfillment, it might seem reasonable to expect that the concepts on which we are now focusing would be accounted for on the basis of the categorial intuition of logical categories. They would be phenomeno-

logically clarified with respect to their non-psychological origin or essence, on the basis of propositional elements and propositional forms of combination whose significational intentions find their fulfillment in the state of affairs proper to the logical categories and combinational forms signified by them. However, we have already seen that Husserl distinguishes the two types of abstraction involved in propositional judgments from the abstraction involved in the formalization of logical categories, among the latter of which are included precisely the categories that interest us.[77] Moreover, he explicitly *distinguishes* the "synthetic forms" of "collectiva" (688/798) from "states of affairs," saying point blankly that they "are not themselves states of affairs." That is, Husserl differentiates 1) the forms of the collections whose "givennesses" are "constituted" in "the acts . . . which furnish a fulfilling intuition" for the conjunction 'and' and 2) the concrete and higher-level categories emphasized in judgments about the content of the object of straightforward perception. They are thus distinguished, even as he recognizes that the former synthetic forms "nevertheless play a large part in connection with states of affairs."

Owing to the role that collections play in Husserl's account of the peculiar unities of the wholes of pluralities and cardinal number concepts in *Philosophy of Arithmetic*, we shall take a very close look at what he has to say about them in the *Logical Investigations*. He writes that "What intuitively corresponds" to the word 'and'

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77. In the first volume of his *Ideas* from 1913—*Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie, I. Buch. Allgemeine Einführung in die reine Phänomenologie*, ed. Karl Schuhmann (The Hague: Nijhof, 1976); English translation: *Ideas Pertaining to a Pure Phenomenology and to a Phenomenological Philosophy. First Book*, trans. Fred Kersten (The Hague: Nijhoff, 1982); henceforth cited as *Ideas I* with German page reference, which is included in the margins of the English translation—Husserl thematizes the distinction between the types of logical categories involved in these different types of abstraction under the heading of "Generalization and Formalization" (26). There he says: "One must sharply distinguish the relationship belonging to generalization and specialization from the essentially heterogeneous relationships belonging, on the one hand, to the *universalization of something materially filled into the formal in the sense of pure logic* and, on the other hand, to the converse: the *materialization* of something logically formal. In other words: generalization is something totally different from that *formalization* which plays such a large role in, e.g., mathematical analysis; and specialization is something totally different from *deformalization*, from 'filling out' an empty logico-mathematical form or a formal truth." Husserl does not, however, discuss here or elsewhere in *Ideas I* the abstractive (or any other type of) processes that would initially yield the "*pure logical* essence" belonging to formalization or the "essential *genera*" belonging to generalization. Rather, he appeals to the authority of "eidetic intuition": "To verify this radical separation we must, as in all such cases, go back to eidetic intuition which at once teaches us that logical form essences (e.g., the categories) are not 'inherent' in the materially filled singularizations in the same manner in which the universal red is 'inherent' in different nuances of red" (27).

> is not anything, as we rather roughly put it above, that can be grasped with one's hands, or apprehended with any [inner or outer] sense, as it can also not really be represented in an image, e.g., in painting. I can paint *A* and I can paint *B*, and I can paint them both on the same canvas: I cannot, however, paint the *both*, nor paint the *A and* the *B*. Here we have only one possibility which is always open to us: to perform a new act of conjunction or collection on the basis of the two single acts of intuition, and so *mean* the *connection* of the objects *A and B*. (688–89/798)

What Husserl says here is very close to what he says in *Philosophy of Arithmetic* insofar as the source of the unity of a collection is not attributed to anything that can be manifested in or by the relationship of its contents but to the act that conjoins or collects them. However, whereas in *Philosophy of Arithmetic* a reflexion on this act itself was characterized as the origin of the collective *relation* belonging to the unity of the collection, as can be seen from the last part of the foregoing quotation, in the *Investigations* this act is characterized as *meaning* the connection and hence the unity of the collection. Thus, Husserl observes:

> That we speak of an act which unites these perceptions, and not of any connection or mere coexistence of these perceptions in consciousness, depends on the situation that a *unitary intentional* relation is here given, and a unitary object which corresponds to it; the object can only be constituted in such a connection of acts, just as a state of affairs can only be constituted in a *relational* act-connection. (689/798–99)

§ 169. The Unity of the Collection as Such Is *Neither* Isomorphic *Nor* Homologous with the Unity of the State of Affairs

Based on what Husserl says in the *Investigations*, however, the exact character or status of the unitary object that the unitary intentional relation of the act of collection *means* and to which it is therefore related *intentionally* is not readily discernible. (Indeed, as we shall see, what he says on this matter in his subsequent works does not make things any easier in this regard.) Clearly, the intended unity is *not* supposed to be a *concept* that originates in a relation that is generated by the psychic *act* of combining together the items that belong to a collection. The unity is not supposed to emerge from the *reflexion* on the generic similarity of the acts of collecting, as Husserl had thought in *Philosophy of Arithmetic*, because he now realizes that such a unity—owing to its psychological origin—cannot be logical; it cannot be logical in the precise sense of what is *meant* by logical unity, namely, a relationship that is valid in a manner that *ipso facto* excludes any psychological component's being responsible for, and indeed being involved in, its validity in *any* way. *Logical categories arise not through reflexion on certain psychical acts but in*

acts that present the objective correlates of logically signitive intentions—and Husserl tells us that the unity belonging to the collective concept of *plurality* counts among the logical categories.

Does this mean, then, that the unity of the plurality meant by the intentionality proper to the act of collection is a categorial form that "gives itself" in an act that presents the fulfillment of the collecting act's signitive intentionality? (This is how its unity would have to be given if its givenness were akin to that of the categorial objectivities corresponding to the unity of propositional elements or propositional forms of combination.) Not at all, if we follow what Husserl, in the *Logical Investigations* and elsewhere, has to say both about logical judgments and what in the *Investigations* is referred to as "the character of an authentic intuition of the collection as such" (690/799).

Husserl is quite clear that logically signitive intentions, together with their signification categories, are propositional components of logical *judgments*, judgments that *also include* the acts of signification fulfillment. This fulfillment terminates in the formal categories (and relations) that compose the objectivity of the state of affairs of a judgment, and Husserl also quite clearly states, as we have just seen, that collectiva are *not* themselves states of affairs. This immediately gives rise to the question of whether Husserl understands the intentionality proper to acts of collection or conjunction to have its basis in logical acts of judgment. And this is to ask whether Husserl gives any indication or otherwise provides any evidence that the *way* in which the act of collection *means* its intentional object is isomorphic to or otherwise homologous with the *way* a logical judgment *signifies* its intentional object. In the case of a logical judgment, the intentional relation to the intended logical object (the formal categories and the forms of their combination) is clearly inseparable from the signification categories that belong to the signitive proposition. But is this the case with the intentional relation belonging to what Husserl calls "the authentic intuition of the collection as such"? Is the collection as such, which is to say, its unity *as distinct* from any act or acts of collecting, something that has its counterpart in some kind of *significational* intention that *means* it?

In answer to this question, it must be said that Husserl not only does not give any such indication or provide any such evidence, but, as we shall see,[78] in his most detailed and mature consideration of this issue, he draws a sharp contrast between "the judgment ordinarily favored by logic" and "the predicative 'propositions' in the broader sense, in which there is a connection in the form

78. See §§ 173–74 below, esp. n. 85.

of 'and', 'or', etc." (*EJ*, 254/215). In making this distinction, Husserl rejects precisely what would have to be supposed as the case in order for the structural identity or isomorphism under consideration to obtain. He writes that the predicative prepositions 'and' and 'or' "do not confer on what is formed by them an independence of the same kind as does the copulative connection" (254/214); they do not confer "the '*copulative' form of unity*" of the "*truly apophantic predicative judgment*" that is favored by logic. This form of unity, "which attains clearest linguistic expression in the connection of the subject and predicate in the 'is' form," "does not take place" (135/121) in "the synthesis which occurs in the *grasping of plurality*" (134/119–20). Hence, in *Experience and Judgment*, Husserl characterizes the connection expressed by the conjunction "and" as a connection that is founded in something other than the explicative grasping of the concrete categorial structures (as he refers to it in the *Investigations*).

Thus, it is certain that for Husserl the relationship between the intention belonging to the act of collection that *means* the collection, and so means it in a manner that employs or otherwise makes use of the "*and* as an objective logical form" (*LI*, 689/799), is related to the collection as such in a manner that is *different* from the manner in which the signification intention in a logical judgment is related to the state of affairs as such. How, exactly, the collective intention is related to the collection as such in a manner that maintains the latter's independence from the act of collecting, however, Husserl does not make clear in the *Investigations* or—as we shall show—anywhere else in his writings.

§ 170. The Contrast between Straightforward Perceptions of Sensuously Unified Multitudes and Authentic Intuitions of the Collection as Such

In the *Logical Investigations* Husserl contrasts "the conjunctive perceptions, in which alone the consciousness of plurality is authentically constituted," with "the *straightforward perceptions of sensuously unified groups*, series, swarms, etc." Referring explicitly to the analysis of the latter in *Philosophy of Arithmetic*, he claims that the straightforwardly apprehended multitudes do "not possess the character of authentic intuition of the collection as such" (690/799), because, as "'figural' or 'quasi-qualitative' moments of sensuous intuition" (689/799), they merely "serve as signs of plurality." These *sensuously* apprehended signs therefore function as "sensuous points of departure for the cognition of the pluralities in question in a manner that is mediated signitively—such that cognition now has no need for an articulated grasp

and knowledge of the single items [in the plurality]" (689–90/799). And because the cognition in question is signitive, it is something that "for that reason still does not possess the character of authentic intuition of the collection as such" (690/799).

Husserl's reference here to *Philosophy of Arithmetic* provides us with the occasion to recall the fact that the analysis of figural moments in that work was intended to establish a basis in sensible presentations for the symbolic presentation of groups. Moreover, it allows us to recall that Husserl there expressed the view that "the symbolic presentations of groups form the foundation for the symbolic presentation of numbers" (*PA*, 222).[79] What Husserl says here in the *Investigations* about the signitive cognition of the collection that takes place in the straightforward perception of pluralities is, of course, consistent with the view presented in *Philosophy of Arithmetic*. It therefore accords with the view that the significative dimension of such cognition permits the apprehension of the plurality as such in a manner that dispenses with the need to grasp its items singly. The further suggestion here in the *Investigations*, however, that apprehension in this sense still falls short of an authentic intuition of the collection *as such*, is something that is *not* found in *Philosophy of Arithmetic*. That is because in the latter, authentic cognition was the intuitive grasping of the single items of a group or plurality while simultaneously grasping them *as* members of the group in question. By contrast, in the *Investigations* Husserl's reference to authentic intuition explicitly refers to the authentic intuition of the plurality *as* a 'collection as such', namely, to the authentic intuition of the concept or categorial from of 'collection'. In *Philosophy of Arithmetic*, this concept was said to arise from the reflexion upon the act of collection, that is, on the inner sense perception of the generic unity of the psychological acts proper to collective combination. In the *Investigations*, of course, precisely this account of the 'collection as such' has been ruled out by the critique of psychologism.

§ 171. The Radical Departure of the *Logical Investigations* from *Philosophy of Arithmetic*'s Account of Number as the Delimitation of an Indeterminate Plurality of Units

Ruling out the psychological origin of the unity of the concept or formal category of a collection is not the same thing, however, as accounting for the non-psychological origin or phenomenological essence of this unity. In *Philosophy of Arithmetic*, the recognition of the peculiar character of the collec-

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79. See Part III, § 48.

tion's unity, its function to unify items that (considered either in themselves or in relation to one another) are manifestly *unable* to support or otherwise provide the foundation for their belonging together as a collective unity, motivated Husserl's attempt to *exhibit* this unity on the basis of its origin in the psychological act of collecting. Husserl's interest in collective unity in *Philosophy of Arithmetic* was guided by that work's *logical* investigation of the fundamental concepts of universal arithmetic, the concepts of multiplicity, cardinal number, and the 'anything', all of which were there characterized as formal categories—and so characterized because of the ability of pluralities and cardinal numbers to unify collectively any arbitrary objects whatever.

In the *Logical Investigations* the most sustained discussion of the arithmetical dimension of the collection occurs in the *Prolegomena*, which means that it comes *before* Husserl's discussion in the Sixth Logical Investigation of categorial intuition. The *Prolegomena*'s discussion itself is noteworthy because, among other things, it makes no mention of the peculiar character of the collective unity in multiplicities and cardinal numbers that, as we have seen, led Husserl in *Philosophy of Arithmetic* to attempt to account for its logical status by appealing to acts of collective combination. Moreover, it makes no mention of the combining function of the 'and' in acts of counting. Husserl situates the *Prolegomena*'s discussion by affirming that "The formations produced in all arithmetical operations refer back to certain psychic acts of arithmetical operation" (*LI*, 173/179). He goes on to say, "only in reflexion on these can we 'exhibit' what a cardinal number, a sum, a product, etc., is." Nevertheless, he maintains that "In spite of the 'psychological origin' of arithmetical concepts," it would be a fallacious "μετάβασις εἰς ἄλλο γένος[80] to demand that mathematical laws should be psychological." Thus, "Numbers, sums, products and so forth are not such causal acts of counting, adding and multiplying etc., as proceed here and there." Husserl attempts to clarify this fundamental distinction between the psychological and the mathematical by articulating the difference between 1) the presentation and the counting of five and 2) the ideal species of the form: number five. He writes:

> The number five is not my own or anyone else's counting of five, it is also not my presentation or anyone else's presentation of five. It is in the latter regard a possible *object* of acts of presentation, whereas, in the former, it is the ideal *species* of a form, which in certain acts of counting has its concrete instance on the side of what is objective in the constituted collection. (173–74/179–80)

By 'possible object of a presentation' Husserl means to stress that the number 'five' (or any other number) cannot "be regarded without absurdity as a

80. That is, "change into some other genus."

part or a *side* of a psychic experience, and therefore it cannot be grasped as something that is real" (174/180).[81] This objective independence of the number 'five' from psychological reality, which for Husserl means from acts perceived in reflexion or inner perception, is no doubt that for which his appeal to the *counted* number 'five' as an instance of the *ideal* species of the number 'five' is intended to account.

Husserl articulates the relationship between number species and instance in the case of the number 'five' as follows:

> If we clearly make present to ourselves what the number five authentically is, we likewise engender an authentic presentation of five, which means we must first form an articulated act of collective presentation of any five objects whatever. Intuitively given in this presentation is the collection in a certain articulated form and with this an instance of the number species in question.

Noteworthy here is, first of all, the close correlation that is maintained between the authentic number 'five' and the authentic presentation of "five" objects, which would accord with *Philosophy of Arithmetic*'s account of the authentic concept and presentation of number. However, the account of the collection being intuitively given as an "articulated form" that instantiates the number species 'five' represents a radical departure from the account in *Philosophy of Arithmetic*. That is because, in *Philosophy of Arithmetic*, authentic number species were accounted for in terms of the delimitation of an indeterminate multiplicity, where both the multiplicity and its delimitation were accounted for on the basis of the collective combination—expressed by the 'and'—of units characterized as arbitrary objects falling under the formal categorial concept of 'anything'.[82] It is thus likewise noteworthy that in the *Logical Investigations*' account of authentic numbers there is no mention of the 'and' in connection with the collective unity of numbers nor of the formal concept that allows any objects whatever to be so unified.

§ 172. Husserl's Account of "Absolute" Numbers Leaves Unanswered the Question of Whether Either Kind of Abstract Number Has the Status of a Formal Collection

Husserl's account goes on to articulate how it is that the ideal form-species of the number in question comes to be apprehended as it is *apart* from its presentation in this or that singular act of intuitively constituted collection:

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81. In the first edition of the *Logical Investigations*, Husserl writes the following after this: "Indeed, in the act of counting we find the individual, as a singular instance of the species, possesses an ideal unity. However, this unity is not a piece of the singularity."

82. See Part III, § 45.

> Looking at this intuited instance we perform now an "abstraction," i.e., we extricate not only the non-independent moment of the collection-form from what is intuited as such, but we grasp in it the idea: the number five as the species of the form appears in consciousness' sphere of meaning. What is now meant is neither this instance nor what is intuited as a whole, and likewise not the form inherent in and inseparable from it; what is meant is rather *the ideal form of the species*, which in the sense of arithmetic is absolutely unitary, and in whose acts an instance of itself may also be intuited in the constitution of collections, and intuited thus without any part of it being touched by the contingency of such acts with their temporality and transience. Acts of counting come and go and cannot be meaningfully mentioned in the same breath as numbers. (174–75/180)

From this account of the abstraction of the ideal form proper to the species of number, it appears that unlike in *Philosophy of Arithmetic*, where the form of the species of an authentic number is nothing other than the form of the conjunction of ones that arises in their collective combination, the species—as what it is, namely, something that is absolutely unitary—is now something quite literally *separate* from such acts. Therefore, the "form" of the five objects intuitively given in a counted collection is decidedly *not* the "ideal form of the species," because the latter in being what it is remains totally indifferent to and therefore different from the acts in which it is nevertheless first apprehended—albeit indirectly, through its "instance." Based on what Husserl says here about the abstraction that extricates the "ideal form of the species" from this instance, it appears to be unlike the formalizing type of abstraction discussed elsewhere in the *Logical Investigations* and closest in character to the type that places emphasis on the non-independent moment of a content and thereby apprehends in ideation the pure species itself.

However, what is apprehended in the abstraction of the ideal form species proper to numbers is already formal in the sense in which the pure forms yielded by formalizing abstraction are formal. This is evident from Husserl's account of the "ideal singular unities" (175/180) that belong to the ideal form of the species that is characteristic of each of the numbers with which "arithmetical propositions are concerned." These singular unities are "lowest species in a distinctive sense that is sharply differentiated from the species of empirical classes," a sense that "holds both of numerical propositions (which are arithmetically singular) and of algebraic propositions (which are arithmetically general)." As distinct from the species of empirical classes, either type of proposition "by no means tells us anything about what is real, neither the real things counted, nor about the real acts in which they are counted, in which such and such indirect numerical characteristics are constituted for us." Consequently, "They are rather concerned with absolute numbers and number-combinations in their abstract purity and ideality" (175/180–81).

The contrast here with *Philosophy of Arithmetic*'s account of the universality of number concepts is striking; it includes no discussion of the formal universality of the units whose collective combination was there held to belong to the authentic cardinal number concepts, nor of the indirect apprehension of such collections by systematically ideal symbolic number concepts; moreover, the eventual replacement (in calculation) of the latter by "surrogate" signs (i.e., the signitively symbolic numerals) that function as "normative" numbers receives no mention. While it can be inferred from Husserl's account of absolute numbers in the *Prolegomena* that he still holds instances of absolute numbers to appear or otherwise show up in the intuitive acts of collecting any objects whatever, that is, in acts of counting, it seems that what renders such acts capable of combining such arbitrarily considered objects is *not* that each is considered abstractly (in the sense of *Philosophy of Arithmetic*'s characterization of abstraction) as 'anything', a 'certain one'. On the contrary, it seems that in the *Prolegomena* it is precisely the abstract status itself of each singular form of the number species, rather than, as *Philosophy of Arithmetic* presents it, the formal universality of the units putatively combined into collections that is responsible for the formal universality of numbers—as well as for the arithmetical propositions that Husserl clearly thinks are related to such numbers. What is not clear from this account, however, is whether Husserl understands such absolute numbers, either in themselves or in their instances, to have the status of formal collections.[83] As we have seen, in Husserl's Sixth Logical Investigation, the authentic intuition of

83. Barry Smith draws the proper *logical* conclusion from Husserl's account here in the *Prolegomena* of numbers as "species," when he comments on "how well this Husserlian view of species in terms of perfect identity works for species from the realm of categorial forms" ("Logic and Formal Ontology," 65 n. 34). That is, the perfect identity of two *instances* of the absolute number five, which emerges when "two separate collections of five items are compared," points to there being "indeed something perfectly and straightforwardly identical in the two collections: their respective individual fivehoods are absolutely alike, and this regardless of all differences in their underlying matters." Smith's faithfulness to Husserl's account here, however, has as one consequence the elision of precisely the issue that we are maintaining comes to the fore when the *Prolegomena*'s account of absolute numbers as species is considered in light of *Philosophy of Arithmetic*'s (along with the *Logical Investigations*' proper and Husserl's later logical works') characterization of authentic cardinal numbers and authentic cardinal number concepts as collections, namely, whether (to follow Smith's phrasing) the "fivehoods" in question, being materially empty and therefore formal categorial structures, present formal collections.

Insofar as the "instances" of the fivehoods that are compared become objective in certain acts of counting, the 'and'—though *not* mentioned in this connection in either the *Prolegomena* or by Smith—would (for Husserl to remain consistent with his accounts—in *Philosophy of Arithmetic*, as well as in his later logical works—of cardinal numbers as collections) have to be a factor in the categorial structure of the numerical unity of the five counted

the collection as such is considered in connection with the logically syntactical form of the 'and'," though without considering the collection's delimitation (or constitution) as a number. Moreover, we have pointed out not only that Husserl does not clarify the character of such an intuition, except in the negative sense of being distinguished from the relational categorial intuition belonging to logical judgments, but also that his discussion of numbers in

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items as an instance of fivehood. Moreover, the materially empty, formal categorial status of the 'and' would also have to be a factor. The question, then, is what happens to the 'and' and the collective categorial unity it indicates when the compared instances of fivehood are grasped (or otherwise apprehended) as instances of the same species? Does the 'and' together with the collective unity of the five items disappear in what Smith (35) refers to as the "ideal objects, which are what result when such empirical groupings [i.e., the counted instances of five items] are treated *in specie*, disembarrassed of all contingent association with particular empirical material and context"? To maintain, as Smith does, that "Two different sorts of objects are then involved: empirical objects, which get counted, thereby yielding empirical groupings (as, e.g., when we talk of there being 'a number of objects on the table') and ideal objects," not only does not solve the problem but raises the very specter of the Platonism that Smith—though not Husserl, to judge by his account of his self-development in the "Introduction to the *Logical Investigations*," as we have seen—is so keen to avoid in his account of Husserl's characterization of species in the *Logical Investigations*.

The specter of Platonism, of course, is Smith's problem only insofar as he embraces Husserl's account of the species in the *Prolegomena*, which, it bears pointing out again, occurs at an earlier point in his investigations than its presumably more mature treatment in the *Logical Investigations* proper, especially in the Sixth Logical Investigation. However, even leaving aside a developmental approach to this issue, Husserl's account breaks down for internal reasons alone, reasons that Smith does not consider when he maintains that "Husserl's theory of linguistic meaning, like his theory of logic, is therefore non-Platonistic in the sense that it is free of any conception of meanings as ideal or abstract objects hanging in the void in a way which would leave them cut apart from concrete acts of language use" (33). Smith maintains that this is the case because Husserl's account of meaning as species, as illustrated in the above numerical example, accounts for the connection between concrete linguistic and meaning acts and their "empirical" objects on the basis of both being "instances" of "any given meaning-species, namely that part or moment which is responsible for the act's intentionality." In connection with this, Smith endorses Husserl's view "'that the Platonistic [Smith's addition] idea of a straightforward perception'" (57), or a "straightforward intuition," of the species "'is absurd.'" Rather, he holds that, on Husserl's view, "a fulfilled directedness to a species" (54), as the moments of identity in concrete acts, occurs "in a process of what Husserl calls 'ideating abstraction.'"

There are two immediate problems with this view of the matter, one that is Husserl's and one Smith's. Regarding the latter, as we have seen (§ 164), Husserl quite clearly distinguishes the formalizing abstraction that yields the categorial structure of the formal universality at issue in pure logic as *mathesis universalis*, and, therefore, the formal universality of the pure species of numbers, from the status of the universality yielded by ideating abstraction. As Husserl puts it in *Ideas I*, generalization, which in the *Logical Investigations* is achieved through ideating abstraction, is heterogeneous to formalization (see n. 77 above). Smith's account of the formal status of the species, as "disembarrassed of all contingent association with particular empirical material and context," appeals therefore to a process that Husserl himself distinguished from the process that yields the formality of the species. The other immediate problem, however—and *is* Husserl's—is reproduced by Smith in his account of the two kinds

the *Prolegomena* maintains that they belong to a different genus than that of the psychological acts of collection proper to counting.

Husserl's reasoning behind the move to absolute numbers in the *Prolegomena*, of course, is clear: he no longer considers the appeal to the generic unity of psychological acts of collective combination to account for the logical unity of numbers, subsequent to his critique of psychologism, capable of sustaining the *logical* meaning that this appeal was intended to sustain. However, as we have just seen, one result of the *separation* of the unity belonging to both the ideal instances of numbers and to the ideal forms of their species from acts of collective combination is that the formal status of the numerical collection is no longer clear. Another result is that it is difficult to harmonize Husserl's account of the logical form of the 'and' in the Sixth Logical Investigation, as something that is apparently *meant* in the intentional acts that characterize conjoining and collecting, with the *Prolegomena*'s account of the *total* separation of the unity belonging to numbers from the acts of collecting that nevertheless must first execute the collective presentation of a number of any objects whatever—in order for an instance of the number species in question to be given intuitively in the collection presented in these acts. Even if we make the reasonable assumption that the Sixth Investigation's account of this issue is the more developed one, this resolves nothing, but only highlights the incompleteness of Husserl's thought on the matter in the *Logical Investigations*.

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of objects involved in the relationship between the instance of the species and the species itself. Briefly put, to characterize the former as 'empirical' and the latter as 'ideal' is indeed typical of the most traditional form of Platonism, namely, that of its so-called "two world" theory. In the case at hand, the fivehood of the empirical collections of five, as ideal, must nevertheless be what is responsible for the unity of the collected empirical items *as* five. (This is the case since, otherwise, the unity of the empirical collection would be *empirical*, which is the very thing the appeal to the ideal species is attempting to forestall.) Not only, as we have suggested, does the appeal to the relationship between empirical and ideal objects here not address the problem of whether the ideal objects are formal collections, but neither does it address the matter of the origin of the materially empty and therefore formalized ideality of such ideal objects. In other words, what fails to be addressed and therefore resolved in Husserl's account of number as species in the *Prolegomena* is how what, in his "Introduction to the *Logical Investigations*," he unabashedly referred to as the "Platonism" of the non-empirical content of formalized ideality of the species, is not overcome so much as brought back into a relationship with thinking. (See § 191 below, where we trace Husserl's attempt to do just this and assess—on internal grounds—his success in doing so.)

Chapter Thirty-three

Husserl's Account of the Constitution of the Collection, Number, and the 'Universal Whatever' in *Experience and Judgment*

§173. The "Turning of the Regard" to Grasp the Collection as *One* Object

Husserl's *Experience and Judgment* contains discussions that address both how the collection comes to be *apprehended* as a collection and how, from the "indeterminate plurality" (446/368) constituted in particular judgments, cardinal numbers "are originally and directly produced" (447/368). Neither discussion, however, addresses the issue that we have found to be problematic in the *Logical Investigations*: the precise character of the intentional *referent* intended by the relation proper to the conjunction 'and' operative in acts of collecting and counting; the precise character, in other words, of the collectively objective categories and categorial relations intentionally signified by such acts—what the *Investigations* terms the 'collection as such'. Nevertheless, because the discussions in *Experience and Judgment* represent Husserl's most mature thought on these issues, they bear close scrutiny.

Husserl's account of how a collection becomes an object begins by reaffirming the *Investigations*' distinction between the objectivities proper to states of affairs and to collectiva. In line with this, he maintains that "States of affairs are not the only objectivities of the understanding which are constituted in predicatively productive spontaneity" (292/244). That is because the collective as such, the "object *set*," is also constituted in "the predicative judgment," albeit in a manner that is different from how states of affairs lead to logical formations of sense. The difference concerns primarily the pre-predicative and, in this sense, "pre-constitutive" level of the acts in which the object substrate posited in the copulative judgment is formed, and the higher level of acts in which the collective is formed as an objective substrate. Both

forms of predicative spontaneity, the "narrower" copulative linkage and the "broader" conjunctive linkage, are "founded" judgments and therefore lead to the preconstitution of their respective objectivities on the basis of acts of pregiven syntheses together with their contents. However, Husserl maintains that the collective linkage "does not lead to the logical formation of sense, to deposits of sense in object-substrates in the same way as copulative spontaneity." The collective as an "objective substrate" is *not* what is preconstituted in the predicative spontaneity that "leads, like all predicative spontaneity, to the preconstitution of a new objectivity, that of the object 'set'" (292/245); rather, what is preconstituted in collective predicative spontaneity is "the noetic unity of a consciousness but not yet the unity of an object in the proper sense, that is, in the sense of a thematic object-substrate" (294/246).

The collective initially emerges in what Husserl refers to as "the domain of receptivity" (292/245), wherein "there is already a plural contemplation [*mehrheitliches Betrachten*] as a collective taking of things together." Involved here is "not merely grasping one object after the next, but a hanging on to the grasp of the one object with the grasping of the other, and so forth." However, in "this unity of taking objects together, the collection is still not *one* object." That is, in the plural contemplation of objects "the pair, the collection, more generally, the set of both objects" is, properly speaking, not constituted; rather, "we have, more than ever, only a *preconstituted object*, a '*plurality*'" (293/246). Thus, for Husserl, "as long as we carry out a merely collective grasping together [*kollektives Zumsammengreifen*]," the apprehension of the *collection as such*, as an "authentic object, something identifiable" (293/246) "as *one* object" (293/245), does not come about.

In order for the collection, say, "the *pair*," to be grasped as such, that is, as a "total object *A* + *B*," a "*turning regard* is first required" (293/246), by which Husserl means to indicate a "*retrospective apprehension*" in which the set, as a "thematic object-substrate," is apprehended following its preconstitutive "active formation" as a plurality. The active formation of a plurality comes about insofar as

> we can direct the regard of advertence and apprehension toward the *pair*, toward the one and the other of the pair, whereby *these* are objects. If we do this, then the repeated individual concentration, the concentrated partial apprehension, now of the *A* and then of the *B*, functions as a kind of explication, as an act of running through the total object *A* + *B*. (293/245).

Only in this manner, in what Husserl calls "the act of plural explication" (293/246), can the assemblage of the total object "be given, in order that it may be apprehended in self-givenness and contemplated as such." Husserl

goes on to characterize the "active formation" that leads up to the self-givenness and contemplation of the collection as a total object as a "collective synthesis" (294/246). In this regard he characterizes, for instance, the collective synthesis '*A* and *B* and *C*' as "the noetic unity of a consciousness, but not yet the unity of an object in the authentic sense." By 'noetic unity of a consciousness' Husserl understands the aspect of "the colligating consciousness" that, in its act of plural explication, "contains a plurality of objects encompassed in unity." According to Husserl, the noetic unity at issue here is *not* the collective, in the sense of "a unique object that has many members": it is *not*, therefore, the "unity of an object in the authentic sense, namely, in the sense of a thematic objective-substrate." Rather, the unity of an object in this sense is only *pre*-constituted in the synthetic collecting, such that the "presentation (A, B) has priority over the collection ($A + B$) in which the *assemblage* is an object" (293/245). In order for the pre-constituted plurality of the "collective combination" (293/246), "originally sprung from the plural explication of *A* and *B*," to become an objective "substrate" and thus an authentic object, something else is required. What is required is "a *retrospective apprehension*" (*EJ*, 294/246) that "follows the completion of the colligation," and this renders the set thematic in a manner that "is given to the ego as an object, as something identifiable."

As Husserl presents it, the manner in which the noetic unity of the pre-predicative plurality is transformed into the authentically objective (and presumably noematic) unity of the set is the same as that of "all objects produced in predicative spontaneity: a syntactical objectivity is pre-constituted in a spontaneity, but only after it is completed *can it become a theme, it being an object only in retrospective apprehension*" (293–94/246).

§ 174. The Proximity of *Philosophy of Arithmetic*'s Psychologism to *Experience and Judgment*'s Account of the Objective Constitution of the Collection

Of course the *content* of what is pre-constituted in the case of the objectivity of a collection is *not* the same as what is pre-constituted in the case of a state of affairs, because the relational syntax of the latter objectivity is founded in prepredicative relational syntheses, while the collective syntax of the former objectivity is founded in prepredicative collective syntheses.[84] Indeed, beyond affirming that, subsequent to the presentation formed by collecting in

84. Regarding the syntactical difference, see § 168 above. See the following note for a detailed explication of Husserl's account of the synthetic difference.

the mode of plural explicating, the collection as such becomes a thematic object (the set) and therefore an objective substrate, Husserl sheds no light on how the retrospective apprehension transforms the *noetic* "unity" belonging to the "predicatively productive spontaneity" of *collecting* objects together into the *noematic* "unity" of the *collection* itself.[85] The objective unity of the

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85. Abraham Stone has recently argued that when Husserl singles out the noetic unity of the collective synthesis, he is not suggesting, in the case of collecting, that there is a noetic process without a corresponding noematic correlate. Rather, not only is this impossible once Husserl introduces (in *Ideas I*, § 88) the noesis-noema correlation, but also the analysis of conjunctive unity in *Experience and Judgment* tells against this being Husserl's meaning. (Abraham D. Stone, "Comments on Burt's Paper," unpublished, presented at the 35th Annual Husserl Circle meeting, University College, Dublin, June 2005.) Specifically, Husserl clearly understands the conjunctive unity in the form of the 'and' to fall among "the predicative 'posita' in the broader sense" (*EJ*, 254/215), which means that a *noematic* unity, posited in the object, is clearly involved here. Stone's argument presents with exemplary clarity the "standard" view of the applicability to the unity proper to collections of the categorial unity proper to "states of affairs." (Because the thesis we are advancing here challenges this view of the matter, we shall discuss Stone's argument in detail.)

This being the case, Stone's argument continues, there is nothing *uniquely* problematic in Husserl's claim that the "preconstitution" of the unity of the collection itself, as a set, comes about in "a kind of judgment [that] posits, in spontaneity, a new unity—based on, but not to be identified with, a preceding prepredicative unity." The predicative judgment in the broader sense responsible for collective linkage, just like the judgment in the narrower sense responsible for copulative linkage, preconstitutes the unity of an object—the set and the state of affairs, respectively—that is then thematized in the "retrospective apprehension" as a new object.

If there is a special problem in Husserl's account of the unity of sets, Stone concludes, it has to be found at an earlier level of Husserl's analysis, in what Husserl, at one point, refers to as "the domain of receptivity" (292/244). Moreover, Stone points out that in § 24d Husserl (at this level) marks "an essential difference" (135/120) between 1) the explication of a particular object, which Husserl calls "explicative contemplation" (124/112), and 2) "the apprehension of plurality" (134/119–20), in which "the unity of configuration" (134/120) of a plurality is "apperceived from the first as existing in a pluralistic way, as a plurality of objects, and this apperception is 'realized.'" And Stone thinks that any special problem in Husserl's analysis will have to be found in this difference, specifically in the role that Husserl assigns to "passive synthesis" in the "combination" that preconstitutes the latter. We shall return to consider the precise locus and character of what Stone signals as problematic here, after first taking a closer look at Husserl's characterization of the difference in question.

What is different here concerns not only the kinds of objects involved (particular in the former case and plural in the latter) but also the sources of the intentional unities proper to the respective "single activities" (135/121) that, respectively, explicate or apprehend these objects. Husserl articulates these differences in the context of a specific similarity, the "analogy" (135/120) between the succession of activities involved in the explicative contemplation of a particular object and in the apprehension of a plurality. In both cases, an "*intentional modification*" (131/118)—which Husserl characterizes "precisely as a still-retaining-in-grasp" (see 135/120)—of the succession of singular acts composing the apprehension of their respective objects is involved. By linking these acts together in a manner such that each subsequent act retains something of the previous act, this modification yields a synthetic unity of their activity that "*persists throughout the succession*" (135/120) of "partial apprehensions" of

the respective objects in question. Husserl refers to this synthesis as one in which "*an overlapping takes place*" (128/115), and characterizes it as follows: "The ego plays a continually active role through the series of steps run through; in the second, it is still directed toward the object of the first; it is directed, therefore, in spite of the privileged position of the new object of primary apprehension, on both of them together: with the new and, through the new, on the old. The two are together actively taken up by the ego; the indivisible ego is both. The succession of the rays of attention and of apprehension has become *a single double ray*" (128/115). Or, more precisely, he characterizes three of the four modes of overlapping (considered in *Experience and Judgment*) as involving a thematic object (or objects) that belong(s), as a substrate, to the synthesis in the manner described here; the fourth type (the one in which we are primarily interested) of synthesis, which Husserl characterizes as "*the unity of the activities in the running-through of a plurality*" (135/121), is not united according to a principle that has its source in "*activity*"—as are the first three types—but in "*passivity*" (136/121). To highlight Husserl's account of this difference, and hence the "essential difference" between "explicative contemplation" and the "apprehension of a plurality," we need to consider briefly his accounts of the first three modes of synthetic overlapping, which will allow us 1) to clarify the specific mode of overlapping he attributes to explicative contemplation and then 2) to lay out his account of its essential difference from the overlapping composing the apprehension of a plurality.

The first distinction Husserl makes in the modalities of overlapping concerns those in which "a synthesis of *coincidence* comes about" (128/115) and those in which it does not occur. When, "*according to the objective sense*" of the synthetic activity, such a synthesis takes place, Husserl characterizes it as having the status of a "special identity-synthesis," one in which like or similar moments overlap in the object. He uses the example of passing "from one color to another" (129/115–16) to illustrate an identity-synthesis. To the identity-synthesis Husserl contrasts the mode of synthetic overlapping that occurs without identity, giving the example of passing "from a color over to a sound." (For the transition of apprehension from a color to a sound to occur, there must be an overlapping of the apprehension of the former in the apprehension of the latter and hence a "single double ray" of apprehension, but no identity—because the objective senses of color and sound are manifestly different.) And to both of these he contrasts "a completely unique synthesis of the coincidence of identity" (129/116), namely, the "*explicative coincidence*." This unique mode of overlapping concerns "the synthesis 'thing and property of the thing,' and, in general, the synthesis 'object and objective property,'" both of which "should not be confused with the *total coincidence of identity* with regard to objective sense." That is because in the latter identity "we pass synthetically from one presentation (mode of givenness) to others of the same object and thereby identify that object with itself," while "in the case of explicative coincidence, it is a question of an identification which is wholly other, completely unique, in which continuity and discreteness are bound together in a remarkable way."

Husserl goes on to characterize the coincidence involved in the synthesis of explication as a two-sided affair, in which "[s]ubstrate and determination are constituted . . . as correlative members of a kind of coincidence" (129/116). On the one (substrate) side, there is the coincidence proper to the "actually holding-in-grasp" (133/119) of the substrate object (*S*), in which—"up to the point in which the explication begins" (133/119)—there is continuous "grasping and a having-in-the-grasp." Once the explication commences, the activity on the substrate side becomes modified as "the secondary still-retaining-in-grasp," an activity that coincides with that of the other (explicated) side, where the coincidence proper yields the still-retaining-in-grasp of each explicate ($\alpha, \beta, \ldots$) belonging to *S* "as *a sense-determinate precipitate of S*." The coincidence of these two syntheses—the grasping in advance and retaining of the substrate *and* the progressive apprehension of its determinations—yields the partial identification characteristic of the substrate's relation to its determinations; it yields,

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in other words, what Husserl calls the 'explicative coincidence'. He characterizes the "continuity" belonging to the explication as "a persisting synthesis of coincidence which concerns the content of apprehensions as well as the activities themselves." Regarding the activities themselves, Husserl holds that "the active ego is and remains constantly turned toward S," in the sense that in "the active apprehending and being-directed toward the 'whole,' or, to speak more precisely, the being directed toward the substrate S, [it] is implicitly 'co'-directed toward the α, . . ."; regarding the content, he maintains that "in the 'emergence' of the α, it is the S which is apprehended and displayed 'in its relation to' α" (133–34/119).

Now it is precisely this "partial identification" (135/121) belonging to the content of the explicative coincidence, that is, to the substrate whole of a particular object and the parts composing its internal determinations, that "does not take place" in the synthesis proper to the apprehension of a plurality. To be sure, the latter also involves a "whole" related to the partial apprehensions belonging to "*a unity of activity*," an activity that—like the activity of explication—"*grows up*" and "*persists throughout the succession*" of the synthesis. However, both the whole in question (the plurality) and the unity of the activities that persists in "*running-through*" it must, according to Husserl, be "rigorously differentiated" (134/119) from the whole and activities of explicative contemplation. In the case of the plurality as a whole, he maintains this because, unlike the whole of explicative contemplation, which is the "*goal of active doing*" (135/121) (in the sense of "a goal of knowledge gained through experience"), the whole to which the apprehension of a plurality is related "is not seized in advance and retained in active grasp in particular apprehensions." And in the case of "the unity of the activities" themselves, activities that partially apprehend the plurality "within an intentional whole" (135/120), he claims that their unity "*is produced, not by activity, but by a combination arising from sources of passivity*" (135–36/121).

These differences are rooted in Husserl's account of the contrast in the modes of overlapping synthesis that characterize the explication of a particular object and the apprehension of a plurality. In the former, the overlapping synthesis is articulated as the partial coincidence of the unity belonging to the part-whole structure of the "internal determinations" (125/113) proper to the object (substrate) itself. This unity comes about in the *active* taking together of the substrate and its explicates, in a manner that concerns both the apprehensions and their contents: the contents, e.g., $S\alpha$, $(S\alpha)\beta$, and so on, emerge as the continuous internal transformation of the substrate, a transformation that occurs in conjunction with its active apprehending, which is at once directed toward the substrate whole and co-directed toward its explicates. In contrast to the active taking together of the part-whole contents in this mode of overlapping synthesis, the overlapping synthesis of the activities belonging to the running-through of a plurality that are characteristic of its apprehension do *not* involve part-whole contents; what overlaps here is solely the unity of this running-through, in the sense that the individual activities belonging to this running-through coincide as a unity that persists with their successive "realization" of the plurality. Thus, in marked contrast with the explication of a particular object, where the object apprehended (as a substrate) is explicated as a whole together with its progressively explicated parts, the apprehension of a plurality does not apprehend the plurality *itself;* the content of the overlapping synthesis that composes its (the plurality as a whole) *partial* apprehensions therefore *does not include* the "object" of these apprehensions. This is why Husserl stresses that in the synthesis that apprehends a plurality, the individual activities involved in this apprehension do not partially coincide with the plurality itself. He stresses this because it is precisely at this point that the "analogy" between the two modes of overlapping synthesis breaks down. In other words, the activities composing the apprehension of a plurality are *not actively* taken together with the plurality itself, in a manner "in which the plurality is constituted as a specific object, as a 'set'" (136/121). This is also why Husserl stresses that the "combining [*verbindende*] activity" in which a plurality is not only "run through" but "also actively taken together" with the unity of the single activi-

ties that apprehend it is "completely other than that [i.e., the combining activity] which gives unity to an explication." That is because (as we have seen) the *unity* of an explication occurs *at the same synthetic level* as the overlapping synthesis in which the partial coincidence of the substrate (whole) and explicate (part) identities occurs, while the combining activity in which the apprehension of a plurality is united with the plurality "constituted as a specific object, as a 'set,'" is characterized by Husserl as "an activity of a higher level." (This activity culminates in the "retrospective grasping" discussed in §§ 173–74 above.)

The difference between the unity of the content belonging to the "objects" proper to explicative contemplation and to the apprehension of a plurality is likewise reflected in the "principle" (135/121) that unites their single activities of explication and apprehension, respectively. Thus, Husserl holds that, "From the very first, the explication has its unity in that the object is continuously the theme and as such remains constantly in grasp in a modified activity" (136/121). In other words, the principle of the unity of the single activities that compose the explication is rooted in the "thematic object which is explicated" (134/120), and it is so because it "belongs to the explication and in it assumes the status of substrate for its explicates." By contrast, Husserl maintains that the plurality does *not* belong—in this manner—to the succession of the single activities that compose its partial apprehensions, because these apprehensions are *not* directed toward it as a whole but rather occur *within* it, as the unity of their "running-through activity moves constantly on the persistent background in which plurality continually appears in a unitary configuration." It is for this reason that Husserl characterizes the source of the principle of the unity of these partial apprehensions as "*not in activity itself, but in combination arising from passivity*" (135–38/121). Thus, what is thematized in the activity of running-through a plurality, namely, its "*individual* members . . . which excite the interest in advance and which are immediately thematized as individuals" (134/120), is not the source of this activity's unity. Rather, it is "the likeness or similarity already given by an association in the background," which yields these individuals "not as mere isolated individuals but as individuals linked together thematically." Husserl maintains that "to the extent that the interest" follows this (passively) already given likeness or similarity, the plurality "is apperceived from the first as existing pluralistically," in the sense that "the configurative unity" of the passively associated likeness or similarity is realized "as a plurality of objects." Hence—and in striking contrast to the objective source of the unity of the single activities belonging to explicative contemplation, namely, the substrate that is thematically grasped in advance and retained in the active grasp of particular explications—the source of the unity of the single activities composing the particular apprehension of a plurality is, however paradoxically it may sound, *not* the objectivity of the plurality. This is the case for Husserl because the "coming-into-prominence of plural existents does not lead to a unitary objective turning toward" the plurality. Hence, not only is the whole of a plurality something that is *not* unified by the synthesis that occurs—as partial apprehensions—within it, but (in the domain of receptivity) the synthetic unity of these partial apprehensions has precedence over the objectivity of the whole of which they are the apprehensions, and it does so because the unifying activity that constitutes the plurality *itself* as a whole, as an object (set), is (as we have seen) "an activity of a higher level." It is explicitly otherwise with the explication of a particular object: "But in explication we do not perform separate acts taking the explicates together; it requires no special interest of a new kind" (136/121).

Returning now to Stone's argument, we shall first address his claim that Husserl's singling out of the noetic unity of a collective synthesis does not mean, in the case of the preconstitution of the collection as an object, that Husserl thinks that a (collective) noematic unity is not correlated to this noetic unity—which would be a situation radically *unlike* the one involved in the explicative contemplation that preconstitutes the state of affairs. (To the noetic unity of the "single double ray" co-directed to the substrate and its internal determinations there corresponds the noematic unity of thing and property.) Rather, Stone maintains

that Husserl "is taking it for granted that a *noetic* unity of consciousness has its correlate in a *noematic* unity, and hence in a unity posited in the object." As we have just seen, what complicates this situation in the instance of the apprehension of a plurality is that Husserl characterizes its difference from the explicative contemplation of a particular object as involving *both* the sources of the unities belonging to their respective modalities of overlapping synthesis *and* the relation of the unity of each synthesis to its content.

This last point means that, in addition to the difference (pointed out by Stone) between the active source of the synthetic unity proper to the explication of a particular object and the passive source of the synthetic unity proper to the apprehension of a plurality, there is the difference between the *contents* of each synthesis. And it is precisely in terms of the difference in contents that Husserl's reference to a "collective synthesis as a noetic unity of consciousness," which is made *without* mentioning a correlate in a noematic unity posited in the collective object, should *not* be seen as taking it for granted that a noematic unity is being posited in the "object" in a manner analogous to the way in which the positing of a noematic unity of a particular object corresponds to the noetic unity of the consciousness of it. The reason for maintaining a disanalogy here is rooted in Husserl's account of the radically different relationship between the particular "object" to its explicative contemplation and the "plural" object to its apprehension in a plurality: the particular object *belongs* to its explication in the precise sense that to the unity of the overlapping synthesis that explicates it there corresponds the thematic unity of the explicated object *as a whole*; the plural object, however, in a marked contrast that Husserl himself emphasizes, does *not* belong to its apprehension in the precise sense that to the unity of the overlapping synthesis that apprehends it there does not correspond the thematic unity of the apprehended object *as a whole*. Thus, as we have seen, Husserl calls explicit attention to both the need for acts of a higher level to apprehend the plurality as a thematic object (set) and to the absence of such a need to apprehend the particular object as a thematic object. Hence, while it is clearly Husserl's view that at this level (the domain of receptivity) of analysis, the (noetic) activities of explicative contemplation and apprehension of a plurality have objective correlates, it is also his view that these correlates are not analogous: the thematic objective correlate of the apprehension of a plurality are the individual objects that "realize" it, *but not the plurality itself as a whole*, while the thematic objective correlate of the explicative contemplation of a particular object is this object itself as a whole.

Husserl stresses (in *EJ*, § 61) that the plurality itself as a whole is not thematically apprehended, when he maintains (as we have seen in § 173 above) that the "presentation of (*A*, *B*) has priority over the collecting of (*A* + *B*) in which the *assemblage* is an object." Achieving the latter involves something more than the "apprehension of plurality" described and analyzed in *EJ*, § 24d, which, as we have just seen, is *not* directed to the collection as an object. Husserl (as we have seen above) calls what is initially involved here "plural explication" (293/246) and characterizes it in unmistakably psychological terms: "we can direct the regard of advertence and apprehension toward the *pair*, toward the one and the other of the pair, whereby *these* are objects. If we do this, then the repeated individual concentration, the concentrated partial apprehension, now of the *A* and then of the *B*, functions as a kind of explication, as an act of running through the total object *A* + *B*" (293/245). Only in this manner, he continues, can the assemblage of the total object "be given, in order that it may be apprehended in self-givenness and contemplated as such" (293/246). Of course, this total object can only be given with the "*turning of regard*" that Husserl characterizes as 'retrospective apprehension', which means that the pair, the total object as aggregate, is not really constituted in the "unity of this apprehension of the two objects" that characterizes the newly introduced plural explication, but rather that "the new object is preconstituted as a result, so to speak, as something which we now [i.e., via retrospective apprehension—*not* plural explication] apprehend as one and which we can explicate in the individual apprehension of *A*, *B*. . . ."

The upshot of all of this is that when Husserl says (in *EJ*, § 61) that prior to the "retrospective apprehension" of the objective unity of the collection itself only the noetic unity of a collective consciousness is preconstituted, he means it, that is, he means that "in" the preconstituted unity established in collecting *A and B and C* there is *neither* the thematic *nor* the objective unity of the collection: A + B + C, and this means that *the (collective) unity of the objects colligated, as a unity that is at once thematic and objective, occurs at a higher level than the unity proper to their colligation*. It also means that the character of the preconstituted "object" that the retrospective apprehension apprehends as the collection as such is not only different from that of the preconstituted object that the retrospective apprehension apprehends in the case of the particular object as such (the former being plural and the latter singular), but also that their statuses are different, because the particular object is already thematically apprehended (as a whole) by the preconstitutive synthesis that explicates it, whereas "in order for the collective combination, originally sprung from the act of plural explication of *A* and *B* to become a substrate—i.e., a true object, something identifiable—a *turning of regard* is first required." Thus, in the case of the collective object, the retrospective apprehension does double duty, both *thematizing it as a whole and objectifying* it, while in the case of the particular object, it objectifies a whole that has already been thematized—because the particular object has already been apprehended thematically and therefore preconstituted as a (whole) unity. Finally, this means that the content of the collective unity as an object (the collection itself or set) is not only directly related to the act of synthesis composing the collection but, moreover, *that it is precisely the retrospective thematization of the collectively relational unity established by this act that transforms the noetic unity proper to it into the noematic unity of the posited collection (set)*. In other words, the status of what is preconstituted (and therefore founding) for the noematic unity posited by the judicative spontaneity characteristic of the broader sense of judgment at work in conjunctive unity is different from what is preconstituted and founding for the noematic unity posited by the judicative spontaneity of the narrower sense of judgment at work in copulative unity—and this difference goes beyond the difference in character belonging to their respective plural and singular objects. Specifically, this difference must be articulated as follows: The preconstitution of the noematic unity of the collection as such (set) has its locus in the noetic act of collecting *and not the pre-objective correlates of this act* (the individual objects which are the objects of the collecting); the preconstitution of the state of affairs has its locus in the pre-objective correlates to the acts of explicative contemplation (the substrate together with its internal determinations).

Thus we contend that the act of thematic apprehension accomplished in the retrospective apprehension by which, as Husserl puts it, "the set is given to the ego as an object" (294/246) is an act that he clearly understands to be directed to a "synthetically unified consciousness" and not, as in the case of the retrospective apprehension of a state of affairs, to the object of a synthetically unified consciousness. It is for this reason that we hold that the act of retrospective apprehension is being called on to do something different in the case of sets than in that of states of affairs, namely, to transform a unity whose locus is in the act rather than the object of a consciousness—which is, therefore, and for that very reason, a non-objective unity—into an objective unity. This is, of course, strongly reminiscent of *Philosophy of Arithmetic*'s discredited attempt to account for the objective unity of the collection on the basis of the psychological act of collecting (which we argue below). The "passive synthesis" that Stone has singled out as problematic in Husserl's account of the preconstitution of sets is, on our view, related to but not identical with the problem we have identified here. That is because (as we have seen) Husserl identifies the source of the "principle" that unifies the synthetic unity of the apprehension of plurality as the "combination" arising from the "passivity" belonging to associations of similarity and likeness, which gives rise to the "configurations" that comprise the background basis for apprehending individual objects together "as individuals linked thematically." Even if the claim (which Husserl does not make) were to be made

collection, the "set," must have the status of an object that stands out from both the *collecting* that pre-constituted it *and* the *individual* objects that now belong to the set—individual objects that, prior to the constitution of the set as an objective unity, were "encompassed" by the (pre-objective) unity of the "colligating consciousness." The necessity that the set's objective unity possess this status stems from the dictates of Husserl critique of *Philosophy of Arithmetic*'s psychologism and the logical problem that is presented by the peculiar character of collective unity as a whole. In the case of this critique, its most basic tenet is that the content belonging to logical unity per se *cannot be established on the basis of reflexion on psychological acts, processes, or contents*. In the case of the logical problem, collective unity as a whole cannot be established by qualities inhering in either the individual members or in the relations between the members that belong to the collectivity as its parts.

In other words, what remains obscure is precisely how it is that a redirection of the regard of consciousness is able to turn the non-objective unity characteristic of the *presentation* of collected objects into the *objective* unity proper to a collection of objects, a unity that "is an object like any other." Husserl does not say, either here in *Experience and Judgment* or anywhere else in his works, *how* this is possible. Of the collection as an objective unity, he does say, "not only can it be totally identified as the identical element of many

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here that it is the "intentional whole" of this background that comprises the preconstitutive unity of the set, the problem we have identified in Husserl's account of retrospective apprehension in the case of sets would remain. For in order to be able to constitute the unity of the set as an *object of the understanding*, the retrospective apprehension of such configurations would be again charged with the task of both thematizing and objectivating a "noetic" unity, albeit the noetic unity would in this case be the passive "bond" of the associations that compose these configurations.

Finally, two more points need to be made in connection with Husserl's discussion of these issues. First, the discussion in § 24d mentions neither the 'and' nor—because the associative principle of thematic linkage is "likeness or similarity"—the apprehension of pluralities composed of thematically *different* objects. And second, the discussion in § 61, which explicitly refers to § 24d, introduces both the 'and' and pluralities composed of different objects (e.g., '*A* and *B* and *C*') into the discussion—without, however, providing an account of the transition to either from the domain of receptivity analyzed in § 24d. These inconsistencies may be partially explained by the different source material that Landgrebe used to compose the text. According to Lohmar, the first three subheadings (a–c) of § 24 are based on Husserl's "Lectures on Transcendental Logic," delivered during winter semester 1920/21 (Lohmar, "Entstehung," 45, 62), although they include supplementary additions written by Husserl at the end of 1928 and the beginning of 1929, after he had seen Landgrebe's first draft of *Experience and Judgment* (50). The entirety of § 24's subheading "d" (on the difference between the unity and content of particular and plural syntheses) was also composed by Husserl as a part of these additions. All of § 61 is based on the 1920/21 lectures, with no additions by Husserl (63). Thus, the places in *Experience and Judgment* where we have noted the inconsistencies related in these two points have their sources in material composed at different times.

modes of givenness, but it can be explicated in an ever renewed identification, an explication that is again and again a process of collecting" (294/246).

The particular force of the question being raised here derives from the strong suspicion that Husserl's appeal to the ability of the retrospective apprehension to apprehend a *collection as such* by, in effect, "thematizing" the presentation yielded by a completed process of *collectively combining* objects is vulnerable to the very critique of psychologism that he himself leveled against *Philosophy of Arithmetic*'s account of the origin of the objectivity proper to collective unity. Namely, the basic claim here is that the result of a synthetic *process* that does not have a proper *objective correlate* nevertheless yields or otherwise originates—when grasped *post factum*—a synthetic *object* as its proper objective correlate. Properly speaking, the objective correlate of the act of collecting is the objects collected into a multitude, not their collection as such. Husserl is both clear and consistent on this point. There is nothing *in* these objects, taken in either their individuality or their relations to one another, that can be considered to pre-constitute the collection to which they belong only *once they are colligated*. That is why Husserl characterizes the status of the preconstitution of the collection as such as a "noetic" unity. The unity in question is therefore manifestly not noematic at this stage in *Experience and Judgment*'s account of the constitution of the collection as an objectivity, that is, an object capable of functioning as a substrate in predicative judgments. Again, and on the contrary, Husserl is quite clear that this comes about only when the *presentation* yielded by the act of collecting, and *not* its objective correlates (which, as we have seen, are individual objects and their relations), is thematized and posited in a "retrospective grasping."

This account of thematization and positing must be radically distinguished from the one Husserl gives of the relational synthesis that pre-constitutes the objectivity belonging to a state of affairs. What is preconstituted in this case is the overlapping synthesis of a *noematically* given thing and its property (or properties), which function as the *objective* substrate for the thematization and consequent positing that is characteristic of the predication belonging to the copulative judgment. In the collective synthesis that pre-constitutes the collection as such, as we have seen, there is no objective substrate in this sense. Rather, Husserl maintains that it is the collective *presentation* yielded by the *act* of collecting that pre-constitutes, as a *noetic* unity, the collective unity as such. Husserl therefore characterizes the "objectivity" of the collection (i.e., the set) as originating in the thematization of a pregiven unity that is manifestly *not* presented as the objective correlate of an act. Rather, it is in the presentation that is inseparable from and therefore characteristic of the act itself that Husserl says the preconstitution of the

collection as such resides. And it is precisely this claim that justifies the suspicion that his account of the origin of the objective unity proper to the collection as such in *Experience and Judgment* does not advance beyond the discredited account in *Philosophy of Arithmetic.* Of course, the language differs in *Experience and Judgment* and *Philosophy of Arithmetic:* 'colligating consciousness' instead of 'psychological acts', 'retrospective grasping' instead of 'reflexion', 'noetic unity' instead of 'collective presentation', and so on. But it is difficult to avoid the conclusion that the basic account remains the same: that from a post hoc attentiveness to the (*noetic*) unification of objects that occurs in the act of combining them into a "collection" there arises the (*noematic*) unity of the logical form of the collection "itself." Moreover, Husserl's account here does not even address, let alone provide clarification of, what role, if any, signification and meaning *intentions* play both in the initial process of collecting objects together and in the retrospective apprehension of the results of this process. More precisely, the issue of whether symbolic acts and their intuitive fulfillment (to use the language of the *Logical Investigations*) are involved in the *preconstitution* (to use the language of *Experience and Judgment*) of the collection as an object accomplished in the process of collecting, or in the retrospective grasping of it as an objective categorial form, is passed over in silence.[86]

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86. Dieter Lohmar addresses precisely these issues in his "Husserl's Concept of Categorial Intuition." He inquires into what is "sufficient to fulfill the categorial intention of the multiplicity" (144) and suggests that "the categorial intention 'and' itself can be viewed as a non-sensible content" and as such "can serve as the fulfilling content of the intention of the collection" (144–45). He admits, however, that "The idea of an intention that contributes to its own fulfillment might give the impression of circularity" (144).

Specifically, because in "collections we cannot do without the contribution of the synthetic categorial intention 'and' itself," it follows that "Collections owe their intuitivity solely to the fact that we synthetically combine objects, that we collect them." Nevertheless, Lohmar argues against there being any circularity in this by pointing out, "it is the synthetic activity of combining the objects of the founding acts into a new object, the collection, that brings about the fulfillment." This, of course, is the view that we have just criticized as being, in essence, no different from Husserl's discredited psychologistic account in the *Philosophy of Arithmetic*. Lohmar appears to recognize this, and writes on "questions about what kind of fulfilling . . . contents are found in collectiva" that "[w]e might suppose that what serves" in this regard "is the experience of the performance of the act of collection (in inner perception)." However, he rejects this view, arguing that "it seems more reasonable to accept the fact" about the 'and' being, in effect, both "an intention of the collection 'a and b'" and its fulfillment, insofar as it is also "viewed as a non-sensible content." Finally, Lohmar suggests that the situation in which "the will to perform a synthetic intention is enough to fulfill an intention" (145) is, "in the realm of intentions and categorial intentions," a "special case," "an important exception," and one that does not concern "knowledge in the narrow sense" insofar as a "collection is . . . itself not a contribution to knowledge at all, though it can be an important element in knowledge if we continue to perform judgments with respect to the collectiva (or set)."

§ 175. *Experience and Judgment*'s Incomplete Account of the Constitution of Cardinal Numbers

Husserl's discussion of cardinal numbers in *Experience and Judgment* does not resolve the issue of the constitution of their objectivity as determinate collections, and it also departs from *Philosophy of Arithmetic*'s account of the generic emptiness (and therefore formal universality) of the contents belonging to their concepts. The account of cardinal numbers in *Experience and Judgment* occurs within the context of Husserl's consideration of the pluralities that fall under the heading of what he calls the "particular judgment" (*EJ*, 446/367). He distinguishes particular judgments from "*singular judgments*" (446/368), inasmuch as the latter "refer to individually determined terms, e.g., 'this rose is yellow'," while the former refer "to some *A* or other in general" (447/368). Hence, "the forms 'an *A*', 'an *A* and an *A*', or, likewise, 'an *A* and another', 'an *A* and another *A*, and again another *A*, and so on', and likewise the indeterminate plurality" (446/367–68) are particular judgments in which "we stand with them near the *origin of the primitive numerical forms*" (446/368). That is because, for Husserl, these "arise here as formations having the function of indicating the 'something or other'." Nevertheless, with both the formation indicative of the 'something or other' and the 'indeterminate plurality', we are only near but not yet coincident with the origin of such forms, because Husserl maintains (in *Experience and Judgment*) that the "*Cardinal numbers* are *determinate pluralities of particular terms*."

Particular judgments emerge according to Husserl when the "*direction of interest*" (445/367) shifts, from what it is when "the intention is involved

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In the context of our analysis, Lohmar's clear presentation and discussion of these issues raises two questions. First, is his suggestion that the 'and' fulfills the function of both the categorial intention of the multiplicity and its objective fulfillment really so reasonable? Second, is it the case that collections do not concern knowledge in the narrow sense?

Regarding our first question, there does not seem to be any textual evidence that Husserl considered the logical status of the 'and' to be, in effect, both that of a signification category and an objective category. Moreover, such a suggestion appears to reduce the objective categorial content of a set to a non-independent moment of the collection as such, namely, to the syntactical connective responsible for uniting its elements. Such a reduction would seem to exclude from the concept of the objective category of a collection the concept of members or elements, which seems extremely problematical.

Regarding our second question, if the authentic cardinal number concept is characterized as the delimitation of a plurality, that is, as a finite set, and if it is the case that cardinal numbers play a role in knowledge in the narrow sense, it would follow that collections do indeed represent a most important contribution to knowledge. Indeed, the contribution would appear to extend beyond the narrow mathematical sense of quantitative knowledge, since, as we have suggested and shall explore in more detail below, Husserl understands the full unfolding of the pure *mathesis universalis* to involve the fundamental unity of the objectivity treated by logic and mathematics, namely, the pure *manifolds* investigated by formal ontology.

in the explication of individual objects" (445/366) to "an other form of intention" (445/367). In the explication of individual objects, the intention is directed to an individual object in a manner that allows the progress of predication to unfold, to "judge predicatively" (445/366) about the object's specific qualities. In the particular judgment, the interest of the intention is "indifferent" (445/367) to the "individual specificity" (446/367) of an individual object, as it is instead "constituted as a form of meaning singulars in which it is only concerned with the identical validity of *any one A or other*," as a "general type." Such an intention thus no longer judges "'the rose is yellow', but 'a rose'," or perhaps "'still another'," or "'some roses are yellow'—some meaning one and one, and so forth." For Husserl, then, it is "this active and productive attitude which determines the activity of [the particular] judgment and saturates it in a characteristic manner" (446/368). Indeed, it is precisely this manner that is responsible for what "arises here as formations having the function of 'something or other'," formations that bring judgment "near the origin of primitive numerical forms."

The cardinal numbers arise when a particular plurality yielded by the "formations having the function of indicating the 'some or other'" is rendered a "determinate particular plurality"—*and* when the latter "is brought under a corresponding form-concept." The latter "belongs to the meaning of a cardinal number" in the sense that "by way of comparison and concept formation," for example, "some apple or other and some apple or other, some pear or other and some pear or other, and so on," the form-concept of 'some concept or other' emerges. It emerges insofar as what "is conceptually common" to the compared items in the determinate plurality "expresses itself as some *A* or other and some additional *A* or other, where *A* is 'some concept or other'." Husserl explicitly states: "That is the cardinal number concept two," which means that this concept is the conjunction of 'some *concept* or other' *and* (another) concept or other—and, he goes on, "likewise for three, etc."

Husserl's account here of cardinal number concepts as determinate particular pluralities composed of some concept or other *and* some concept or other, and so on, does not resolve the problem of the constitution of the *objectivity* of the collection under discussion. No description is provided of an objective referent that would correspond to the "collection" as something that is irreducible either to the items that are combined by the 'and' or to the *noetic* "unity" of the combining intention. Thus, neither the objectivity of an indeterminate plurality nor that of the determinate pluralities is established as something that, on the one hand, is other than the individual items belonging to either type of collection. Nor is the objectivity in question established, on the other hand, as something that encompasses these

individual items as items that *belong* either to the set or to the cardinal number. Rather, what Husserl's descriptions articulate is the combination by the 'and' of particular judgment forms proper to "some object or other" and, on the basis of the comparisons of such forms, the combination (again effected by the 'and') of judgment forms proper to 'some concept or other'. His claim that the conjunction of "some concept [A] or other, and some concept [A] or other" is the concept of the cardinal number two, and that with the conjunction of an additional 'and' together with another 'A', the concept of the cardinal number three is generated, and so on, therefore fails to account for the objectivity of the purported concepts in question, 'two' and 'three'.

Stated as succinctly as possible, this failure has two aspects. First, the objective referent of the 'and' in the case of either an indeterminate plurality or determinate pluralities (cardinal numbers) is not established. By this I mean, on the one hand, that the logical problem to which Husserl's psychologism in *Philosophy of Arithmetic* is the response is not resolved. The objectivity of the collection, as a unity that cannot be grounded on predications directed toward individual objects, is not accounted for. On the other hand, I mean that the shortcomings of *Philosophy of Arithmetic*'s psychologism are not transcended, because *Experience and Judgment* does not provide a descriptive articulation of the objectively *collective* referent of the 'and' that Husserl's own critique of psychologism demanded. Second, what J. N. Findlay noted with respect to the account of cardinal numbers in *Philosophy of Arithmetic* likewise applies here, namely, that Husserl's discussion of the determinate pluralities that compose the concepts of cardinal numbers "has not considered what may be involved in the necessary *diversity* of the abstract anythings collected, since something and something and something is not three if the anythings are one and the same."[87] Thus, even if one were to maintain what I have argued cannot be maintained, that Husserl's account of the "retrospective apprehension" of the noetic unity manifest in the act of colligating the individual items unified in an indeterminate collection is sufficient to establish the noematic unity of the collection as the objective correlate of a judgment, the problem of accounting for the diversity of the objectivities proper to determinate collections would remain. Namely, the differentia responsible for the determination of each cardinal number as a well-ordered whole that not only differs from all the other cardinal numbers but also (beginning with 'two') is successively related to the preceding cardinal number cannot be established on the basis of Husserl's descriptions of the combination of *homogeneous* elements into collections.

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87. Findlay, "Translator's Introduction," *LI*, 14.

Finally, in connection with this last mentioned point about the homogeneity of the elements in Husserl's account of cardinal numbers, it is both significant and noteworthy that the account of the scope of their "universality" in *Experience and Judgment* deviates from *Philosophy of Arithmetic*. The terms that compose the determinate pluralities in *Experience and Judgment* are *particular*, namely, *some A* or other, or *some* concept or other, which means that they contrast with the formal *universality* that characterizes the cardinal number concepts in *Philosophy of Arithmetic*. The latter's account of the 'and' characterized it as combining *any* object whatever that falls under the generically empty concept of the 'anything'. Indeed, in his self-interpretation of this matter in the "Introduction to the *Logical Investigations*" and *Formal and Transcendental Logic*, Husserl characterizes the units belonging to cardinal number concepts as falling under the *formal* concept of the 'anything whatever', the meaning of which he explicitly articulates as encompassing *any* arbitrary object or objectivity whatever. This, of course, contrasts with the judgment terms that are presented as belonging to the cardinal number concepts in *Experience and Judgment*, which we have just seen concern *some* concept or other, but not what Husserl will call there the "*arbitrary anything in general*" (452/372).[88] In fact, in *Experience and Judgment*, Husserl refers to the latter as "*a completely new form*," and he not only contrasts it with the particular judgment form, but he also characterizes it as being "*dependent*" upon it.

§ 176. The Original Givenness of the 'Universal Whatever'

Husserl traces the new form at issue in the "universal whatever" (453/372) to "possible simple, predicative judgments" (452/372) of the type that

88. In an early essay entitled "On the Theory of the Aggregate," Husserl makes a distinction between "pure numbers" and "concepts analogous" to these, "only less abstract, through formation of aggregates whose members are taken exclusively from the extension of a certain concept C" ("Zur Lehre vom Inbegriff," in *Philosophie der Arithmetik*, 385–407; "On the Theory of the Aggregate" [Essay 1], in *Philosophy of Arithmetic*, 359–83). (NB: The date of this text is most likely five or more years later than 1891, which is the date given to it by the editor of the Husserliana volume. See Stefania Centrone, *Logic and the Philosophy of Mathematics in the Early Husserl* [Dordrecht: Springer, 2010], 81.) The unity of such numbers, which Husserl calls 'qualified', is said by him to result "from the pure 1 through determination by C." However, Husserl maintains that if the unity C is thought "as undetermined, but as arbitrarily determinable . . . then it is clear that the concept of the pure number can be regarded as a special case of that concept, since the concept of the *one* presents itself, after all, as a special case of the C, of the concept in general" (390). He goes on to say, "considering the matter from this point of view, the pure number is regarded as that special qualified number whose unit is the abstract *one*." Husserl concludes that "This of course in no way conflicts with the fact that given another—namely, our earlier—point of view, any concrete number falls under the concept of pure number, so long as we precisely bring each unity C under the concept *one*."

emerge when "we perceive that this *A* here and that *A* there are *B*" (451/371). If, in "the progress of perception we find another *A* again and again, and again and again find that it is *B*," then, according to Husserl, "In this progress arises an ever stronger presumption with each new occasion, and we expect to find again the newly grasped *A* as *B*" (451–52/372). Moreover, "in this progress there is generated an open horizon of possible *A*'s as real possibilities, presumably just waiting to be found" (452/372). Finally, "we do what we are freely able to do, namely, we relate the supposed some *A* or other to this open sphere," which means that "in the attitude of particularity, 'some' *A* or other" is "thus produced before our regard" as "a presumptive *A*," and "we grasp it at the same time in the form 'something or other, whatever it may be.'" By so grasping it, we are "no longer in this merely particular attitude," because what "we present anticipatively as an open chain of *A*'s" is now grasped as "something or other, something arbitrary which emerges out of this open sphere," something that now has the status of the "thought of a universal 'something or other'" and to which is "attached in its universality a necessity of being *B*."

According to Husserl "the novelty" at issue here is not that we have "merely extracted 'something or other' in this particular form," which, as we have seen, is something that occurs in particular judgments. Rather, it consists in "a completely new meaning form belonging to the states of affairs." That is because "the directly extracted *A*, which is indeed some *A* or other, is something that could have been arbitrarily taken instead of another in the chain," and it therefore becomes, "as it were, the representative for an arbitrary something whatever." Husserl considers this account to be an "exposition of the original givenness of any universal content whatever" (453/372), which, as such, "is *a higher structural form that includes in its meaning the idea of a particular whatever* and raises it to a higher form." As "*an accomplishment of a completely new kind*, it is a judgment that does not simply place a predicate by a determinately given subject, but it generates and grasps the new universal validity belonging to such predications," namely, that when "*AB* is universally given, then universally if something is *A*, it is also *B*" (452–53/372).

Husserl's account here of the original givenness of the 'universal whatever', as a structural form that is higher than that of the idea of the 'particular whatever', raises more questions than it answers when it is considered in the context of our concerns. For instance, is this an account of what he refers to elsewhere in *Experience and Judgment* as 'formalization'? Formalization in *Experience and Judgment* is characterized as an "accomplishment" (435/359) whereby materially determined concepts are "grasped under the formal category 'anything whatever.'" As such, this accomplishment involves

"a disregarding, an emptying of all objective, material determinations." If this account of Husserl's of the shift from the merely particular attitude to the novel attitude wherein the 'universal whatever' is given is indeed an account of this disregarding and emptying, it is hard not to wonder why he makes no mention of the word 'formalization' in connection with it.

§ 177. The Representational Function of the Particular Judgment Form 'Something or Other' to Bring About Universality Is Not Fundamentally Different from the Account in *Philosophy of Arithmetic* of How a Sensuously Unified Multitude Arises

In addition to the question of the relationship between the 'universal whatever' and the formal concept of 'anything whatever', a close look at the content of the *Experience and Judgment*'s account of the origination of the former makes it hard to dispel the suspicion that its appeal to the representational function of the particular judgment form 'something or other' to produce universality is *not* fundamentally different from the signitive account in *Philosophy of Arithmetic* of how the unity of sensuously unified multitudes arises. We have seen that in the *Logical Investigations* Husserl rules out the possibility that the account in *Philosophy of Arithmetic* could possess the status of an "authentic intuition of the collection as such" (*LI*, 690/799), because its access to the unity of the collection as such is "mediated significatively." We have also seen[89] that this significative mediation is said in *Philosophy of Arithmetic* to emerge when, at a certain point in the "term by term" (*PA*, 211) collective combination of the members that belong to a certain sensuous multitude or group, this "rudimentary process then serves as the sign for the full process intended" (213). Because completing the full process would entail grasping *all* the items that belong to the group in question, which is a psychological impossibility, Husserl maintains that at a certain point in the collection process "the group intuition guarantees us that the process begun can be continued." Recall that in the *Investigations* Husserl's criticism of this account of grasping—indirectly and therefore symbolically—the unity of the collection as such was *not* that its account of such a process is inaccurate but rather that it is *insufficient* to establish the *logical* status of the unity proper to the collections in question. And it is not up to this task precisely because its appeal to a *significative* process was deemed incapable of yielding an *intuition* of the collection as such, that is, the intuition of the collection as a log-

89. See Part III, § 46.

ical object whose very meaning as *logical*—according to the critique of psychologism—can in no way be *dependent* upon a significative or particular judgment process.

It would seem that the account, mutatis mutandis, in *Experience and Judgment* is vulnerable to this very criticism insofar as it credits a significative process with the generation of a *logical* structure. Granted, *Experience and Judgment* makes no mention of acts, psychological or otherwise, as does *Philosophy of Arithmetic*. Moreover, the unity at issue in the latter is sensuous or material, whereas that in *Experience and Judgment* is formal. Nevertheless, the target of Husserl's critique of psychologism in the *Logical Investigations* does not distinguish between material and formalized logical meaning: this critique clearly maintains that the logical unity of both types of meaning is independent of the mediation signs for the precise reason that Husserl characterizes its "in itself" status as the *non-symbolic referent* of symbolic presentation. As a consequence, it may well be impossible to harmonize this characterization of the status of logical unity and form with *Experience and Judgment*'s talk of an attitude that generates universal judgment structures, and indeed an attitude whose judgmental form is initially particular, from its freely taking its anticipation of a *particular* judgment form as *representative* of "the progress of perception" (452/372).[90] And it may be impossible because the *Investigations*' critique of psychologism clearly maintains that the locus of logical unity is not signification but the referent of signification, a referent that is neither an act nor its (or its object's) symbolic presentation.

§ 178. Sum and Substance of Husserl's Accounts of the Unity of Indeterminate and Determinate Collections and the 'Anything Whatever' between *Philosophy of Arithmetic* and *Formal and Transcendental Logic*

At present, however, our main intent is not to assess the consistency of Husserl's thought on these issues but to trace the development of its treatment, subsequent to *Philosophy of Arithmetic* and leading up to *Formal and Transcendental Logic*, of the concepts that concern us, namely, the objective unity of pluralities (sets), cardinal number concepts, and the formal concept of the 'anything whatever'. We have seen, first of all, that the context in which Husserl's critique of psychologism occurs is his prior disassociation of

90. According to Lohmar, all the material in *Experience and Judgment* on the generation of universal judgment structures is drawn from Husserl's winter semester 1922/23 lecture course "Einleitung in die Philosophie" (Lohmar, "Entstehung," 48, 68).

what he characterizes as the "merely symbolic" arithmetical and logical techniques of the algebraic calculus from the spheres, respectively, of properly (conceptual) mathematical and logical cognition. Moreover, we have seen that the effect of this disassociation is twofold. On the one hand, the letter signs belonging to the merely symbolic thinking operative in the algebraic calculus are characterized as syntactical derivatives of the "rules of the game," which somehow act as "surrogates" for the genuine thinking and concepts that belong to mathematics and logic. On the other hand, proper thinking in mathematics is now characterized as part of formal logic, whereas the merely symbolic thinking of calculational technique is, strictly speaking, *not part of logical thinking and concepts at all*.

We have likewise seen that the critique of psychologism that occurs within the context of these developments after *Philosophy of Arithmetic* effects the separation *in principle* between concepts that are generated by reflexion on psychological acts, which is to say, concepts generated from inner sense perception, and the pure "in itself" of the concepts proper to the objectivity of logical form. Further, we have seen that the conscious Platonism entailed in this separation renders acute the problem of how, nevertheless, to account for the relationship between thinking and logical form. Finally, we have seen that Husserl's accounts between *Philosophy of Arithmetic* and *Formal and Transcendental Logic* of the concepts that concern us leave four main issues either unresolved or unclear.

1) In the *Logical Investigations*, the answer to the question of whether the categorial intuition of the logical form of the collection as such involves the fulfillment of a signification category belonging to a proposition is not clear. Whether the non-signitive objective unity of the logical form of a collection is something that is intended by a signification intention, that is, by a symbolic presentation, and thus is capable of being given in the concrete or ideating abstraction in which such intentions, according to Husserl, find their ultimate fulfillment, is not addressed. On the one hand, he includes pluralities and cardinal numbers—both of which have the logical status of collections—among the pure logical objects that find their fulfillment in the acts of logical judgment that render intuitively present formal logical categories. This inclusion suggests that the intuition of their objective unity would involve the fulfillment of signification intentions. On the other hand, he distinguishes the logical status of the 'and' involved in the conjunctive unity of collections from the relational unity belonging to states of affairs. This distinction suggests that the intuition of the 'and' is brought to givenness neither by the abstraction that sets into relief the non-independent moment of the categories belonging to states of affairs nor by the ideational abstraction characteristic of the intu-

ition of such categories as such. The distinction does not, however, shed light on how the collection as such is intuited as a logical object.

2) Nor is it clear in the *Investigations* whether it follows from the total separation of psychological reality and logical ideality entailed in the critique of psychologism that the pure species forms of numbers, the instances of which unify the collective presentation of the objects collected in counting, are themselves formal collections. Achieving clarity on this issue is crucial for grasping the relationship between the presentation of a) the numerical unity given (in the intuition of counted collections of objects) as an instance of the pure species form of the logical unity of the number in question and b) this pure species form *itself*.

3) *Experience and Judgment* gives an answer to the question of the status of the objective unity of a collection that is left unresolved in the *Logical Investigations*, namely, that it indeed differs from the objective unity of a state of affairs. Whereas the unity of a state of affairs is constituted in the coincidence of the propositional intention and the intuitive fulfillment that is characteristic of the "truly apophantic judgment," the partial identity and copulative form of unity of the subject and predicate "does not take place" (*EJ*, 135/212) in the synthetic grasping of a plurality. An answer to the question of how, then, the unity of the collection itself is given is also provided in *Experience and Judgment*. The collective unity is said to be given in a retrospective grasping of the noetic unity of the synthetic collecting of a plurality of objects, the noetic unity being characterized as the preconstitution of the objective unity that is grasped with its thematization. How such a grasping is able to transform the noetic unity, which is inseparable from the process of collecting, into an objective unity remains unclear in *Experience and Judgment*. It is therefore not apparent how this account of the genesis of the objectivity of the unity of the collection as such is able to meet the *sine qua non* of Husserl's critique of psychologism, that the objective concept of logical unity is something that *cannot* be generated from the abstractive reflexion on the activity of collecting.

4) Questions about the formal concept 'anything whatever' go unanswered in Husserl's analyses in the *Logical Investigations* and *Experience and Judgment*. In the *Investigations*, precisely what the process of formalizing abstraction involves, especially in relation to the two other types of abstraction from which Husserl distinguishes it (i.e., explicating and ideating), is not clarified. Beyond the claim that formalization involves abstracting contents from their material specificity, no light is shed on how this occurs. In *Experience and Judgment*, the accomplishment that generates the original givenness of the judgment form 'universal whatever' is discussed, but two questions

remain open in connection with its relation to formalization. One is whether this account is actually an account of formalization, which is characterized elsewhere in *Experience and Judgment* as a disregarding and emptying of objective material determinations, since no explicit mention of formalization occurs in the account of the original givenness of the "universal whatever" as a judgment form. The other question is how it is that a significative process, rooted in the attitude belonging to the *particular* judgment form, is nevertheless able to generate a universal logical object form. What makes the role of the particular judgment in this process especially questionable is that such a process (mutatis mutandis) was characterized as unable to accomplish precisely this in the *Prolegomena*'s critique of psychologism.

Husserl's accounts between *Philosophy of Arithmetic* and *Formal and Transcendental Logic* of the objective unity of indeterminate and determinate collections and the 'anything whatever' provide no definite, let alone unproblematic, solutions to the logical problems that *Philosophy of Arithmetic* confronted with respect to these concepts. These problems not only remain, but when the measure of the critique of psychologism in the *Prolegomena* is applied to the *Experience and Judgment*'s investigations and conclusions, the suspicion that psychologism has not been overcome cannot be avoided. In light of Husserl's own observation in the "Introduction to the *Logical Investigations*" that the problem of psychologism cannot really be overcome prior to the transcendental turn taken by his phenomenology in 1908, this conclusion should not be so surprising, especially since *Experience and Judgment* does not mention the transcendental reduction at all. However, as we shall see, Husserl's investigations in *Formal and Transcendental Logic* are likewise unable to stand up to his own critique of psychologism. And while this, too, might seem surprising, we shall suggest, to the contrary, that it is hardly surprising at all when one considers that the measure that makes Husserl's critique of psychologism possible is none other than that of Platonism.

Chapter Thirty-four

Husserl's Investigation of the Unitary Domain of Formal Logic and Formal Ontology in *Formal and Transcendental Logic*

§ 179. The Focus in *Formal and Transcendental Logic* on the Relationship between Formal Ontology and Apophantic Logic

We have already seen that Husserl is quite explicit about *Formal and Transcendental Logic*'s reliance upon the *Prolegomena*'s formulation of the relationship between formal mathematics and formal logic, a formulation that the former work characterizes as unsurpassable. The idea taken up in the former is that of building a "full and entire *mathesis universalis*" (*FTL*, 87) in which formal mathematics appears as "*the highest level of logical analytics*" (86), an idea that is rooted in Husserl's characterization of the unitary province of formal mathematics and formal (analytical) logic as that of 'anything whatever'. But unlike the *Prolegomena*, where this idea remains undeveloped, in *Formal and Transcendental Logic* Husserl begins to articulate the concrete steps necessary for its development. And he does so by focusing on a problem that "is not yet propounded in the *Logical Investigations*" (75), the "problem of the relationship between formal ontology and apophantic logic."

Husserl's characterization in *Formal and Transcendental Logic* of the status of this problem in the *Investigations* is significant, especially when it is considered in the light of our presentation of the unresolved issues in Husserl's accounts between *Philosophy of Arithmetic* and *Formal and Transcendental Logic* of the objective unity of the concepts with which we are concerned. Its significance is found in the fact that this problem is none other than that of the nature of the relationship between 1) the formal categorial intentions that in the *Logical Investigations* Husserl maintains are inseparable from predication significations and 2) the objective forms belonging to the materially

indeterminate categorial objects themselves to which these intentions are related. Husserl's characterization in *Formal and Transcendental Logic* of the exact nature of what is problematic in this relationship begins with a reference to the concluding chapter of the *Prolegomena*, which he credits with the discovery that "The *formal ontological* apriori arises in an inseparable coupling with the *apophantic* apriori (that of the predication significations)," a discovery that "necessarily brings with it a sensitivity to the problem of precisely how this inseparability should be understood." Husserl's admission here that this problem "is not yet propounded" in the *Investigations* is, of course, very important for our own purposes, since it confirms our analyses' conclusion of its lack of resolution in the *Investigations*.

In *Formal and Transcendental Logic* Husserl approaches the problem of how to understand the intentional relationship between predication significations and the formal categories to which they are (somehow) related by focusing on the relationship between formal logic—what he calls variously 'formal apophantics' or the 'apophantic apriori'—and formal ontology. As we have already briefly indicated, by the latter Husserl understands an a priori theory of objects, objects that are formal in the sense of being "modes of anything-whatever" (68). By the former, he understands traditional Aristotelian logic, albeit with a structure that has been "formalized" (80) in a manner that contrasts with the "concrete relation to reality" (70) of Aristotelian logic.[91]

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91. Olav Wiegand has called attention to Husserl's bewildering use of the term 'formal logic', pointing out that "On the one hand Husserl speaks of a 'fully developed formal logic' [*EU*, 2 (OW)], which, as *mathesis universalis*, would encompass *abstract* formal logic as well as abstract mathematics," while "On the other hand he speaks of formal logic as a *special science*." See Olav K. Wiegand, "Phenomenological-Semantic Investigations into Incompleteness," in O. K. Wiegand, R. J. Dostal, L. Embree, J. J. Kockelmans, and, J. N. Mohanty, eds., *Phenomenology on Kant, German Idealism, Hermeneutics and Logic: Philosophical Essays in Honor of Thomas M. Seebohm* (Dordrecht: Kluwer, 2000), 101–32, here 105 n. 26. (By '*abstract* formal logic', Wiegand means the logic that has broken with "the *traditional logic* that has ultimately not severed its connection to the grammar of natural language" [104].) Owing to these two ways in which Husserl refers to formal logic, Wiegand concludes, "it is up to the reader to distinguish on every occasion whether Husserl is referring to traditional (Aristotelian) formal logic [i.e., formal logic as a special science] or to modern mathematical logic as a discipline within the *mathesis universalis*" (105).

In addition to agreeing with Wiegand's account of this ambiguity, we also want to stress that what underlies it is precisely Husserl's recognition that Aristotle's logic (and, therefore, traditional "Aristotelian logic") is *not* formal in the sense that modern logic is formal, because the latter but not the former operates in the "sphere" (*FTL*, 42) that arises through the processes of formalization or algebraization. What it means to operate in that way, and indeed how these processes may be related and what a phenomenological account of their constitution should look like, is, of course, the focal point of our concern with Husserl's account of the origin of the logic of symbolic mathematics. Moreover, on our view, the failure to main-

Husserl characterizes Aristotelian logic from the perspective of "a formal logic, as we understand such a discipline today and as Leibniz already understood it in effecting his synthesis of formal logic (as apophantic) and formal analysis to make the unity of a *mathesis universalis*" (42). Thus, on the one hand, Husserl credits Aristotle with being "the first to bring out the idea of form which was to determine the fundamental sense" of formal logic as it is understood today. He regards Aristotle as "the first . . . to operate in the *apophantic* sphere." (Husserl characterizes the latter not only as "the sphere of assertive propositions {judgments in the sense expressed by the word in traditional logic}" but also as the sphere of "'formalization' or algebraization" that "makes its appearance in modern algebra with Vieta.") On the other hand, however, Husserl recognizes that Aristotle "lacked formal ontology, and therefore lacked also the cognition that formal ontology is intrinsically prior to the ontology of realities" (70). The main consequence of this lack is that "Aristotle relates his analytics to the real world and, in so doing, has not yet excluded from his analytics the categories of reality" (43).

By approaching Aristotle in this manner, Husserl clearly exemplifies the pattern for interpreting ancient science that Klein claims began with Vieta, namely, viewing it from the perspective of its modern formalization. Husserl therefore measures Aristotle's achievement in terms of how close he comes to the modern understanding of "formal 'analysis'" (42) as something that is distinct "from all material mathematical disciplines." In consistency with this approach, Husserl understands Aristotle's use of letters in his logic as substituting "algebraic letters for the words (terms)" in "materially determinate propositions," that is, in propositions that refer "to divers material provinces or single matters" (42–43). Husserl says that this use "implied that he substituted the moment 'arbitrary anything' for each materially filled 'core' in the judgments, while the remaining judgment-moments were held fast as moments of form." Notwithstanding this implication, however, Husserl also notes that "in Aristotle the variability of the terms is not completely free, and consequently the idea of form is not quite pure" (43). Aristotle's logic is not pure because, as we saw above, on Husserl's view he related

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tain the radical distinction at stake here between the fundamentally different "formalities" of Aristotelian and modern logic conceals the important historical aspect of this problem, which is tacitly recognized in Husserl's account of this distinction (without, to be sure, being pursued by him either in *Formal and Transcendental Logic* or elsewhere). It is for this reason that we cannot follow Sokolowski's recent suggestion that "Aristotle carried out his formalization and analytics" within the horizon of "the 'real world,'" because, strictly speaking, Aristotle's logic did not carry out any formalization (see § 207 below). See Robert Sokolowski, "Review of Dieter Lohmar, *Edmund Husserls 'Formale und transzendentale Logik'*," *Husserl Studies* 18 (2002), 233–43, here 241.

his logic to categories of reality and, as such, "had a universal ontology of realities only" (70).[92] Moreover, on Husserl's view "*this* was what he accepted as 'first philosophy.'"

§ 180. Modern Logic and Mathematics Are United by the Formalization of Their Relation to Reality

The problem that is taken up in *Formal and Transcendental Logic*—the relationship and indeed unity between a) formal logic and b) mathematics understood as formal ontology—is one that "could not confront the ancients," because neither their logic nor their mathematics was formal. The problem of how to understand properly what the *Logical Investigations* characterized as the relationship between signification intentions and their fulfillment in formal categories is what is at stake in this relationship. The *Investigations*' formulation of this relationship is a consequence of Husserl's critique of psychologism, and because of this he emphatically characterizes it in a manner that is doubly non-psychological. Thus, in the *Investigations* these categories can neither originate in inner sense perception nor depend on any aspect of, or connection with, empirical psychological reality for their logical meaning. We have seen that the fact that Husserl treats this problem as unresolved in his logical works prior to *Formal and Transcendental Logic*[93] supports our claim that these works do *not* resolve satisfactorily the question of the relationship between symbolic presentations or thinking and the formal *mathematical* categories to which they *must* be related in order to circumvent—on his own terms—the critique of psychologism.

For Husserl the key to establishing the unity of formal logic (understood as formal apophantics) and mathematics (understood as formal ontology) lies in what he takes as the *fact*[94] that both are related to the realm of the

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92. Having made this qualification of the degree to which Aristotle's logic implied the formalization and algebraization that makes modern analysis possible on his view, it is interesting to note that Husserl does not similarly qualify the status of Aristotle's logical employment of letters. Such a qualification appears to be called for, because once it is admitted that the formalization that makes algebra possible was not accomplished by Aristotle, it would follow that the function of the letters in his logic should no longer be characterized as 'algebraic'.

93. In this connection, the fact that *Formal and Transcendental Logic* grew out of what Husserl initially intended to be an introduction to the analyses presented in *Experience and Judgment* is significant, because if the latter work was understood by him to have resolved this problem, it is more than likely that Husserl would have given some indication of this in *Formal and Transcendental Logic*.

94. In his introduction to *Formal and Transcendental Logic*, Husserl characterizes his method as "presupposing the sciences, as well as logic itself, on the basis of the 'experience' that gives them to us beforehand" (8). While he immediately qualifies the nature of such ex-

empty 'anything whatever'. Indeed, we shall show that it is because of this conviction, *and not because of any phenomenological analyses that he has performed*, that Husserl thinks that the objective unity of pluralities (sets) and cardinal numbers has its foundation in formal logic. Husserl therefore thinks that the mathematical unity of sets and cardinal numbers has its basis in their relation to the objective unity belonging to the realm of the empty 'anything whatever'. He characterizes this relation as one that they have in common with the logical unity of each level of what *Formal and Transcendental Logic* identifies as a "*threefold stratification of the fundamental concepts—and therefore the disciplines—of formal logic*" (10). These concepts concern, respectively, categorial forms, the principle of non-contradiction, and pure manifolds. The disciplines devoted to each, again respectively, are the pure theory of categorial form, including the pure logical grammar of meaningful and meaningless propositions in apophantic logic, the pure logic of consequence or validity, and pure analysis (otherwise referred to by Husserl as '*pure mathesis universalis*'). Thus, rather than "appear as unquestionably separate sciences" (70), as the ancients thought, logic and mathematics are united by the formalization of their relation to reality. What is problematic for Husserl is *not* this formalization per se, since it is something that is unquestionably given beforehand by the sciences of logic and mathematics, but *how* its meaning and sense is to be properly understood by the *mathesis universalis*, the essentially new science their unity makes possible.

What distinguishes this new science from "[t]he earlier level of logic" (79) is its theme. The earlier logic, by which Husserl presumably means traditionally developed non-(algebraically) formalized Aristotelian logic, "had taken as its theme the pure forms of all significational formations that, as a matter of a priori possibility, can occur *within* a science." By contrast, what "now becomes the theme" for the pure *mathesis universalis* is "the *judgment system in its entirety*, which constitutes the unity of a possible deductive theory" and which, as such, is "a possible 'theory in the rigorous sense.'" It is this theme, which Husserl also characterizes as the "idea of a formal theory of theory forms or the formal theory of manifolds," that he says he first caught sight

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perience as involving "cultural formations given to us beforehand and bearing within themselves their meaning, their sense," it is clear that the characterization of the "realm of the empty 'anything whatever'" as *the* object of logic and mathematics, or, in a formulation that makes no difference at all to Husserl because it has essentially the same meaning, as the *object* of modern formal logic and formal mathematics, has its basis for him in its "pregiven" meaning and sense. Sokolowski considers these to be "remarks on Husserl's 'hermeneutics'" (*Husserlian Meditations*, 272 n. 2), because "Husserl says he intends to accept 'intellectual formations' from the tradition and radically investigate their sense by bringing them to original clarification" (271).

of in the *Prolegomena* and that now, in *Formal and Transcendental Logic*, he is attempting to develop. For our purposes, it will be sufficient to show that Husserl's discussions of the unity of the province of formal logic and formal mathematical analysis are not only presented by Husserl himself as provisional, but also that, in light of our guiding concern, they leave open more questions than they resolve. Questions regarding the putative non-psychological origin of the objective unities of sets and cardinal numbers, as well as the objective unity of the formal category 'anything whatever', will be shown to remain unanswered in *Formal and Transcendental Logic*. We shall also examine Husserl's account of what he calls "'[t]he most universal idea of a theory of manifolds'" (80) and show that his attempt to establish a transcendental phenomenological program of research to account for the origin of this idea's most basic concept, the formal category 'anything whatever', *presupposes* rather than delineates (in either a programmatic or more developed fashion) the formalization—which we are *not* disputing—that makes this concept possible. And, finally, we shall show that the origin of the objective unity of pluralities (sets) and cardinal numbers likewise remains *presupposed* and that, related to this, the precise nature of the relationship between the signification intentions belonging to pure apophantic logic and their putative fulfillment in the formal categories of formal ontology remains unresolved.[95]

§ 181. The Concept of a Singular Mathematical Manifold and the Most Universal Idea of a Theory of Manifolds

Husserl's articulation of the most universal idea of a theory of manifolds begins with an account of a singular mathematical manifold, characterized as "'[t]he objective correlate of the concept of a possible theory, determined only in its form'" (79).[96] As such, a manifold "'is the concept of *any possible province of cognition that would be governed by a theory having such a form*.'" Because it is a province of cognition, Husserl understands objects to belong to the manifold. Indeed, it is precisely the condition "'that among the objects belonging to the province, *certain* connections are possible,'" connections that "'come under *certain* fundamental laws having such and such a *determi-*

95. This is to say, using Husserl's earlier formulation of these issues, it is the relationship between symbolic thinking in mathematics and the non-symbolic categories symbolized by such thinking that remains unresolved.

96. Here and below, Husserl is quoting what he himself wrote on the topic at hand in the *Logical Investigations*. Thus, Husserl's conviction that the *Investigations*' articulation of the idea of a theory of pure manifolds cannot be improved upon is very much in evidence, for the discussion of these issues in *Formal and Transcendental Logic* takes as its point of departure the status of the problem established in the former text.

nate form,'" which Husserl characterizes as "'the sole determining feature'" of the condition that brings about a manifold. Being "'uniquely and solely determined by its falling under a theory of such a *form*,'" the composition of the objects of a manifold are "'neither determined directly as individual or specific singularities, nor indirectly through their material species or genera.'" Rather, they are determined "'exclusively through the *form* of the connections'" attributed to the manifold by the theory to which it is the objective correlate. Husserl takes such form connections to pertain to the "'objects belonging to the province'" proper to a manifold, connections that, like the objects they lawfully connect, have no determinate content and therefore "'remain completely indeterminate with regard to their matter.'"

Husserl distinguishes the *theory* of manifolds from this account of a singular mathematical manifold on the following grounds. Whereas the singular manifold is the objective correlate of the concept of a possible formal theory, the theory of manifolds "'develops in a determinate manner the essential types of possible theories and correlative provinces and explores the manners in which those types are lawfully related to each other'" (80). It is able to do so, because the laws that determine a singular manifold, "'as they determine a *province* and its *form*, likewise determine the theory to be constructed, or, more correctly, *the theory's form*.'" Therefore, "'In the theory of manifolds, e.g., '+' is not the sign for numerical addition, but for any connection for which the laws of the form $a + b = b + a$, etc., hold.'" That is, while in a singular manifold '+' may indeed function as a sign for numerical addition, in the *theory* of this singular theory form the plus sign signifies merely the possibility for the lawful connection of the formal objects belonging to the manifold in question. And this means that their specific *formal* determination—in the numerical example, their determination of quantities—is of no account for the theory of manifolds. Specifically, "'the manifold is determined by the fact that its thought objects render possible these and other 'operations' that can be shown to be compatible a priori with them,'" and it is the task of the theory of manifolds to explore and establish just such a priori compatibilities among *theory forms*.

From the perspective of the most universal idea of a theory of manifolds, Husserl draws the following conclusion: "'All actual theories, then, are specializations or singularizations of corresponding theory forms, just as all theoretically fashioned cognitive provinces are *singular* manifolds.'" Moreover, from this same perspective he also concludes, "'If the formal theory in question has been actually worked out within the theory of manifolds, then all the deductive work for the construction of all actual theories having the same form has been done.'"

§ 182. The Idea of a Theory of Manifolds, as the Form of a Deductive Theory, Is "Naturally" Founded on the Categorial Concepts Belonging to the Lower Level of Deductive Theories

In *Formal and Transcendental Logic* Husserl supplements these "'elucidations'" (81) by characterizing "[t]he new supreme concept of the discipline here in question" (80), that is, the pure theory of manifolds, as a "*form of a deductive theory*, or of a 'deductive system.'" Especially important for our purposes is Husserl's understanding that this concept, "naturally," is "founded on the categorial concepts belonging to the lower level" of deductive theories or systems, and his view that, "in addition to the task of providing a formal definition for the new supreme concept, there are infinite tasks" closely connected with providing such a definition. Husserl enumerates these tasks as follows:

> not only of differentiating the forms subsumed under the new supreme concept, of projecting, in their explicitly systematic developed state, possible forms of deductive theories, but also of recognizing various deductive theory forms of this sort as singularities subsumed under higher theoretical form universalities, of differentiating in a systematic theory the particular determinate forms subsumed under each of those higher form universalities—and ultimately under the highest idea itself, that of any theory form, and deductive theory, whatever.

Husserl thinks "the modern mathematical theory of manifolds and, ultimately, the whole of modern formal analysis, is already a realization—partial, to be sure, but in living development—of this idea of a science of possible deductive systems" (80–81). Hence, he characterizes his account of the theory of manifolds as "a necessary explication of the sense that Leibniz had in mind" (81) of a *mathesis universalis*. Husserl explicates this sense, in conformity with his understanding of the *non-logical* status of the signs and calculational operations employed by both the modern mathematical theory of manifolds and modern analysis,[97] as follows:

> In the definition of a manifold we must not define merely in terms of signs and calculational operations—for example: "It shall be allowed to manipulate the given signs in such a manner that the sign $a + b$ can always be substituted for $b + a$." Rather we must say: "There shall obtain among the *objects* belonging to the manifold (conceived at first as only empty anythings [*leere Etwas*], as "objects of thinking") a *certain combination* form with the *law* form $a + b = b + a$—where *equality* has precisely the signification of actual equality, such as belongs to the categorial logical forms. Which logical categories are to be introduced by the definitions is a matter of choice, though the choice is restricted by the requirement of non-contradiction; but in any case they must be meant, and designated, as those entirely determinate categories. (88)

97. See § 156 above.

Three things need to be highlighted here for our purposes. First, Husserl explicitly understands the referent of the signs that express the law form proper to the combination form belonging to a manifold as the formal concept *Etwas*, or, more precisely, as a *plurality* of such concepts, namely, 'empty anythings'. Second, he explicitly understands the form of the lawfulness characteristic of their combination form *to signify* the "actual" equality of categorial logical forms. And third, the logical categories introduced into the definition of a manifold are themselves non-contradictory determinate categorial forms, *and not merely the signs that*, *Husserl maintains, express the law forms pertaining to such categories*.

If we reformulate the three points we have just highlighted in the idiom of the *Logical Investigations*, the first would have to be expressed as: the signs 'a', 'and',[98] 'b' all belong to a signification intention that finds its fulfillment in a conjunction, which is to say, in the categorial form that yields a lawfully formed combination of materially empty formal concepts. The second as: the signs $a + b = b + a$ belong to a signification intention that finds its fulfillment in categorial logical forms that are presented as identical in *categorial intuition*. And the third as: the non-contradictory determinate categories introduced into the definition of a manifold bring about a singularization of the manifold, that is, they bring about an actual theory whose form and corresponding province of determinate categorial objects belong to what is now a manifold with formally *determinate* categories.

§ 183. The Categorial Unity of the Collection Itself and the Origination of the Form-Concept 'Any Object Whatever' Are Relevant to the Task of Accounting for the Relationship between Formal Apophantics and Formal Ontology

On the basis of this reformulation, the following is apparent. First, Husserl's account of 'empty anythings' as the referents of the algebraic letter signs 'a', 'b', and the word 'and' means that he understands the theory form that composes a manifold to establish the relationship among a *plurality* of *objects* to which this theory form refers.[99] These 'empty anythings' belong together in the objective unity of a materially empty but nevertheless lawfully deter-

98. Note that 'and' is being considered here as the sign for the conjunction of that to which the signs 'a/b' refer, namely, the combination form or collection of formal concepts belonging to the domain of 'empty somethings'.

99. On this point Sokolowski writes, "A theory form is correlated to a multiplicity [i.e., manifold]. This a group of objects determined only by the characteristic of being governed by that theory form" (*Husserlian Meditations*, 285).

mined collection of categories. This means, in turn, that the problem of accounting for the ground of the logical unity of the theory form proper to the relationship of these 'empty anythings' remains relevant to the investigations in *Formal and Transcendental Logic*. In other words, the problem of accounting for the objective unity of the collection itself, which Husserl maintains in his investigations prior to the latter work is founded in *neither* a state of affairs *nor* signification intentions,[100] must remain relevant to the stated problematic of that work.[101]

Second, the 'empty anythings' that comprise the *objective* content of the manifold, and whose relationship (as a function of a combination form) is defined by the laws composing the manifold's *theory form*, must be—as the *elements* that are unified by the laws of the manifold to which they belong—somehow already available to and therefore in some sense independent of the laws that govern their unification into a manifold.[102] That is because the combination form responsible for this unification must signify *actual* categories on Husserl's view, in the sense that this form must pertain to formal categories that are both determinate and non-contradictory. This means for Husserl not only that the domain of the form-concept 'any object whatever' is part of the concept of the pure manifold but also that, as such, this domain's origination is therefore tied to an actual manifold and its origination. In other words, the origination of the objective unity of the combination *form* that composes a given (singular) manifold, as well as of the pure combination *form* as such that composes the pure form of any manifold whatever, is somehow distinct from the origination of the domain of the form-concept 'any object whatever' whose lawful combination is governed (immediately) by both the former combination form and (mediately)

100. See § 170 above.

101. That is, it remains relevant if the investigations in *Formal and Transcendental Logic* are to be consistent with what we have shown is ruled out in his earlier logical investigations, i.e., either the "state of affairs" or signification intentions being the source of the objective (i.e., substrate) unity proper to the collection itself.

102. On this point, Sokolowski writes in his "Review of Lohmar," 241: "A theory form is a systematic group of judgment forms, proof forms, etc., and a multiplicity [i.e., manifold] is the set of objects to which the theory forms refer. The two, theory form and multiplicity, are positioned over against one another. A multiplicity is a range of objects that are characterized by being the domain to which a given theory form applies. A multiplicity is the set of objects, formally considered, that a given theory governs." Whether the theory form and manifold are in fact "two," and what exactly is involved in the formal consideration of a "set" of objects, is precisely what we are emphasizing remains unresolved in *Formal and Transcendental Logic*, and remains so when measured against Husserl's own stipulated terms for their resolution. See below (§§ 208–9), where we show that Husserl's analyses fall short of his own criteria for meeting adequately these terms, i.e., overcoming psychologism and providing a phenomenologically genetic account of the origin of the formally empty concept 'anything whatever'.

by the latter. The origination in question is distinct, notwithstanding the inseparability of a) the principles of the combinational form's unity and b) the unitary form elements that compose any manifold whatever.

The realization of the task of accounting for the relationship between formal apophantics and formal ontology, which has as its goal the provision of a phenomenological foundation for the development of the idea of a pure *mathesis universalis*, therefore also involves the task of accounting for the logical unity of the combination form that lawfully determines a multitude of formal categories. That is because it is this form that is the referent of the signs composing the symbolic calculus. The task of providing a phenomenological foundation for the formal objects that belong to the domain of the 'anything whatever' is likewise inseparable from the development of the idea of a pure *mathesis universalis*, as is the task of establishing this formally objective domain as the unitary province of the two sciences that investigate it, formal apophantics and formal ontology. In other words, the stated goal of *Formal and Transcendental Logic* to develop the idea of a pure *mathesis universalis* hinges upon 1) resolving the question of the origin of the *logical* unity of a multitude, 2) accounting for the formalization that yields the materially empty domain of 'any objects whatever', and 3) establishing that these formal objects compose the unitary province of both formal logic and formal mathematics.

Regarding (1), Husserl's account in *Formal and Transcendental Logic* remains consistent with that in *Experience and Judgment*. He thus notes that in the plural judgment, "the plural . . . is not the object in the precise sense, it is not the object 'about which' judgment is made, and thus the plural is not the substrate of determinations" (*FTL*, 69). The transformation of the plural into the object about which judgment is made, as the substrate of determinations, requires "operations" that are found "in the formal theory of judgments, as a theory of pure forms." Husserl maintains that in this theory "operations are present by which the plural judgment form can be transformed into the form of the singular predication about the collection." His term for these operations is 'nominalization'.

Considered in the context of our discussion of the account of the constitution of the collection as an object as such, three things stand out in what Husserl writes here about nominalization. First, neither plural judgments nor singular predications about collections themselves are his concern. Rather, at issue are the *forms* of such judgments and predications. Second, the transformation Husserl characterizes here *presupposes* that the collection, as the result of a plural judgment, has already been constituted. This is evident in his talk of "operations" that yield the (form of the) singular predi-

cation *about* the collection. Hence, what is at issue in nominalization is *not* the constitution of the objectivity of the collection, as a logical structure whose unity is distinct from i) the objects that fall under its unity *and* ii) the act of collecting in which this unity is presented. Third, Husserl does not describe the operations that he credits with bringing about the transformation of a plural judgment form into a singular judgment form about the collection as such. Regarding the operations belonging to nominalization, what he does discuss concerns the universal judgment form *S* is *p*. He says that this form "can be converted, by 'nominalization,' into a judgment about the state of affairs, *S* is *p*, or into the judgment about the quality *p*, in the form *p belongs to S*" (69–70). But neither does he elaborate here (or elsewhere) how this occurs.[103]

Later in *Formal and Transcendental Logic*, Husserl returns to the topic of nominalization, and he again mentions how "the plural that makes its appearance in judging and, on being 'nominalized,' on being transformed into the object in the preeminent sense (substrate, the 'object about which'), yields the *set*" (95). As in the earlier discussion, nominalization is characterized here under the rubric of the 'theory of the forms of judgments,' which again means

103. In a footnote to this discussion, Husserl refers to passages in the *Logical Investigations* and *Ideas I* that deal with what the former calls "nominal formations" (*LI*, 685/796) and the latter the "*law of* '*nominalization*'" (*Ideas I*, 286). Those in the *Investigations*, however, do not deal with the question of transforming a judgment about plurality into a predication about a collection, while those in *Ideas I* assert but do not elaborate how "*the plural consciousness can be essentially transformed into a singular consciousness*, a transformation that draws from out of the plural consciousness the plurality as *one* object, as something single." The discussion of this in *Ideas I* follows Husserl's articulation of the law of nominalization as a law "evinced in logic" according to which "something nominal corresponds to every proposition and to every component form distinguishable in the proposition." This means that just as "the nominal that-proposition corresponds to the proposition itself, let us say, to 'S is P'" (*Ideas I*, 276), e.g., "in the subject-place of new propositions being-P corresponds to 'is p'," so too "*plurality* [corresponds] to the plural form" (277/287). How this occurs in any of these cases, but especially the case that concerns us, namely, how from the plural consciousness the plurality as a singular logical object is constituted, that is, the plural as a singular objectivity or what in the *Investigations* is referred to as the 'collection as such', Husserl does not say.

Bearing in mind that *Formal and Transcendental Logic* was originally intended to be an introduction to *Experience and Judgment*, one could speculate that Husserl's reticence here regarding *how* nominalization occurs is explained by the fact that the latter text explicitly discusses the thematic apprehension of the collection as such. Indeed, *Experience and Judgment*'s discussion of precisely this issue contains a reference to the second discussion (mentioned immediately below) in *Formal and Transcendental Logic* of nominalization (*EJ*, 247 n. 1/201 n. 1). An Author's note in the first discussion in *Formal and Transcendental Logic* of this topic also contains a reference to the second (*FTL*, 70 n. 1). In connection with this, however, see § 174, where we have discussed in detail *Experience and Judgment*'s account and its shortcomings when measured against Husserl's own standard of the critique of psychologism.

that the pre-predicative constitution of the objectivity of the collection is something that is *presupposed* rather than accounted for in his talk of the plural being nominalized. This is particularly evident when Husserl acknowledges "that one can collect and count without forthwith incorporating the produced formations[104] in actual predications." This clearly implies that the "plural," in the guise of collections and cardinal numbers, has the status of formations that are distinct from (the activities of) collecting and counting, and are such *prior* to being nominalized (incorporated in actual predications). In keeping with this, Husserl continues: "Collecting and counting are 'objectivating' (doxic) activities like the predicative activities" (95–96). That is to say, "they have the same modalities of believing as predicative activities, as they can be brought to bear on all conceivable substrates (anything whatever), their formations consequently being modes of the same formal categories" (96). Husserl does not mention how this happens, that is, *how a collective modality of belief can be objectifying in a manner that yields collections themselves as formal categories*, but instead refers (in a footnote) to *Philosophy of Arithmetic*,[105] where "already essentially the same point was made." Yet he does point out that "the essential nature of these formations is such that all of them can be incorporated into predicative judgments and given additional forms in these." And this again bears out the point that rather than account for the constitution of the objectivity of the collection (and cardinal numbers) as a logical structure distinct from 1) the act of collecting (and counting) and 2) the individual objects that compose the collection (or the cardinal number), the logically formal operations of "nominalization" presuppose both this objectivity and its constitution.

Given our current concern, this point is absolutely crucial, of course, since what is at stake is Husserl's claim (and the possibility of establishing it), that "*all the forms belonging to doxic 'positings' and doxic posita*—all the forms that we ever call *formal-ontological*—must *also* occur in the course of that universal treatment of all apophantic forms which is demanded for a formal logic." They must do so because

> Ultimately *all the forms of objects*, all the derivative formations of anything-whatever, do make their appearance *in formal apophantics itself;* since in-

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104. That is, the "formations" brought about by collecting or counting, which are, respectively, multitude and number.

105. Specifically, he refers to the discussion of the concept of 'one and something' in *Philosophy of Arithmetic* (84–85), though this does not address collecting and counting as objectivating activities. Rather, what is addressed is the one, multiplicity, and cardinal number as the "most general of all concepts, and most empty of content—as form concepts or *categories*" (84), the "all-encompassing character" (85) of which Husserl explains "in the fact that they are concepts of attributes which originate in reflexion directed toward psychical acts."

> deed, as a matter of essential necessity, determinations (properties and relative determinations), states of affairs, combinations, relationships, wholes and parts, multitudes, cardinal numbers, and all the other modes of objectivity, *in concreto* and explicated originaliter, have being for us—as truly existent or possibly existent modes—as only making their appearance in judgments. (69)

It must be added at once that what Husserl elsewhere refers to as the need to "heed this belonging together" (96) of what "seem to be two different sciences, separated by their provinces" (69), namely, traditional formal analysis in mathematics and the traditional theory of judgments in logic, he sees falls short of solving the problem of the unity of all apophantic forms with all formal-ontological forms of objects. Thus, Husserl remarks, "With the consideration of this,[106] the problem of the unity or diversity of logical analytics and formal mathematics can by no means be regarded as already solved, although the thought of their unity does receive some force from this quarter" (70).

§ 184. The Transition to the Critical Attitude in §§ 44–45 of *Formal and Transcendental Logic* Does Not Account for the Unity of Formal Ontology and Formal Apophantics

Husserl does not elaborate here on the reason or reasons why these considerations fall short of resolving the problem of the unity of formal logic and formal mathematics, a unity, it bears repeating, whose establishment is tantamount to the realization of the idea of a pure *mathesis universalis*. Some commentators have concluded that Husserl's characterization of the emergence of the "critical attitude" (111) in two closely related sections of *Formal and Transcendental Logic* (§§ 44–45), the first of which is entitled "The Shift from Formal Analytics as Formal Ontology to Analytics as Formal Apophantics," provides analyses that do resolve the problem.[107] They base their conclusion on an interpretation that equates the shift announced in the heading with a change in the modality of the object of cognition's existence, from its naive acceptance as existing to its becoming questionable, such that it is merely *supposed* to exist. The change in modality is characterized as entailing a change in intentional focus, from an uncritical grasping of objectivities as

106. Namely, 1) that all the forms of objects only make their appearance in judgments and 2) that judgments, via "nominalization," can be converted into judgments about "states of affairs."

107. See Sokolowski, *Husserlian Meditations*, 271–81, and "Review of Dieter Lohmar," 237–39, as well as Wiegand, "Incompleteness," 109–10. See also n. 118 below, which discusses their interpretations of this section.

simply straightforwardly there to a critical treatment of them in which they are posited in reflection as mere suppositions. The identification of formal ontology with the straightforward focus and formal apophantics with the critical focus is held to account for the unity of and the distinction between formal analytics as formal ontology and as formal apophantics (apophantic logic): they are united insofar as they both investigate the same objectivity; they are distinct insofar as formal ontology investigates it with a straightforward focus and therefore as an actuality and formal apophantics investigates it with a critical focus and therefore as a proposition.

A close look at these sections, however, reveals that only the heading of § 44 makes explicit the problem of the relation between formal ontology and formal apophantics, because in the analyses of both sections neither formal ontology nor formalization is mentioned in relation to formal apophantics.[108] Moreover, Husserl's account of the critical focus (or attitude) of judging neither opposes it to a judging whose attitude is straightforward nor equates it with a focus on objectivities in a manner that, by apprehending them as supposed, brings about the judgment or proposition as such in a manner that would account for the shift announced in the section heading. Rather, Husserl's account of the critical attitude is more complex, as it presents both straightforward judging and the reflexion on meaning as components of a critical attitude that *moves between* these objective and propositional modes of judging. Thus, in Husserl's analyses, it is only when the *pre-critical* distinction between "actual" and "supposed" objectivity is transcended that the critical attitude and with it, the "supposed objectivity, as supposed" (the judgment in the traditional Aristotelian sense), is secured. And, finally, Husserl articulates "narrower" and "broader" concepts of "supposed categorial objectivity, as sup-

108. Both the preceding section in *Formal and Transcendental Logic* (§ 43), entitled "Analytics, as Formal Theory of Science, Is Formal Ontology and, as Ontology, Is Directed to Objects" (106), and § 44a make explicit mention of formal ontology and (in the case of § 43) of formalization. However, the remainder of § 44 mentions only "categorial intuition" (108), "categorial formations" (or "categorial form"), "categorial objectivity" (112), and "formable categorialia," with no further mention of formalization or whether the "objects" (and the intuition of them) being discussed have undergone formalizing abstraction. One consequence of this is that it is not clear whether the "objectivity" and "states of affairs" to which Husserl is referring in connection with the "critical attitude" (111), and the "transition through" (112) it that he maintains is "necessary" for scientific cognition and scientific judgment (see our discussion below) have been formalized. The absence of such clarity, we want to emphasize, means that Husserl's discussion of the change in "*thematizing focus*" in § 44 sheds no light on the topic announced by this section's heading. And while the closely related § 45 *does* distinguish the "narrower" concept of judgment in "apophantic [and presumably formalized] logic" from the "broader" one of "traditional logic," it does so without addressing how formalizing abstraction is related to or otherwise connected with the apophantic concept of judgment under discussion.

posed," in a manner that appeals—at most—only indirectly to the distinction between the incompletely formalized logic of the tradition and the algebraically formalized logic proper to the *mathesis universalis*. Because of this, the problem of the relationship between analytics as formal ontology and analytics as formal apophantics is not addressed in these sections, notwithstanding the title of § 44.[109]

§ 185. The Analyses in §§ 44–45 of the Transition to the Critical Attitude and the Emergence of the Broader and Narrower Concepts of Judgment

For Husserl, "That which comes into being and has come into being as judged in the judicative accomplishment, that which then, as an ideal objectivity, is always reidentifiable" (107), can be focused on in one of two ways: either as "the actuality existing" (112) for the judger "in his straightforward judging" or as "a *supposed objectivity as supposed*." In the straightforward judging, "judging is always believing something, having something 'before one' as existent, whether one has it there intuitively or non-intuitively" (108). By contrast, "the categorial formations which previously were simply existing objectivities for the judger" (108) can be focused on as something that "must be verified by going over to the evidence, the 'categorial intuition,' in which they would be given originaliter as they themselves, verified, cognized as *truly and actually existing*." That which is focused on as being in need of verification has for Husserl the status of "a *supposed objectivity as supposed*" (112), and is characterized by him as "the mere correlate of the 'supposing' or 'opining' (often spoken of as the opinion, δόξα)."

Husserl characterizes the "continual alternation" (112) between the actuality existing for the judger in straightforward judging and the supposed objectivity as supposed as "the *transition through the critical attitude*." He maintains that this is "a transition necessary to every scientific cognition and therefore one that every scientific judgment must undergo." It is within the context of this transition that he characterizes both "the broadest concept of a 'supposed categorial objectivity as supposed,'" the judgment as it functions in the sciences—"disregarding logic itself"[110]—and the "*narrower concept*,"

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109. According to Lohmar, "there is no doubt that they [the section headings of *Formal and Transcendental Logic*] are done by Husserl alone. At least I know nothing that points in another direction" (private correspondence).

110. This is an example of a reference to 'logic' wherein Husserl's failure to distinguish terminologically between Aristotelian (traditional) and modern, completely formalized logic (see n. 91 above) creates significant challenges for the reader. Indeed, this whole passage (see

in which "apophantic logic" prefers to "frame" its "theme," namely, its theme of "*what is called the judgment* (*apophansis*) *in traditional logic*."[111] Husserl maintains that the narrower concept "completely includes" the broader one, "though of course not as a specific particularization." Rather than one of particularization, the relationship between the broader concept of the judgment operative in the sciences (which is also understood by Husserl to be what traditional logic calls the judgment) and this very concept of judgment *as it is made the theme* of apophantic logic, is one of modalization.[112] In the (narrower) concept of judgment proper to apophantic logic, the "supposed categorial objectivity as supposed," which characterizes the judgment in the sense of traditional logic, is itself "supposed" (and hence modalized). Or, more precisely, "the *predicative judgment* (the apophansis as a *self-contained unity of determination*)," which, according to Husserl, is "constantly privileged" by scientific judging, because it "is directed to the specific scientific region it is

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also the following note) is illustrative of this. We surmise that the logic Husserl is "disregarding" here is *both* its traditional and modern formalized versions. We base this on the distinctions Husserl makes in this section (*FTL*, § 45) between 1) the broadest concept of judgment operative in the sciences, which is "*called the judgment*" (112) in traditional logic, 2) the "*predicative judgment*," which treats (1) "as a *self-contained unity of determination*" in a manner that is "constantly privileged" in scientific judging, and 3) the judgment "in the sense proper to apophantic logic," which treats the supposed predicative states of affairs in (2) *as* something that is, in turn, supposed. Thus, the "logic" we take Husserl to be "disregarding" in his discussion here of judgment as it functions in the sciences is both traditional and modern (apophantic) logic. We should also mention that the full context of this discussion (which is discussed below) is Husserl's claim that it belongs to the status of "all formable categorialia" to be "component parts within these [i.e., scientific, in the broadest sense] judgments."

111. The distinction that Husserl is drawing here, between judgment as it is characterized by traditional logic and this very same judgment as the theme of apophantic logic, is obscured by his inclusion of the Greek word *apophansis* to designate the traditional concept, which no doubt invites a denominative association with the adjective 'apophantic' used by him to designate the non-traditional logic in whose judgment the "empty concept anything" (*FTL*, 98) makes "its appearance." Apart from Husserl's discussion that characterizes the narrower concept of judgment in the sense of apophantic logic as the higher-order modalization of the broader concept of judgment in traditional logic, Husserl's recognition that traditional (Aristotelian) logic lacks the completely formalized (and therefore empty) concept of the 'anything' (see § 180 above), together with his understanding of the material determination of the concepts of most sciences, presents strong evidence that he is indeed making a distinction here between two concepts of logical judgment. Namely, he is distinguishing—or, better, appealing to the distinction—between the incompletely formalized logic proper to the Aristotelian tradition and the algebraically formalized logic proper to modern logic (see also n. 91 above).

112. Albeit, the modalization belonging to the apophantic judgment for Husserl is of a higher order than that which belongs to the existent's modalization on the basis of its givenness in straightforward judging. This is because, as we shall see below, in the former modalization what is modalized is the existent, while in the latter what is modalized is the "intention aimed at verification" (*FTL*, 109).

able to determine cognitively," is what is thematized by apophantic judgment. The consequence of this, for Husserl, is "all [the] formable categorialia," which as parts of the broader concept of judgment "are called on to function within predicative judgments" and which, therefore, in turn, "present themselves as component parts within these judgments," are now parts of the narrower concept of judgment. Husserl thus writes: "In other words, judgments, in the sense proper to apophantic logic, are supposed predicative states of affairs[113] as supposed; and, more particularly, they are self-sufficiently complete ones. All other categorial suppositions function as parts within such 'judgments.'" The higher-order modalization involved here for Husserl—no doubt like all modalization for him—"expresses a *modification of meaning*" (109), whereby the predicative judgment of traditional logic, namely, "a supposed objectivity as supposed," is *itself* treated as a supposition. Thus, while in the case of the predicative judgment of traditional logic it is the *objectivity* that belongs to the traditional scientific judgment that is supposed, in the case of the higher-order modalization proper to apophantic judgment it is precisely this state of affairs *itself*, as predicatively supposed, that is (in turn) supposed. This is why Husserl says that all the categorial suppositions of the former now function as parts of the latter. And it is also why he considers the apophantic concept of judgment to be narrower than the traditional concept thereof, since what its higher-order modalization "frames" is the predicative judgment that is "constantly privileged" by the traditional, broader concept of judgment.

§ 186. Critical Elaboration of §§ 44–45: The Absence of a Simple Opposition between Straightforward and Critical Foci Precludes Phenomenological Clarification of the Shift from Formal Ontology to Formal Apophantics

What section heading § 44b announces as the "Phenomenological clarification of this change of focus" (108), namely, "the change of thematizing focus from object-provinces to judgments as logic intends them" (107) announced in § 44a's heading, is something that § 44 as a whole addresses only in terms of the broader sense of the concept of judgment, the traditional sense of apo-

113. To capture in English Husserl's distinction between the traditionally broader and the apophantically narrower concepts of judgment, a distinction that he formulates on the basis of the latter treating the predication of the 'supposed objectivity as supposed' of the former *as itself* supposed, we have used a variation of Cairns's sometimes maligned English rendering of *Sachverhalt* as 'predicatively formed affair-complex'. We have done so because this is the only way to preserve in English the fact that what is being supposed in apophantic logic is, according to Husserl, already itself a *predicative* supposition.

phansis. This becomes apparent when we consider precisely what Husserl clarifies under this section's various subheadings. He begins (§ 44bα) by describing how, *in* the course of straightforward judging, the object (substrate objects and categorial objectivities) of such judging "'modalizes itself.'" Specifically, the "existent" (*Seiendes*) that is there (either intuitively or non-intuitively) for the straightforward judging as something that it "accepts in its being" (108) undergoes a modification: it no longer "stands firm" in the judging, and consequently becomes modalized "in the doubtful, the questionable, the possible, the supposed, or even in the null." However, "so long as nothing like that happens and the accepted objectivities remain for the time being in straightforward acceptance of their being, they simply *are* for the judging." Husserl calls this "judgment continuity" "*harmonious*" because, in the absence of the existent modalizing itself, "For the style of the further activities of judging . . . each one of these objectivities is posited over and over again in connecting identifications as 'one and the same' throughout all of its further syntactical changes."

Husserl does not consider this "passing through modalizations" of the existent in the course of straightforward judging to be tantamount to the emergence of a new focus on the part of the judging.[114] Instead, he considers

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114. Sokolowski writes that when "one of the states of affairs that we have categorially constituted in the world may become questionable to us . . . it takes on a new sense; it becomes, as Husserl says, taken as 'merely proposed'; we now deal, as Husserl puts it in the plural, with *vermeinte Gegenständlichkeiten als vermeinte*" ("Review of Lohmar," 237). From this Sokolowski infers: "It is at this point, also, that a concern with the truth of what we are asserting comes into play; until this point, we were just naively directed toward things and the question of truth, of criticism, simply did not arise." We shall show below, however, that Husserl's phenomenological clarification of the change of focus from objects to logical judgments traces the emergence of the state of affairs as "previously merely supposed" (*FTL*, 109) to a modification of the "intention aimed at verification," *not* to a modification of the modality of the state of affairs' existence. Moreover, we shall see that Husserl does not *exclusively* identify the modification of this (just-mentioned) intention with either the scientist's attitude or the critical change of focus (or attitude) that makes the scientist's attitude possible. (It can also occur "even in everyday judging" [108].) Finally, Husserl's distinctions (discussed below) between a "supposed objectivity," "*a supposed objectivity as supposed*" (112), and (as we have already suggested) the broader and narrower concepts of the latter, are passed over by Sokolowski.

What Sokolowski writes here seems to represent a modification of his earlier views on these matters, when he maintained not only that "such disturbances [i.e., modalizations] need not change our focus" (*Husserlian Meditations*, 277) but also that on the basis of such disturbances, "a new mode of consciousness is possible in which we become concerned with verification" (278). However, these earlier views, like the later, do not appear to distinguish between a 'supposed objectivity' and a 'supposed objectivity as supposed'; rather, he seems to treat as interchangeable 1) the "focus on some objects . . . as supposed" and 2) the becoming "aware of what we supposed as supposed." And, significantly, Husserl's discussion of the broader and narrower concepts of judgment is passed over in these earlier discussions as well.

this passage to provide only "a motivation" (110) for the eventual emergence of such a focus, a focus that will be characterized as "critical" because it is directed to the *alternation* between the actually existing objectivity characteristic of straightforward judging and the supposed objectivity as supposed correlated to supposing. Before the critical focus is able to emerge, however, Husserl says that the following has to occur: 1) the "intention aimed at verification" (109) belonging to the everyday, occasional interest in positive verification is disappointed, such that what this intention was aiming at is not fulfilled as such, which results in the judger's realization that "'the state-of-affairs is not *as I supposed*'"; 2) a theoretically interested, vocational way of life emerges which, instead of occasionally taking prior judgments as provisional (i.e., those that have been disappointed), "takes all prior judgments *as only provisional*" (111); and 3) the distinction between "*supposed and genuine evidence*" is recognized. Only subsequent to the realization of these three conditions does Husserl talk about "*a peculiar judging procedure on the scientist's part*," one he characterizes as "a *zigzag* judging, so to speak: first making straight for the givenness of something itself, but then going back *critically* to the provisional results already obtained—whereupon his criticism must also be subjected to criticism, and for like reasons."

Husserl says that the disappointment of the intention aimed at verification belonging to everyday judging occurs when "'what is actually there'" (109) does not fulfill all aspects of this intention. Thus, judging is disappointed when only certain of its "component positings" are fulfilled. By contrast, where "judging goes right ahead naturally," and the judger, "even while he is being guided by his need to verify," is able to say, "when he ends up with the evidently seen object 'itself,'" that "'[*t*]*he* object is actual, is actually qualified thus, stands actually in these relationships,'" judging is "harmonious." In the disappointment of judging, then, the harmonious "transition" from the continued acceptance of the straightforwardly accepted objectivity, as this acceptance is guided by the everyday judger's (occasional) need for verifying its acceptance, is disrupted. This means that what Husserl refers to as "an *identifying coincidence* between the objective affair (and ultimately the complete judgment complex, the state of affairs) that formerly was already believed-in, and the believing that is now given in the evidence of the fulfilling of the cognitive intention," does not come about. With this, Husserl holds that partial fulfillments of the original, now disappointed intention aimed at verification "supplement themselves in the direction of the things themselves, toward the positing whole of a categorial objectivity that 'conflicts' with what was believed in previously, a conflict that originally makes its cancellation necessary." In other words, the fulfilled partial positings be-

longing to the intention as a whole[115] begin to yield, on the ground[116] of their partial fulfillment, a new categorial whole wherein the disruption of the identifying coincidence emerges; and it is precisely the emergence of this disruption that makes necessary the cancellation of the belief in the whole of the initial intention. When this occurs, Husserl maintains, "the judger now says, for example, 'the state of affairs is not *as I supposed*.'"

Even though Husserl maintains that what the judger now says is something that "*expresses a modification of meaning*," he nevertheless does not characterize the need to cancel the initial belief as a whole or the "added phrase, 'as I supposed,'" as a shift in "focus" or the emergence of a new, "critical" attitude in contrast to the straightforward judging that underlies the harmonious judgment complex. In fact, he qualifies this by saying that "our reference to 'verification' proper (which leads to trueness derived from an evidence that gives the object itself)" is "like every preference of an *ideal case*," namely, "a simplification." Moreover, he states that this simplification is "one that is indeed privileged for the sake of our future exposition but not absolutely necessary, as though it were the only case in which there is a motive for distinguishing between something supposed, as supposed, and something actual." The other cases Husserl mentions either refer to such "verification proper" as "the ideal case, that of perfection" (110), or "adjust the concept of verification proper in a different direction," namely, as "an *adaequatio*," which is "perhaps an imperfect one." The "motive" in question in this passage, even in its simplified form, is therefore not presented by Husserl as being a sufficient phenomenological condition for arriving at the distinction "between something supposed, as supposed, and something actual."[117] Rather, as the beginning of § 44bγ makes clear, "*with the differentiation between something supposed and something actual*, the differentiation between the sphere of *mere judgments* (in the broadened sense) and the sphere of *objects* is likewise prepared for."

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115. It is important to note that what has not been fulfilled is precisely this intention as a *whole*, which is why Husserl can speak of parts belonging to it as nevertheless finding fulfillment.

116. Here the ground of the contrast between the aspects belonging to the intention as a whole that have not and those that have been fulfilled is precisely the partially fulfilled aspects of the original intention.

117. Husserl does not present the "motive" in question in this passage, even in its simplified form, as being a sufficient phenomenological condition for arriving at the distinction at issue. This will become even more apparent below, when we elaborate his account of both the scientist's vocational focusing on cognition—in contrast to the everyday judger's occasional focusing being described here—and the existence of the distinction between supposed and genuine evidence, both of which Husserl maintains are required to bring about the scientist's judging procedure and, therewith, the distinction in question.

By writing here that the latter differentiation, which involves not simply the differentiation between something supposed and something actual but rather the differentiation between something supposed, *as* supposed, and something actual, is "prepared for," Husserl is clearly indicating that he does not consider the modification of meaning that occurs with the disappointment in the everyday (and, therefore, occasional) intention aimed at verification to be what it is that brings about the judgment in the sense of traditional logic. Indeed, in connection with precisely this issue, he writes that "to advance further, we direct our regard to the *sciences*." More precisely, Husserl directs his regard to the scientist's "vocational judging," because it is this—in contrast to everyday occasional judging—that "is always ruled completely by *intentions aimed at cognition*," and therefore it is *this* judging that eventually brings about the sphere of mere judgments—in the broadest sense. The scientist's living a "'*theoretically interested life*,' *with vocational consistency*" means "nothing else but accepting no judgments as scientific except those that have shown their '*correctness*,' their 'truth,' by an *adequation to the things themselves* and [that] can be originally produced again, at any time, with this correctness" (111). This does not mean, however, "that the scientist does not judge at all before such a having of the 'things,' of the objectivities 'themselves' belonging to the respective categorial level." Rather, it means that "he takes all prior judgments as *only provisional*; the categorial objectivities in them, as only provisionally accepted, as merely supposed objectivities." Moreover, even as the scientist takes the categorial objectivities as merely supposed, "The cognitive intention passes clear through them, as supposed, and aims at the things themselves, at their self-givenness or evidence."

Taking all prior judgments as only provisional, and doing so while the cognitive intention is directed to either the self-givenness of things themselves or the evidence for this, is nevertheless not the exclusive difference that "distinguishes the cognitive striving of the scientist from the naive cognitive striving of the non-scientific thinker." This is the case for Husserl because the scientist, unlike the non-scientific thinker who "'merely looks and sees'" whether his cognitive striving has been satisfied, "has long been apprised not only that evidence has degrees of clarity but also that it may be deceptive evidence." Hence, for the scientist "there exists the further distinction between *supposed and genuine evidence*." And it is precisely this distinction that Husserl credits with being what "brings about a *peculiar judging procedure on the scientist's part*, a *zigzag* judging, so to speak," which he characterizes as "the *transition through the critical attitude*" (112), and which he maintains is what initially yields "a *supposed objectivity as supposed*."

Husserl characterizes the "zig" aspect of the critical attitude as "first making straight for the givenness of something itself" (111), while the "zag" aspect is characterized as "then going back *critically* to the provisional results already obtained." This "criticism must also be subjected to criticism, and for like reasons," which means that "the scientist is guided by the *idea* of an evidence that is perfect or perfectible by systematic stages, and attainable by means of criticism, an evidence having as its correlate an attainable, or approachable, true being." Husserl adds that the critical attitude "concerns all judging activities with respect likewise to the *modalizations* occurring in them and the distinctions between evidence and non-evidence that are peculiar to such modalizations themselves; but the cognitive intention aims through these modalizations, through the questionabilities, possibilities, probabilities, negations, toward evident certainties." Consequently, in "every scientific cognition" (112), and, therefore, in "every scientific judgment," what "the scientist has before him, in continual alternation," is, on the one hand, "an *objectivity simpliciter* (as the actuality existing for him in his straightforward judging or else aimed at by him as a cognitive subject) and, on the other hand, *a supposed objectivity as supposed*." Husserl includes among the latter "a supposed consequence, a supposed determination, a supposed multiplicity, cardinal number as such, and so on."

Because the cognitive intention of the critical attitude aims through the modalizations occurring in all judging activities, Husserl is not identifying the critical attitude with a focus whose exclusive concern is the suppositional character of these modalizations. Indeed, his account of the "zigzag" nature of this attitude's judging and the alternation between straightforwardly judged objectivities and supposed objectivities as supposed confirms this, for it is precisely *within* the continual alternation between these two *modes of giving objectivities* that Husserl locates the critical *transition* necessary for scientific cognition and judgment.[118] Moreover, it is important to note

118. Husserl's articulation of the critical attitude (in §§ 44–45), and the transition through it necessary for scientific cognition and judgment, is therefore made on the ground of a distinction that occurs *within* judicative accomplishments, i.e., the distinction between *judgments* that yield, in "continual alternation," 1) straightforward objectivities and 2) supposed objectivities, as supposed. Husserl does not scruple to refer to the straightforward attitude of the "judging" of the "judger," and he also explicitly refers to the judging involved in this attitude as "always believing in something." Hence, it is surprising to read Sokolowski's claim that, for Husserl, "in our original and naive focus on things and states of affairs, judgments have not yet arisen. In this original focus there are only objects, states of affairs, groups, and other things and structures in the world. Judgments arise only when the intended states of affairs become taken as proposed, as supposed" (Sokolowski, "Review of Lohmar," 237–38). Having drawn this sharp distinction between objects and judgments, Sokolowski goes on to criticize the characterization of "the judgment, as well as the entire apophantic domain, as already therefore beforehand" (238), in the sense that "they are already there waiting to be re-

flected on" (239). In contrast to this view of the judgment, Sokolowski claims "that judgments are not there until we reflect on a state of affairs and take it as proposed." This claim is consistent with his overall interpretation of these passages in *Husserlian Meditations*, where he maintains that "Through § 44 Husserl makes us aware of how we focus on judgments. He discloses the origins and the primitive presence of the apophantic domain" (279). Moreover, it is consistent with Sokolowski's claim that in accordance with Husserl's distinction between objects and judgments, "In its origins and in its finality, the domain of meaning is derivative from the domain of objects." And, finally, with this radical distinction between the apophantic domain and the domain of objects, Sokolowski believes that Husserl has resolved the issue of the relationship between formal logic, as apophantic logic, and formal ontology, because "formal ontology is the sheer absorption in given categorial objectivities themselves, without explicit concern with the critical apophantic domain" (288).

On our view, Drummond's account of what goes on, according to Husserl, when we "reflectively direct our attention to the judged as such, to the judged state of affairs precisely as supposed" (Drummond, "Paving the Way to a Transcendental Logic," 39) captures the judicative context within which Husserl articulates the critical attitude. Drummond writes, "In the straightforward focus on objects, we apprehend the categorial objectivity or state of affairs as such; in the critical focus on the state of affairs as supposed, i.e., on the supposition itself, we apprehend the judgment or proposition." Rather than signal that Husserl is involved in a "genetic analysis, one that shows how the judgment (and the apophantic domain) is founded upon and motivated by the ontological state of affairs (and the ontological domain)" (Sokolowski, "Review of Lohmar," 239), Drummond holds that in this shift in attitude or focus, "the judgment takes on for us a double character, that of the categorially formed, judged state of affairs and that of the judgment merely as such, the supposition as supposed, the proposition, the judgment in the logical sense" (Drummond, "Paving the Way to a Transcendental Logic," 39). Drummond's characterization of the "double character" of the judgment allows us to see that rather than account for the distinction between objects and judgments (as well as the origin of the latter in the former), as Sokolowski argues, Husserl's "*differentiation between something supposed and something actual*" (*FTL*, 110) is one in which "the distinction between the sphere of *mere judgments* (in the widest sense) and that of *objects* is also prepared for." (It is interesting to note in this connection that Sokolowski modifies the translation of this passage in a manner that elides Husserl's preparatory characterization of the relationship of the modal difference between "supposed" and "actual" and the ontological difference between 'mere judgments' and 'objects'. Namely, "'with the distinction between the proposed and the actual, we have also worked out the distinction between the *sphere* of mere judgments . . . and that of objects'" [Sokolowski, "Review of Lohmar," 238; his ellipsis and emphasis].)

As we have just seen, Husserl's reason for qualifying as preparatory the relationship between the modal and ontological differences involved here is connected with his view that what is required for the latter distinction—*as a theoretical distinction*—to emerge is the "vocational judging" (*FTL*, 110), the "scientific judging," which marks "the cognitional striving of the scientist" (111). The distinction between "mere judgments" and "objects," between, more precisely, "mere" judgments *as apophantic* and judgments "that have shown their '*correctness*,' their 'truth,' by an *adequation to the things themselves*," therefore involves for Husserl something more than the modal distinction between the 'supposed' and the 'actual'. What is important to emphasize here is that Husserl not only characterizes scientific judging as a "*zigzag* judging" (111), namely as a judging that involves the "confrontation" with *both* straightforward objectivity and supposed objectivity as such, but also that it is precisely *within* the "alternation" from the one objectivity to the other that Husserl identifies the scientist's "critical attitude." Thus, strictly speaking, Husserl's account of the judgment in the sense proper to logic (in either its broader or narrower concepts) does *not* oppose a straightforward judging or attitude to a critical judging or attitude; *rather*, he characterizes the critical attitude *it-*

self in terms of its confrontation with the alternation between objectivities that are modalized either as actually existing, straightforward objectivities or as supposed objectivities, as supposed. Therefore he does not maintain that the supposed objectivity, as supposed—and, with it, the proposition—is something that originates (per Sokolowski) or otherwise is "properly distinguished" (Drummond, "Paving the Way to a Transcendental Logic," 39) by "a difference in the way we *focus* the meant objectivity," a difference in focus that is "critical" in contrast to another—straightforward—way we focus the meant objectivity. On the contrary, it is precisely the encounter with the modalized differences in the *meant objectivities themselves*—as 1) straightforwardly judged or 2) supposed—that characterizes for Husserl the critical attitude. The zigzag judging belonging to the critical attitude's *awareness* of the constant alternation between straightforward objectivities and supposed objectivities, as supposed, therefore does *not* arise on the basis of a putative change in the way the critical attitude focuses on the meant objectivity, specifically a change in focus from the straightforward attitude's apprehending it "as such" to the critical attitude taking it as "supposed." Rather, the different modes of meant objectivity themselves emerge on the basis of the modalization of both 1) the intention aimed at cognition and 2) the *evidence* in which the objectivities so aimed at are given. The former modalization, as we have seen, is brought about by the scientist's vocational consistency, which, in contrast to the transformation by everyday judging of some prior judgments (i.e., those that have been disappointed) into only provisional ones, transforms *all* prior judgments into only provisional ones. (Hence, Husserl traces the motivation for this modalization to pre-scientific judging and the modalization of existent *objectivities*—not attitudes—that occurs in such judging.) The modalization of existent objectivities, then, gives rise to the distinction between objectivities *as* "supposed" and objectivities as "actual"—but not, as we have stressed, to the distinction between supposed objectivities as supposed, and straightforwardly existent ones. This latter distinction, as we have also seen, arises on the ground of the existence of the scientist's distinction between "supposed and genuine evidence," which Husserl credits with initiating the zigzag movement of *judgment* from the evidence in which straightforwardly existent objectivities are given, to the provisional judgment about the objectivities already obtained, and then back again to the straightforward evidence, etc. It is precisely this movement, *between* two different modalities of *objectivity*, and not any change between two different *attitudes* or *foci*, that Husserl characterizes as the "critical attitude." The crucial question of *how* the scientist's distinction between supposed and genuine evidence comes into existence, we want to emphasize, is *not* raised by Husserl. (As we have seen, Husserl introduces the distinction by crediting the scientist with having "long been apprised" of it.) However, from what has been said here, it should be apparent that its coming into existence cannot be properly characterized, or otherwise accounted for, on the basis of the modalization of objectivities judged to be existent by everyday, non-scientific straightforward judging. Again, this is because the everyday modal distinction involves the opposition between 'merely supposed' and 'actual' objectivities; and this means that the possibility of the modalization of the putative evidence that yields the latter in terms of the distinction between "supposed and genuine *evidence*" is patently *not* a factor at this level of Husserl's analysis.

That Husserl himself does not consider his account in §§ 44–45 of the modal distinction between 1) the objectivity simpliciter manifest to straightforward judging and 2) the supposed objectivity as supposed manifest to supposing, to be equivalent to an account of the distinction between straightforward and critical attitudes and their respective differences in focusing on meant objectivity, can also be seen by a brief consideration of § 48 of *Formal and Transcendental Logic*, which provides "a more penetrating phenomenological clarification" (*FTL*, 116) of what has already been partly attained by the analysis in Chapter 4"—and, therefore, in these earlier sections. In § 48, Husserl distinguishes "a *judging at a second level*, in which what was judged straightforwardly, and was therefore an existing objectivity for the judger, is no longer posited, but rather *the judged as such* is posited in a reflexion" (117). Husserl charac-

terizes the distinction involved here as "the *transition* from a judgment (a supposed objectivity simpliciter) to a judicial meaning" (my emphasis) just as he does in § 45. And again, as in §§ 44–45, he does not identify "the going over" (118), from the "straightforward asserting-judging attitude" "into the reflected, in the attitude in which the corresponding object meanings, state of affairs meanings, become apprehended or posited" (118), as a transition involving the critical attitude. In fact, he explicitly rules out that his account of this transition says anything "about whether or not the straightforward (unreflected) judgments and the reflexive meaning judgments are evident, whether or not they bear within themselves cognitive intentions, which eventually proceed to their fulfillment." He does so on the ground that "in both judgment foci," i.e., "in the 'straightforward' and in the reflexion on meaning [*Sinnesreflexion*]," the "modes of modification" involved here, "the evidential having of something itself and the merely having it in belief," are modes that "obviously can occur." This means for Husserl that in the case of either judicative focus, straightforward or reflexion on meaning, "there is the difference between" fulfilled and unfulfilled cognitive intentions. In other words, in *both* modes of judicative focusing, "there is modalization, in particular, eventually cancellation, verification, evident refutation (as negative verification), and so forth." Finally, "criticism [*Kritik*]"—as in §§ 44–45—is not identified *exclusively* with the reflexive focus on meaning, but rather is explicitly identified with what occurs "as a result of going back," after the modalization of *either* mode of judicative focusing, "to the higher level meaning." In other words, Husserl distinguishes between *Kritik* and *Sinnesreflexion*, such that while to all criticism there belongs reflexion on meaning, the converse does not necessarily hold, i.e. reflexion on meaning is not necessarily critical.

As a result of these considerations, we cannot follow Wiegand's account of this issue, which claims that for Husserl "the *critical attitude* of the logician is tantamount to an act of *reflection*, which is the necessary condition for encountering a judgment *as judgment*" (Wiegand, "Incompleteness," 106). We cannot do so because even though, as we have just seen, the judged as such is indeed posited in a reflexion, Husserl does *not* posit criticism as tantamount to the reflexion (or, an act of reflection) that yields the judgment as such, but rather characterizes it as the zigzag movement between the reflexive apprehension of the judged as such and the non-reflexive straightforward judgment, a movement whose zig *and* zag are both informed by the cognitive intention and, intrinsic to this intention, the difference between its fulfilled and unfulfilled modalities. Again, what we are taking issue with is not whether Husserl, in *Formal and Transcendental Logic*, comes "to speak of logic treated in two attitudes" (133) but whether he characterizes these attitudes in terms of an opposition between the critical focus on judgments and the un- (or pre-) critical focus on objectivities. Husserl's summarizing statements on this topic, which are taken as definitive by Sokolowski and those who follow or otherwise take Sokolowski's interpretation as their point of departure, do indeed articulate "*two correlative senses of formal logic*" (131), one of which is characterized as "starting with traditional *focusing on judgments* as apophantic meanings or opinions—that is to say, giving preference to the attitude of criticism," through which "we acquire an *apophantic logic*." The other of which is said to "give preference to the *focusing on possible categorial objectivities themselves* or their forms," whereby what is pursued from the very beginning is "a *formal-ontological logic*." Nevertheless, on our view, the interpretation of this as meaning that the critical attitude is essentially characterized by the reflective focus on judgments while the objective attitude is focused on objects misses Husserl's crucial account of the movement between judgments and objectivities that is essentially characteristic of *both* attitudes. Hence, he writes of "the formal logic of possible truth" (129), which "has (so we explained) a *critical* attitude," that "we regard *judgments as our exclusive theme*—even though we bring in the corresponding possible objectivities that they might fit, thus taking on predicates of 'correctness,' of truth." This bringing in "corresponding possible objectivities," we stress, is expressive of what Husserl calls the "zig" modality of the critical attitude, which, as this quote makes plain, is directed to the possible objectivities that corre-

that the "merely supposed objectivities" that follow from the scientist's taking as provisional all prior judgments do not yet have the status of "merely supposed objectivities as supposed." That is because in the absence of the further distinction between supposed and genuine evidence, and the *criticism* of evidence that follows from this distinction in the form of the zigzag judging back and forth between 1) the givenness of something itself, 2) the provisional results already obtained regarding its givenness, then back again to (1), and so on, the "merely supposed objectivities" remain just that. This is to say, for Husserl a "merely supposed objectivity" becomes something that is "supposed as such" (112), namely, "a supposed objectivity as supposed," only on the basis of the continual alternation between the givenness of something and the provisional results already obtained by which he characterizes the "transition" through the critical attitude. Therefore, merely supposed objectivities, and the cognitive intention passing through them, as supposed, to the givenness of the things themselves is *not* sufficient to bring about this attitude—though it is obviously, on Husserl's view, an essential component of it. It is not sufficient, Husserl points out, because the distinction between "merely supposed" objectivities and the givenness of the things themselves is something that characterizes the cognitional striving of the non-scientific thinker as well. Hence, it is Husserl's identification of the existence of the further distinction between supposed and genuine evidence, a distinction that is proper to the cognitive striving of the scientist but *not* to that of the non-scientist, that is bound up with the critical movement back and forth between straightforward and merely supposed objectivities. And it is precisely from out of *this* movement that the "supposed objectivity, as supposed" is held by him to emerge, and to emerge as the "mere correlate of the 'supposing' or 'opining,'" which he maintains is "often spoken of as the opinion, δόξα." It is the "mere" correlate and not simply the correlate of supposing or opining, because unlike the status of the distinction between the supposed objectivity and the actual objectivity that emerges in everyday judging, the supposed objectivity, as supposed, emerges in response to the further *scientific* distinction between supposed and genuine evidence in relation to the putative givenness characteristic of actual objectivity. It is this distinction that engages the critical at-

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spond to judgments. Likewise, he writes of the "[*c*]*ategorially* formed objectivity," which is "*not an apophantical concept*" but "an ontological concept," that "the essence of such an objectivity consists precisely in its being a *fulfilled* judgment having a corresponding sense-form." As a fulfilled judgment, what Husserl calls the "zag" movement of the critical attitude is thus also a factor for "a *formal ontological logic*" (131), which he explicitly affirms "will be forced, for reasons of method, to make judgment senses into objects—though only as a means, whereas its final purpose concerns *the* objects."

titude's zigzag judging back and forth between the supposed objectivity and the actual objectivity, such that the former, *as* supposed, emerges for the scientist by subjecting criticism to criticism; subjecting, that is, the provisional, merely supposed objectivities to criticism by going back to the putative givenness of something itself, then back to the supposed objectivities, in a manner guided by the *idea* of a perfect evidence, an idea wherein the merely supposed objectivity, as supposed, finds its fulfillment in the *true*—as opposed to supposed—givenness of the actual objectivity itself.

With the "supposed objectivity, as supposed," we have, according to Husserl, "laid hold of *what is called the judgment* (*apophansis*) *in traditional logic* and is the theme of apophantic logic." We have already discussed above the distinction Husserl makes between the broader concept of judgment connected with the judgment in traditional logic and the narrower concept connected with the modalization of this concept, a higher-level modalization that occurs with its thematization (and presumably formalization) by apophantic logic. And although he does not say it explicitly anywhere in § 45, we want to stress that objects in the sense of formal ontology must be among those "categorial formations" (108) that show up in the "zig" of the scientist's judging.[119] That is, they must show up in the making straightaway for the putative givenness of objects that, Husserl maintains, characterizes one of the two foci whose continual alternation marks the transition belonging to the critical attitude, a transition that "every scientific cognition and therefore every scientific judgment must make" (112).

§ 187. The Unitary Province of Formal Logic and Formal Mathematics and Their Epistemological Distinction Is Not Resolved by the Account of the Alternation between the Attitude of Straightforward Judging and That of Supposed Judging as Supposed

Examined closely, the account of the transition through the critical attitude that is necessary for scientific cognition and judging, a transition that involves the alternation between 1) objectivities manifest as actual to the attitude of straightforward judging and 2) supposed objectivities, manifest as supposed in correlation to what the tradition calls supposing or opining (δόξα), resolves neither the nature of the overlapping conceptual province

119. At the start of § 44's discussion of what its heading refers to as "The Shift from Analytics as Formal Ontology to Analytics as Formal Apophantic," Husserl does explicitly say just this when, in connection with "[a]ll the objectivities with which we ever busy ourselves or ever have busied ourselves" (*FTL*, 107), he mentions "their formal-ontological configurations."

that unifies formal logic and formal mathematics nor their disciplinary, epistemological distinction. To begin with, no mention is made here of the different formal statuses of 1) the objective collective unity proper to pluralities (sets) and 2) the (non-collective) categorial unity proper to states of affairs, nor of their different originations or geneses. Indeed, even though Husserl apparently restricts the proper meaning of the narrower concept of judgment belonging to apophantic logic to "supposed predicative states of affairs, as supposed," he also includes "a supposed plurality, a supposed cardinal number" (112) among the supposed objectivities that he maintains belong, as supposed, to the concept of judgment in the broadest (traditional logical) sense. This means, moreover, that Husserl's account of judgment in §§ 44–45 proceeds as if the question of the overarching unity of the objective collective (formal) unity of pluralities (sets, including cardinal numbers) and the objective (formal) categorial unity belonging to states of affairs has already been resolved, because he holds there that all the categorial suppositions belonging to the broadest concept of judgment function as parts of the narrower concept.[120] In other words, while Husserl's analyses in *Formal*

120. Sokolowski argues that §§ 44–45 represent a "new formulation" (*Husserlian Meditations*, 280) of the "first approach to the problem of formal ontology and formal apophantics" in *Formal and Transcendental Logic*, one that overcomes the first, which "merely accepts both from the tradition and places them over against each other." Sokolowski characterizes this acceptance as one in which "facts"—Sokolowski's translation of *Sachverhalte*—"are relegated exclusively to the objective sphere while judgments are in the apophantic sphere." Our statement of Husserl's account of the relationship between formal ontology and apophantic logic in *Formal and Transcendental Logic* (§ 176 above), which articulates it in terms of precisely *how* the inseparability of the formal ontological apriori and the apophantic apriori should be understood, no doubt falls under the heading of Sokolowski's notion of a first approach. This is because in our development of this account (in §§ 205 and 207 above), what we highlight as unresolved for Husserl is the fulfillment of 1) the symbolic signification intention proper to collections in 2) the categorial form of the collection itself. No doubt, on Sokolowski's view, this way of formulating the problem would be interpreted as relegating categorial states of affairs to the objective sphere and judgments to the apophantic sphere. And for him it is, again, no doubt precisely this opposition—between symbolic judgments and non-symbolic categorial forms—that Husserl overcomes when, in § 45, Sokolowski maintains, "we are told that judgments are facts—as supposed." Continuing with Sokolowski's line of thought on this matter, this opposition is overcome because, rather than attempt to solve the problem of the unity between the judgment and object by beginning from his first characterization's positing of their opposition, Husserl's second formulation begins with the object and shows that the judgment is nothing other than the object itself, albeit modalized as supposed by a change in the cognitive intention's focus.

Granting for the moment Sokolowski's line of thought here that formal logic and formal ontology are distinguished solely by an attitudinally induced modalization and therefore that the unity between the "empty" intentions of judgments and the objectivities they intend is (at least, ideally) insured, the problem of accounting for the relationship between the ontological apriori and the apophantic apriori in the case of the categorial form of the collection itself and the signitive intention proper to it would still not be resolved. As we have seen, not

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only does Husserl consistently *distinguish* collective unity from non-collective categorial unity, and therefore the objective character of the unity that belongs to the collection from the objective unity that belongs to the state of affairs, but he also distinguishes the mode of synthetic linkage that 1) yields the preconstitution of the plurality as such that founds the objective unity of the set and 2) the mode of synthetic linkage that yields the preconstitution of the substrate and its explicates that founds the objective unity of the state of affairs. Moreover, he distinguishes the level of the acts that render thematic the objective (logical) substrate unity of the plurality (set) and the acts that render thematic the state of affairs. Thus, not only does the "copulative" unity posited on the basis of the synthesized partial coincidence between an object and its properties differ from the "conjunctive" unity posited on the basis of synthesized partial apprehension of a plurality, but also collective unity as such, as a logically objective substrate, is constituted by a higher-level act than the acts that are constitutive of the states of affairs as a substrate, specifically by the "retrospective grasping" of the noetically preconstituted collective unity yielded by the plural explication proper to colligation. Sokolowski's account of the genesis of the apophantic judgment, and with it the proposition, on the ground of a change in the modality of an object or state of affairs from simply (straightforwardly) asserted to merely supposed, even if granted, would not be able to account for the logical unity of the plurality (set) as such. For in the instance of the logical unity of the collection itself, i.e., of the formal unity belonging to the collection as *something whose content is irreducible to the activity of synthetically collecting objects*, this very unity is *not* something that is initially straightforwardly asserted, in a manner analogous to the state of affairs, such that it would lend itself to being modalized as 'merely supposed' by a change in attitudinal focus. Rather, instead of a straightforward assertion directed to the object there is *colligation*, which for Husserl is patently *not* something that is logically objective. Hence, to talk of the collection as something capable of being modalized like a state of affairs, which is what would have to be done in order to follow Sokolowski's account, can make no sense. Indeed, if we follow Husserl and consider collecting as a form of "predicatively productive spontaneity," the "predicative judgment" in the case of the formal categorial unity of the collection itself is clearly *not* something that is founded in the (straightforward) assertion of the objectivity of the collection. Rather, this predicative judgment must be constitutively *prior* to the assertion (positing) of the collection's objectivity, because it is precisely the collective synthesis produced by this predicative spontaneity that yields the noetic *preconstitution* of the collection itself, the noema of which is—somehow, in a manner, we must emphasize, unaccounted for by Husserl—accessed in the retrospective apprehension of the judgment, and clearly not in any modalization of an object. The predicative judgment belonging to the collecting must therefore *constitutively* precede the objectivity of the collection itself, because, on Husserl's view, *the collective, as a categorial, i.e., logical, objectivity is not constituted prior to this retrospective apprehension*.

With respect to our main concerns, two important conclusions can be drawn from this:

1) Husserl's articulation of the judgment in the sense of apophantic logic—in either Sokolowski's or our interpretation of it—accounts for neither the unity nor the distinction between the apriori of formal ontology and that of apophantic logic. It fails to do so on the terms Husserl himself sets for resolving these problems, i.e., the issue he lays out (in *FTL*, § 24) of establishing the unitary objective province of the forms of multiplicities that defines non-apophantic mathematics, i.e., the traditional formal analysis of mathematicians, and that of predicative propositions, judgments in traditional logic's sense of the term. Failing to establish this unity, the objective provinces of mathematics and logic remain distinct, or, in an equivalent formulation, the most universal concept that would delimit the collective objectivities treated by mathematics and the judgment unities treated by logic remains (in the sections of *Formal and Transcendental Logic* currently under consideration) elusive.

2) The phenomenological nature and phenomenological conditions of possibility of

and Transcendental Logic recognize, on the one hand, that the unity of apophantic formal logic and formal mathematics (understood as formal ontology) hinges upon establishing what these analyses have *not* yet established, namely, the identity of the most universal concept that would delimit the unitary province of these disciplines, the analyses in §§ 44 and 46, on the other hand, treat the objective concepts of apophantic formal logic (e.g., predicative states of affairs) and mathematics (e.g., plurality, cardinal numbers) *as if this unity had already been established*. However, in the absence of having provided a phenomenological account of what Husserl himself recognizes is necessary in order to establish the formal logical unity of indeterminate and determinate multitudes, an account, that is, of the objective forms that compose collections as such as well as the collections proper to specific cardinal numbers, the following conclusion has to be drawn: "the universal concept that should delimit the unitary province" (68) of the discipline that deals with the "forms of sets" (67)—what he calls "the traditional formal 'analysis' of mathematicians"—and the discipline that deals with "*predicative propositions*, 'judgments' in traditional logic's sense of the word," has not been established. That is because Husserl himself has ruled out accounting for the objective unity of the collection as such on the basis of the logical unity presented by states of affairs. The appeal to the critical attitude, and the transition through it to scientific cognition and judgment, certainly provides an account of neither the objective unity of a collection as such (a plurality as such, i.e., a set) nor a universal concept whose unity would embrace formal ontology and formal logic. Finally, the direction toward which Husserl points regarding such a concept, to the formal concept of the 'anything whatever' that he maintains characterizes the common a priori object of both formal ontology and formal logic, is *not* even mentioned in his account[121] of the broadest and narrower concepts of logical judgments—nor does he mention in this account the formalization that putatively yields or otherwise generates it.

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the relationship between the empty intention of the symbolic presentation belonging to the signitive, i.e., the "apophantic" apriori of mathematical judgments, and the formalized categorial objectivities to which they refer, and *must* refer according to Husserl if merely symbolic mathematical thinking is to establish a relationship to mathematical objectivity without succumbing to psychologism, is not established. In other words, the inseparability of, as well as the distinction between, apophantic logic and formal ontology remains (so far in *FTL*, at least) unaccounted for.

121. Here, in §§ 44–45, and, as we shall see, elsewhere in *Formal and Transcendental Logic*.

§188. Logical Psychologism, Husserl's Logical and Phenomenological Idealism, and His Failure in *Formal and Transcendental Logic* to Address the Distinction between the Ontic Meaning Proper to the *Irrealities* of Collective and Non-collective Categorial Unity

Husserl's discussion of psychologism in *Formal and Transcendental Logic* takes as its point of departure the logical critique of cognition advanced in the *Prolegomena*, expands this critique to formulate a more general (epistemological) sense of psychologism, and then, finally, identifies the non-analytic province of a "*transcendental criticism of cognition*" (152–53). For our purposes, it will be sufficient to show that neither the summary of his initial critique of logical psychologism nor his subsequent critique of what he characterizes as the more general concept of psychologism addresses the distinction between collective and non-collective categorial unity that is our main concern. Rather, even though Husserl's analyses acknowledge that the difference in the kinds of categorial objects given (to experience) in a manner that refutes both logical and the more general psychologism requires the "constitutional clarification" (147) of each such object's "ontic meaning [*Seinssinn*]," he does not address this difference in the case of the non-collective objectivities of apophantic logic and the collective objectivities of traditional mathematics. Indeed, Husserl's analysis (within the context of his discussion of psychologism) of what he calls "*irreale Geistesgebilde*" (138)—conceptual forms that appear to the mind and that therefore exist, even though they are neither sensible nor individuated in space and time—treats indiscriminately the ideality of the logical species, the judgment, and that of a symphony (147).[122]

Husserl summarizes the "highly plausible conception" (138) of logical psychologism as "the *equating of the formations produced by judging* (and then, naturally, of all similar formations produced by rational acts of any other sort) *with phenomena appearing in internal experience*." Logical psychologism is highly plausible because these formations do indeed make "their appearance 'internally,' in the act-consciousness itself," and hence it *seems* to follow that they "would be psychic occurrences, and that logic would be . . . a 'part, or branch, of psychology.'" However, Husserl counters that conception by saying that

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122. Our point is not that Husserl believes or otherwise asserts that the "irreality" of these objects shares the same ontic meaning but only that the critique of psychologism articulated in *Formal and Transcendental Logic*—which can aptly be termed Husserl's "mature" critique—does not address the issue of the differences in ontic meaning between the different irreal objects appealed to in its "refutation" of psychologism.

> there is an original evidence that, in repeated acts, which are quite alike or else similar, the formed judgments, arguments, and so forth, are not merely quite alike or similar but *numerically*, *identically*, *the same* judgments, arguments, and the like. Their "making an appearance" in the domain of consciousness is multiple. The particular formative processes of thinking are temporally outside of one another (viewed as real psychic processes in real human beings, they are outside one another in an objective temporal sense); they are individually different and separated. But not so, the thoughts that are thought in thinking. To be sure, the thoughts do not make their appearance in consciousness as something "external." They are not real objects, not spatial objects, but irreal conceptual formations, whose peculiar essence excludes spatial extension, original locality, and mobility. (138)

The mistake of logical psychologism is that it does not respect what Husserl describes here as the "essential separation between the real and the irreal." This distinction can be maintained only on the basis of the realization that "Perception *alone* is never a full objectivating accomplishment, if we understand such an accomplishment to be indeed seizing upon an *object* itself" (140). Hence, so-called inner or internal perception cannot yield, any more than external perception can, "by itself alone" what Husserl characterizes as "the '*transcendence*' *belonging to all species of objectivities over against the consciousness of them*" (148), including, of course, "the genuine idealities of the amplified Platonic sphere" (152).

Husserl characterizes "the battle against logical psychologism" as intending "to serve no other end than the supremely important one of making the specific *province* of analytic logic visible in its purity and ideal peculiarity, freeing it from the psychologizing confusions and misinterpretations in which it had been enmeshed from the beginning" (154). We have shown that Husserl ascribed the *Prolegomena*'s successful realization of this end to its embrace of Platonism.[123] The manifest logical idealism in this manner of securing the purity of ideal being, however, leaves unresolved the all-important question which psychologism was initially intended to answer, namely, how is it that thinking is related to logical objectivities at all. It is one thing to maintain (as Husserl does in the *Prolegomena*, the *Logical Investigations* proper, and *Formal and Transcendental Logic*) that there is a fundamental separation between the real and the ideal. It is quite another, however, to provide an account of how thinking is nevertheless related to and involved with what *Formal and Transcendental Logic* formulates as the ideality of the irreal. In the latter, Husserl identifies two senses of the criticism of cognition that address and presumably provide his mature phenomenology's answer to this question. Both senses are treated under the heading of "*phenomenolog-*

123. See §§ 160–61 above.

ical idealism" (152), which he claims "gets its *fundamentally different* [i.e., from traditional idealism] *and novel sense* precisely by radical criticism of . . . psychologism," a criticism whose ground is "a phenomenological clarification of evidence."

Husserl designates the first sense of the criticism of cognition as "*analytic*," and he characterizes it as having two sides that are related to the two sides of the possible experience belonging to every science. One side is related to the objective side of the "self-contained realm of possible 'experience'" belonging to every science, that is, to "the ideal cognitional results (those belonging to the 'theory')" of each science. The other is related to "the subjective side of such possible experiences, relating to what is ideal in a correlative sense, namely, the acting (concluding, proving) that corresponds to these idealities." Husserl characterizes either side of the analytic criticism of cognition as "a *secondary thematic sphere*" of each science, with the first thematic sphere being that of each science's "province" (in the sense of its self-contained realm of possible experience). The province of each science, like that of every "'seeing' and, correlatively, everything identified in 'evidence,'" has on Husserl's view its "proper right." Finally, Husserl maintains that "Through this criticism [of the primary thematic sphere], . . . each science gets its relation to analytics as a universal science of theory conceived with formal universality," that is, to "the universal and not merely analytic, theory of science" (154).[124]

On top of this, Husserl articulates a "*third thematic sphere*" belonging to every science, which is "likewise a sphere of criticism, but of a criticism *turned in a different direction*." This direction is characterized in contrast to analytic criticism's directedness to "the prior data, the actions, and the results, that make their appearance openly in the field of consciousness." In the third thematic sphere, criticism is directed instead to "the *effective performances and accomplishments that remain hidden* during the inquiring and theorizing directed straightforwardly to the province" of any given science. Husserl designates such criticism the "*transcendental criticism of cognition*" (152–53) and says that it is "criticism of the constitutive sources from which the positional sense and legitimacy of cognition originate."

For our purposes, three things need to be singled out in Husserl's account of the criticism of the cognition in which "every sort of irreality" (138)

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124. Husserl has in mind here the transcendental project, sketched in the last part of *Formal and Transcendental Logic*, "for developing the idea of formal ontology [i.e., the analytic theory of science] into the idea of an ontology of realities and, ultimately, into that of absolute ontology" (156). We shall deal with this project only insofar as it is directly related to our main concern.

makes its appearance as a numerically identical object. First, he maintains that "By studying and paralleling the evidence of the real and irreal we shall gain an understanding of the universal homogeneity of objectivities—*as* objectivities." Second, "that in every evidential consciousness of an object an intentional reference to a synthesis of recognition is included" (143). Finally, third, the mark proper to the peculiar transcendence of an object's "psychic irreality" (148) is its status as "an object experienced many times or, as we may also say, as an object that '*makes its appearance*' *many times* (as a matter of ideal possibility, infinitely many times) in the domain of consciousness." And it should be stressed that for Husserl this means that "the sequence and synthesis of different experiences of the same [wherein the recognition mentioned directly above occurs]" (145) is a process that makes "evidentially visible *something that is indeed numerically identical* (*and not merely things that are quite alike*), namely *the* object."

The very crux of Husserl's statement in *Formal and Transcendental Logic* about how phenomenology overcomes psychologism appeals to, in his words, "an original evidence" (138) of something that is "*numerically* identical." The difference between the ideal and the real[125] is for Husserl an "essential or principled separation." It is grounded in "a *fundamental law of intentionality*" (143), which states the following:

> Absolutely any consciousness of anything whatever belongs a priori to an openly endless manifold of possible modes of consciousness, which can always be connected synthetically in a unity-form of conjoint acceptance (*composito*) to make *one* consciousness, as a consciousness of "*the Same*." To this manifold belong essentially the modes of a manifold *evidential consciousness*, which fits in correspondingly as an evidential having, either of the Same itself or of an Other itself that evidently annuls it.

The mark of the evidence that confirms 'the Same itself' as something that is had with evidence in the synthetic unity-form of the *one* consciousness is precisely its—the Same's—status as something that appears as numerically identical in the manifold of different modes of consciousness united by this form. The manifold of these modes of consciousness "are individually different and separated" (138), in the sense that they are "temporally outside one another" in "objective time," when "viewed as real psychic processes in real human beings." Not so, however, the "*irreal* objective formations" yielded by these processes: their characteristic essence excludes both spatial

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125. Husserl's discussion of psychologism in *Formal and Transcendental Logic*, in connection with "the distinction between the real and the irreal" (*FTL*, 138), refers to this discussion in *Experience and Judgment* as containing "an exposition that substantiates this distinction" (138 n.1) and notes that this exposition was not yet made in the *Prolegomena*.

and temporal individuation. Husserl therefore refers—in the case of logical judgment—to the "supertemporality" (*EJ*, 313/261) of the temporal manifold that constitutes the unity "of the identical as the correlate of an identification" (316/263). The "concatenation" (310/259) of acts of judgment, each one temporally discrete, "enter[s] into the unity of an inclusive total identification: they are composed of manifold acts, but in all of them there is an identical judicative proposition."

The objectivities of the understanding are therefore "objectivities of a higher level" (310/258) than the objectivities either belonging to, or characteristic of, the temporally discrete acts that belong to a "lower level" in relation to them. In contrast to the localized spatiotemporality of the lower-level acts in which they are constituted, the higher-level objectivities are "'everywhere and nowhere,'" and, in this sense, they are characterized by the "*timelessness*" (313/261) of "*a privileged form of temporality*, a form that distinguishes these objectivities fundamentally and essentially from individual objectivities." For Husserl, then, the total identity of the *irreal* objectivities, which is essentially characterized by the ability of such objectivities to appear in many spatiotemporal positions as "numerically identical as the same" (312/260), is the consequence of a "supertemporal unity." He maintains that this supertemporal unity "pervades the temporal manifold within which it [i.e., the object of the understanding] is situated." Because "this *supertemporality implies omnitemporality*," in the sense that "The same unity is present in each such manifold, and it is such that it is present in time essentially," Husserl stresses that the implied omnitemporality is *not* something that is *outside* of time. Omnitemporality is *in* time, as "*a privileged form of temporality*," in the sense that "the what of the judgment, the judicative proposition, is present to consciousness in the mode of the now." As present to consciousness, however, "it is not at a point in time and is not presented in any such point by an individual moment, an individual singularization." In *Formal and Transcendental Logic*, the What of the judgment is called "a psychic *irreality*" (*FTL*, 148), because it "consists in the '*transcendence*' *belonging to all species of objectivities over against the consciousness of them*." Husserl characterizes the consciousness of these objectivities as involving "the repetition of the subjective life-processes, with the sequence and synthesis of different experiences of the Same" (145). As such, these processes "make evidently visible *something that is indeed numerically identical* (*and not merely things that are quite alike*), namely, *the* object, which is thus an object experienced many times or, as we may also say, one that '*makes its appearance*' *many times* (as a matter of ideal possibility, infinitely many times) in the domain of consciousness."

In light of our reconsideration of Husserl's psychologism in *Philosophy of Arithmetic*, and his subsequent critique of this psychologism, his mature statement in *Formal and Transcendental Logic* on the phenomenological refutation of psychologism invites closer scrutiny. As we mentioned at the outset, this statement identifies three kinds of psychologism. The first kind, "logical psychologism," is characterized as maintaining—on the basis of "their making their appearance 'internally,' in the act-consciousness itself"—that logical concepts, judgments, etc., are "psychic occurrences" (138). This kind of psychologism is supposed to be disposed of by the "omnitemporal" substantiation of the distinction between the real and *irreal*, which establishes—on the basis of the numerical identity of every sort of *irreality*, we want to emphasize—that "the possible participation in reality" of the *irreal* "in no way alters the essential or principled separation between the real and the *irreal*."

The second, "epistemological" kind of psychologism Husserl characterizes as "an extreme *generalization of the idea of psychologism*" (151), in the sense that it represents "[t]he extraordinary broadening and, at the same time, radicalizing of the refutation of logical psychologism." He succinctly sums up this type of psychologism as "any interpretation which converts objectivities into something psychological in the proper sense," and provides the following rationale for it: because "evident objectivities, . . . as is obvious, are constituted in the manner peculiar to consciousness," psychologism in this sense denies "their *sense as a species of objects* having a peculiar essence," such that "they [i.e., the evident objectivities] are 'psychologized.'" Moreover, Husserl remarks that "the *exact* sense of psychologism should be defined accordingly." The cure for this type of psychologism, which is "conceived so universally and (purposely) in hybrid fashion" and which therefore "is the fundamental characteristic of every *bad* '*idealism*' (*lucus a non lucendo!*) like Berkeley's or Hume's" (151–52), is "phenomenological idealism." Crucial to Husserl's characterization of phenomenological idealism is the "*novel sense*" (152) it gets, "precisely by radical criticism of the aforesaid [epistemological] psychologism," a criticism whose basis is "a phenomenological clarification of evidence." The crux of this clarification resides in the "proper right" of "[e]very 'seeing' and, correlatively, everything identified in evidence."

Finally, there is "transcendental psychologism," which is born of the understandable but nevertheless "*falsifying dislocation*" (224), which "mistakes . . . psychological internal experience for the internal experience relied on transcendentally as an evidential experiencing of ego cogito." This mistake is understandable, because "it is a falsification that could not become

noticeable before the rise of transcendental phenomenology." Its basis is a "dislocation," because it fails to realize the truth that transcendental phenomenology establishes, namely, that "Neither a world nor any other existent of any conceivable sort comes 'from outdoors' [θύραθεν] into my ego, my life of consciousness" (221). That is because "Everything outside is what it is in this inside, and gets its *true being* from the givings of it itself, and from the verifications, within this inside—its true being, which for that very reason is something that itself belongs to this inside: as a *pole of identity* in my (and then, intersubjectively, in our) actual and possible manifolds, with their possibilities as my (and our) abilities: as 'I can go there,' 'I could perform syntactical operations,' and so forth."

Conspicuously absent in each of these accounts of psychologism is any response to the problem to which the psychologism of *Philosophy of Arithmetic* was a response: the irreducibility of the logical concept of the multitudinous unity of a multitude to the singular unity characteristic of the individual items that compose it. On the other hand, what is conspicuously present in each of these accounts of the phenomenological refutation of psychologism is the thought that from a manifold a "numerical" unity is "constituted," and in fact constituted in a manner that is irreducible to the manifold's individuated contents. The "mode of being" of the *irreal* objectivity that refutes once and for all the logical bogy of psychologism is therefore characterized by Husserl as *numerical*, in the precise sense of a whole that is characterized by a structural unity that encompasses the items of the multiplicity to which it is irreducible and from which it is, simultaneously, inseparable.

Neither Husserl nor his commentators address either the phenomenological character or the "constitution" of the numerical mode of being of the identity proper to the meaning of the higher-level objects of the understanding to which he appeals in his mature refutation of psychologism. No doubt Husserl thought the "proper right" belonging to every seeing and its corresponding evidence is sufficient to establish (in the transcendental reflection directed toward the evidence in which this identity is given) the phenomenological warrant of its essential transcendence of psychic reality. In the case at hand, this "proper right" would belong to the seeing that grasps the numerical identity of the total meaning that—as the correlate of the temporally manifold acts in concatenated syntheses—constitutes the omnitemporality characteristic of an object of the understanding. Dieter Lohmar, in his instructive study of Husserl's account of the omnitemporality of mathematic objects, rightly stresses that "The claim of the omnitemporal identity of objects of the understanding is not connected with the idea of a separate realm

of being in which these objects exist,"[126] but he does not mention or investigate Husserl's characterization of this identity as numerical.

Husserl's appeal to the numerical character of the omnitemporality that defines the transcendence belonging to psychically *irreal* objects brings into relief the "one over many" unity of their meaning. Lohmar has shown that Husserl's mature account of the categorial intuition of such meaning departs from his earlier, comparative account of generically universal meaning. That earlier account sought to establish the constitution of higher-level objects of the understanding on the basis of an overlapping coincidence of the partial objectivities given to individual acts.[127] Such an account proves limited, however, since it is only able to account for generic universality, for the "eidetic intuition" of the identity presented as an overlapping coincidence of meaning is founded in, and therefore presupposes, the partial objectivities apprehended in individual acts. By contrast, Husserl's mature account involves no such presupposition, because the omnitemporality characteristic of the objects of the understanding's unification "rests on the possibility of total coincidence with their respective meanings." In Husserl's words: "But the proposition itself is, for all these acts and act modalities, *identical as the correlate of an identification and not general as the correlate of a comparative coincidence*" (*EJ*, 316/263).

As we have seen, in *Experience and Judgment* Husserl characterizes this total coincidence as being founded in the "supertemporal" unity that pervades temporally manifold acts of judgment that constitute the same object of the understanding. What are "many" here are not individual acts that intend a comparative unity but temporally discrete acts that intend a synthetic unity; what is "one" is not a generic unity that is "instantiated" in the individual objects of many acts but the non-individuated unity of a meaning that is "numerically" the same in the many acts in which it occurs. We suggested above that the numerical character of the omnitemporal unity of meaning here concerns its structure as a whole, which is both irreducible to and yet inseparable from the many acts that found it.

We now want to make two additional suggestions. First, this structure is *isomorphic* with the aspect of the logical structure of authentic cardinal numbers that Husserl, in *Philosophy of Arithmetic*, attempted to account for by appealing to the abstractive apprehension of presentations that originate in acts of collective combination. That is, Husserl's account of the omnitemporal unity

126. Dieter Lohmar, "On the Relation of Mathematical Objects to Time: Are Mathematical Objects Timeless, Overtemporal or Omnitemporal?" *Journal of Indian Council of Philosophical Research* X (1993), 73–87, here 83.

127. See § 165 above.

of meaning exhibits a logical structure that is "*arithmological*"[128] in the precise sense of a whole that manifests a unity whose *content* has absolutely no basis in the content of the unity proper to each of the individual contents it unifies into a manifold. By calling attention to the isomorphism between the logical structure of the numerical character of the meaning of omnitemporal unity and authentic cardinal numbers, we are *not* suggesting that this omnitemporal structure is itself "arithmetical" in the precise sense of a whole that unifies an exactly delimited multiplicity of generically arbitrary units. For the contents of the multiplicity unified by the omnitemporal unity of the meaning of an object of the understanding are, of course, in no way isomorphic with the contents of the multiplicity unified by arithmetical (cardinal) numbers. The contents of the acts composing the multiplicity that is "pervaded" by the supertemporal unity that "implies" omnitemporality are intentional, while the contents composing the multiplicity unified by authentic cardinal numbers are generically arbitrary units. However, because neither the *content* of the units unified by a cardinal number nor the *content* of the acts that are united in their constitution of an object of the understanding can either ground or be predicated of the content of the respective unities that unite them, the structures (that we have designated as 'arithmological') manifested by these "higher-level" unities are clearly isomorphic. Our second suggestion is that it is precisely what we are here calling the "arithmological" structure of this unity to which Husserl is appealing when he characterizes the meaning of the omnitemporal objects of the understanding as a unity that is "numerically identical" in the manifold acts in which such meaning occurs. Thus, despite the "participation" of such acts in the constitution of the meaning of omnitemporal objects, neither these acts nor their temporally individuated contents form any part of the supertemporal meaning that "pervades" them. And it is precisely this characteristic that reflects the structure of cardinal number, namely, its unification of the multitude of units that compose it in a manner that precludes the content of this multitude from informing the content of its unity. In other words, just as the content of the unity proper to the meaning of an object of the understanding is not individuated in time while the content of the acts that intend this unity is so individuated, likewise the unity of a cardinal number is not a multitude (more than one) while the units that compose this unity are a multitude.

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128. Jacob Klein, "The Concept of Number in Greek Mathematics and Philosophy," in *Lectures and Essays*, 51. Klein uses the term 'arithmological' to refer to the "one over many" structure of the unity that makes possible arithmetical numbers and every kind of "ideal" concept. We shall claim below that the proper philosophical context to understand Husserl's appeal to precisely this structure in his confrontation with psychologism is the reconstruction of Plato's "unwritten doctrine" of *eidetic numbers* (ἀριθμοὶ εἰδητικοί).

We have shown that Husserl's logical investigations of (indeterminate and determinate) collective unity after *Philosophy of Arithmetic* do not measure up to the standard set by his own critique of psychologism. These later accounts retain the appeal to a "higher-level" apprehension of acts of unification (whether collecting or counting) to establish the "constitution" of the objective unity of the whole of a collection. In making this appeal, they thus fall short of Husserl's stipulation that the meaning of logical objectivity is something that is "pure," in the precise sense that the content of its meaning—"in itself"—excludes all reference to an origin in or genesis from psychological acts. Husserl no doubt thinks that his account of the omnitemporality of meaning in *Experience and Judgment* satisfies this criterion, because rather than appeal to the origination of such meaning on the basis of an object that is presented as the intentional correlate of *individual* acts, he maintains that this meaning originates as the intentional correlate of *manifold* acts. Being essentially founded on the possible syntheses of *more than one act*, the meaning at issue here is necessarily apprehended in terms of an object whose "level" is higher than that of the objectivities apprehended by individual, or singular, acts. And with this, Husserl's stipulation that the content of logically pure meaning—"in itself"—must exclude all reference to contents that originate in individual acts or their objects is apparently satisfied. We have shown, however, that Husserl's recognition of the logical purity of the meaning of omnitemporal objects is tied to his appeal to its "numerical identity," the phenomenological status of which Husserl and his commentators do not address.

In light of this last point, we wish to make three more suggestions. First, Husserl's appeal to the numerical identity of the meaning of the omnitemporal objects of the understanding does not address the *constitution* of its "mode of being" as numerical, but, rather, the appeal to its numerical "mode of being" as something that is already *constituted* is made in order to justify the ascription of omnitemporality to such objects. Second, owing to what we have characterized as the "arithmological" structure of the numerical identity that is at issue, an account of its constitution would face the same problem that Husserl originally confronted in *Philosophy of Arithmetic*: how to account for the logical unity of a collective whole that is composed of a plurality of parts that, either individually or in relation to one another, manifest unities that can neither ground nor be predicated of the collectivity proper to the unity of the whole to which they nevertheless belong. And third, that rather than signal the failure to resolve the problem of psychologism, Husserl's recognition of the "numerical" structure of the meaning of the objects of understanding signals the only way out of it. This way out requires that the *supposition* of the numerical mode of being proper to the

structure of the meaning of the omnitemporal objects of the understanding be recognized for what it is, namely, the realization that the logical unity presupposed by all cognition is something that manifests an "arithmological" structure that has priority over the unity that belongs to all individual objects and to the temporally located acts of the living being (*psyche*) that both experiences and cognizes them. Like Plato before him, Husserl's radical account of the conditions of possibility for logical unity exposed the need to suppose its "arithmological" structure. The reconstruction of Plato's "unwritten doctrine"[129] devoted to both making manifest and articulating this structure can be shown to point to the omnitemporal existence of "numbers" (ἀριθμοί) whose structure is not mathematical but *eidetic* (ἀριθμοὶ εἰδητικοί), in the precise sense that is under consideration in Husserl's appeal to the numerical mode of being of the psychically irreal ideality proper to objects of the understanding. The last part of our study will demonstrate that the supposition of such numbers entails neither their metaphysical hypostatization nor "Platonism," as it is traditionally understood.

§ 189. The Formal-Ontological Validity of Formalized Meaning Categories Leads Back to a Possible Individual Whatever or a Possible Experience

We turn now to Husserl's account of formalization in *Formal and Transcendental Logic* and therewith to his account of the origination of the formal concept 'anything whatever'. His account distinguishes between two types of "*essential* universalizations, as brought about, on the one hand, in conformity with the *material apriori* and, on the other hand, in conformity with the *formal* apriori" (*FTL*, 189). The former is drawn "from the individual exemplar" in a manner that "secures, on the basis of its *own essential* contents, the material essences of its genera and species and their materially essential laws." The latter, by contrast, is simply characterized as coming about "in *formalizing* universalization," whereby "each *individual* must be *emptied to become anything whatever*." Husserl does not say how this occurs, by precisely what presumably logically cognitive process "every syntactical fashioning of an object out of individuals, and likewise every categorial formation made out of antecedently present categorial objectivities, must be considered as a mode of the mere anything whatever."[130] What he does talk

129. Klein's reconstruction of this doctrine in *Origin of Algebra* is elaborated above in Chapter 19.

130. In *Experience and Judgment*, Husserl characterizes formalization as an "operation [*Leistung*]" "by which two concepts are apprehended under the formal category of the 'any-

about, however, is the contrast between the material apriori's and the analytical apriori's relationship to the intuition of individual objects. Every material apriori, "in the interest of the production of genuine evidence, demands the return to the exemplary *intuition* of individuals, thus the return to '*possible*' *experience*," while "*The evidence of laws pertaining to the analytic apriori*" (189–90) lacks such a demand, as it "needs no such intuitions of determinate individuals" (190). Thus, even though "categories having indeterminately universal cores (as when propositions about numbers serve as examples) may indeed *point back* intentionally to individuals," it does not follow from this for Husserl that they "need to be *further examined* and explicated in this regard."

The contrast in the relationship to the intuition of individuals between the material and analytic (formal) apriori is not the whole story, however. Husserl also maintains that "nevertheless, the *meaning-relation of all categorial meanings* to something individual, which considered noetically is directed to individual evidences, to *experiences*—a relation springing from the genesis of the meaning in question and one that therefore characterizes every possible kind of example used by formal analytics—surely *cannot be insignificant for the meaning and the possible evidence of the laws of analytics*, including the highest ones, the principles of logic." The meaning-relation of categorial meanings involved in the laws of analytics to something individual and thus to individual experiences cannot be insignificant, because "Otherwise, how could those laws claim *formal-ontological* validity, a validity in concert with every possible predicative truth, with everything *conceivably existing*?" The conceivability at issue here is something that, for Husserl, "surely signifies a possibility of evidence, which leads back ultimately, even though with formal universality, to a possible individual whatever or a possible experience."

On Husserl's view not only do the meanings of the formal categories that compose the logical principles of formal logic derive their formal ontological validity from possible individuals and their possible experience, but he also maintains that what these "principles presuppose as truth and falsity" can be uncovered only by "a critical determination of their genuine meaning, starting from the sources of the formation of this meaning." In other words, the logician does not critically examine but rather *presupposes*

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thing whatever'" (435/359). Significantly, on our view, Husserl pointedly distinguishes formalization from variation. He writes: "It [i.e., formalization] does not consist in imagining that the determinations of the variants are changed into others; rather, it is a disregarding, and emptying of all objective, material determinations." A footnote to this passage contains a reference to *Ideas I* regarding "this difference between generalization and formalization"—see n. 77 above.

the "truth and falsity" of the principles employed by formal logic, and does so because the examples of judgments that provide the basis of these principles "stand before him as finished products of a genesis that, generally speaking, was none of his doing." Specifically, "The *logician* has before his view his logical principles, which have been drawn originally and evidently from whatever judgments (categories) have served as examples." These exemplary judgments provide the basis for "forming the consciousness of "'any judgments' whatever," insofar as the logician "varies them in the consciousness of free arbitrariness." Yet this process is "naive," according to Husserl, because "there is no question of an uncovering of this genesis," that is, the genesis of the examples of judgments that stand before the logician as finished products. Likewise, there is no question of uncovering "its essential structure—to say nothing of bringing the essential content of the constitution of the meaning belonging to any judgment whatever into its essential connection with what the principles presuppose as truth and falsity and what they determine concerning these."

§ 190. The Meaning Relation of Formal (Analytical) Categories to Individuals Is Based Not on Phenomenological Analysis But on an Ontological Argument

Three remarks can be made here regarding Husserl's discussion of formalization and the origination of the formal category 'anything whatever'. First, he treats propositions about numbers as categories having indeterminate universal cores, and these cores themselves as pointing back to individuals. He does so without mentioning anything about the categorial status of the syntactical form that unifies these cores into a determinate multiplicity, which he would have to discuss in order to account for the logical status of the form proper to the numerical collection as such. This is crucial for our concerns, of course, because (as we have shown) Husserl's own analyses of this form have established that it cannot be accounted for on the basis of the non-collective unity of that which it collectively unifies, the arbitrary objects that fall under the materially empty category belonging to the formal province of the 'anything whatever'. Second, he contends that what he calls 'evidence of formal universalization' extends beyond the arbitrary variation in consciousness of exemplary judgments or categories, wherein the consciousness of 'any judgments whatever' or 'any categories whatever' is formed. He now maintains that it extends ultimately to individuals and their experience, albeit to both treated in accord with formal universality and thus to individuals and experiences that are possible. Third, the conclusion Husserl

draws from this, "that, even in the evidence of formal universalization, the [intuitively given] cores are not wholly irrelevant" (191), is established not by an appeal to phenomenological analysis but by an ontological argument.

The meaning-relation of formal (analytic) categories to individuals is maintained on the basis of Husserl's phenomenologically *unsupported* claim that formal ontological validity is inseparable from individuals and the experiences of individuals. Implicit in this is the claim that the logical principles that govern these categories contain *presuppositions* about truth and falsity that likewise lead back to individuals and their experience. From this claim it follows that the genesis of the categorial meanings in question is likewise inseparable from individuals and their experience. But Husserl has not established this claim "phenomenologically." To do so, he would have to *show* that the meaning proper to formal categories points back to possible individual objects, and that the judgment proper to formal judgments likewise points back to the possible experience of such objects. Moreover, he would have to show that not only is the meaning in question itself characterized by formal universality but so is the evidence leading back to the possible individual objects and their possible individual experiences. In other words, to establish the truth of his claim that individual objects and their experience somehow lend ontological validity to the formally functioning ontological validity proper to the possible meaning and laws of analytics, Husserl must provide phenomenological analyses that show how the formalization in which the formal categories and formal laws of truth and falsity originate has its basis in these objects and their experience. However, as we shall see, Husserl's phenomenological analyses of the genesis of the ultimate judgment meanings and ultimate judgments proper to formal analytics do not extend to the genesis of the process of formalization that is presupposed in the genesis of both these meanings and judgments.

§ 191. The Extension of Formal Analytics' Epistemological Investigations to Evidence and Experience and Therefore to Transcendental Logic

Husserl's claims regarding the "reduction" of all formal meaning, formal judgment, and formal truth "back to" individuals occur within the context of his investigation of "a *transitional link*" (*FTL*, 179) between pure logic and what he calls 'truth logic'. As we have seen, for Husserl, pure logic is analytic, in the sense that its cognitive concern is with the *forms* of meanings, of judgments, and, ultimately, of objects. By contrast, truth logic is concerned with original meanings, judgments, and objects that Husserl main-

tains are not formal, notwithstanding their being presupposed by formal logic. In accordance with the very meaning of formal analytics, its "'epistmo-critical' investigations" (188) must nevertheless eventually investigate the material content of these original meanings, judgments, and objects. In Husserl's words:

> Now, in accordance with its meaning, formal logic—and therefore all forming of formal-analytic universalities, as a function of the theory of science—is intended to serve the ends of sciences that have material content. With all its freedom in the reiterative forming of forms, and with all its reflexive relatedness to its own scientific character, formal logic still intends—and even *in* these reiterations and this reflexiveness—not to remain a playing with empty thoughts, but to become an aid to cognition that has material content. (182)

The methodical path to the investigation of such material content leads through the evidence of both judgment meanings and the processes of judging itself to experience that is "pre-predicative," in the sense that as the "founding experience" (188) of formal analytics, "its style of syntactical performances . . . are still free from all the conceptual and grammatical formings that characterize the categorial as exemplified in the predicative judgment and the proposition." However, because "no such thing as evidence and experience is to be found in its [i.e., formal analytics'] province or theory," Husserl characterizes the extension of formal analytics' epistemo-critical investigations to evidence and experience, and therefore to "the subjective," as 'transcendental logic'.

Husserl seeks to establish the link from pure logic, including formal mathematics proper (formal ontology), to transcendental logic and therewith to truth logic on the basis of explicitly *phenomenological* considerations that link the distinction between the different concepts of judgment made earlier[131] with "the *hidden intentional implications* included in judging and the judgment itself as the product of judging" (184). As we have seen, these different concepts differentiated the general theory of categorial form (including pure logical grammar), the logic of consequence, and the logic of truth. Husserl maintains that, when investigated phenomenologically, "The *hierarchy of evidence* goes with that of the judgments and their meanings" (182) belonging to each one of these concepts. His analyses establish this hierarchy by maintaining that: 1) any actual or possible judgment leads back to ultimate judgment substrates; 2) to the reduction of judgments to ultimate judgments and ultimate judgment meanings there corresponds a reduction of truths that relate directly to truths and evidences that are individual; 3) hid-

131. See § 189 above.

den in the judgment meanings is their essentially necessary meaning-history (*Sinnesgeschichte*), "pointing back" to ultimate, non-syntactical "cores," cores that point, in turn, to their origination from experiences; and 4) the experiences in which the ultimate meaning cores of judgment meanings originate are pre-predicative judgments about individuals.

Husserl's understanding of "the formalization which analytics carries out, and which determines its peculiar character" (179) is that it "consists in thinking the syntactical stuffs, or 'cores,' of judgments as a mere anything whatever." Thus, "only the syntactical form, the specifically judicial . . . , becomes determinate for the conceptual essences that, as 'judgment-forms,' enter into the logical laws of analytics." The indeterminate universality of these cores is relative, in the sense that the judgment forms say "nothing about whether the subject or the predicate of the judgment already contains a syntactical form in the core itself" (179–80). However, Husserl maintains that "it can be seen a priori that *any actual or possible judgment leads back to ultimate cores* when we follow up its syntaxes" (180), and therefore that judgment "is a syntactical structure built ultimately, though perhaps far from immediately, out of *elementary cores, which no longer contain syntaxes*." Thus, "always it is clear that, by reduction, we reach a corresponding *ultimate*, that is: *ultimate substrates*—from the standpoint of formal logic, *absolute subjects* . . . , *ultimate predicates* . . . *ultimate universalities*, *ultimate relations*." For Husserl, "the reduction signifies that, *purely by following up the meanings*, we reach *ultimate anything-meanings*," initially "*supposed absolute objects-about-which*," and then, "in the ultimate judgments, the ones on which the other judgments at different levels are built, we get back to the *primitive categorial variants of the meaning, absolute anything*: absolute properties, relations, and so forth, as *meanings*."

§ 192. The Extension of Both Material and Formal Logical Universalities Bears a Relation to Individuals; the Priority of Individual Judgment

On Husserl's view the "ultimate substrate-objects are *individuals*," and even though "very much can be said" about them in formal truth, the *mathesis universalis*, as formal mathematics, has "no particular interest" in them. In fact, the claim that the formal truth of pure analytics has something to say about individuals and, indeed, the claim that "*all truth ultimately relates*" back to individuals "is by no means an 'analytical' proposition" (181). Insight into the evidence necessary to establish this proposition requires that "one must *bring ultimate cores to intuition*," and that this evidence must issue from "the 'things'

corresponding to them." Husserl's attempt to provide the evidence needed to establish this proposition is based on the thesis "that *every conceivable judgment ultimately* (and either definitely or indefinitely) *has a relation to individual objects* (in an extremely broad sense, real individuals)" (181). In order "to ground" this "thesis more rigorously," he points out "that universal judgments say nothing with definiteness about individuals, but that in their extension, according to their meaning, they bear an immediate or a mediate relation ultimately to individual particulars" (181–82). Husserl maintains that this is clear both in the case of universalities with a material content and in the case of formal-analytic universalities, which, according to their meaning, "also involves their possible application to arbitrarily selectable objects with material content" (182). In each case, "However much, as upper-level universalities, they may relate in their extension to other universalities, they evidentially must lead back by a finite number of steps to particulars with a material content that are themselves not universalities but individuals."[132]

According to Husserl, "The ultimate applicability of formal analytics to individuals" means that it is characterized by "a *teleological* relatedness to all possible spheres of *individuals*" and that "these spheres are, for logic, what is first in itself." The significance of this for truth and "the evidence by which it becomes one's own originarily" is the correlation of the hierarchy of evidences with judgments and judgment meanings. And for Husserl this means that "*the truths and evidences that are first in themselves* must be the *individual* ones." Indeed, he maintains that "A priori, the judgments made subjectively in the form belonging to the evidence that is *actually most original*, the evidence that seizes upon its substrates and states of affairs originally and *quite directly*, must be *individual judgments*."

Husserl's conviction here, that the ultimate meaning substrates composing the elementary cores of formal analytics are individuals and that the ultimate judgments to which all higher-level judgments may be reduced (in the interest of truth) are related to individual objects, gives "us access to an understanding of the *proper task of judgment-theory*" (183). On Husserl's view, this discipline has "remained rather fruitless," because "it has lacked all understanding of the specific character of the investigations directed to the subjective that are necessary in the case of judgments, in the logical sense, and in the case of the fundamental concepts relating to these." That is, it has

132. Evident here is Husserl's view of the subordination of extensional to intensional logic, which he first articulated in his Schröder review (see § 150 above) and therefore prior to the phenomenological phase of his thought. As we shall stress below, rather than attempt to substantiate this view by means of phenomenological investigations, Husserl simply assumes its truth here.

lacked all understanding of phenomenology. The phenomenological theory of judgment takes care of the "general confusion" typical of non-phenomenological theories of judgment by 1) distinguishing the ideal formation and stated proposition belonging to the "judgment itself" from the judging, 2) understanding judging as an intentional performance in which all ideal judgment formations and ideal truth-formations are "'constituted' unities" (185), and 3) investigating the meaning-genesis and judgment genesis belonging, respectively, to these constituted unities.

§ 193. Phenomenological Backward Reference to the Genesis of Judgment Meanings and Judgments

According to Husserl, (1) and (2) are the result of "overcoming the psychologistic confounding" of judging and the judgment itself; (3), however, stems from the realization that the a priori reducibility of all judgments (which formal logic treats as objects) to ultimate judgments with an ultimate meaning, and the reducibility of the evidence for all truths to pre-predicative judgments about individual particular objects signify "the phenomenological *backward reference*" to the genesis of judgment meanings and judgments. This "backward reference" for Husserl "is not inferred from an *inductive empeiria* on the part of the psychological observer" (187) but "is an essential component of the intentionality, a component uncoverable among the intentional contents thereof in the corresponding productions of fulfillment." This means, on the one hand, that judgment meanings, "as the finished products of a 'constitution' or 'genesis,' can and must be asked about this genesis" (184). Thus, "they are meanings that bear within them, as a meaning-implicate of their genesis, a sort of historicalness," in which, "level by level, meaning refers backwards to original meaning and to the corresponding noetic intentionality." On the other hand, this means that every judgment, including "the non-evident judgment, even the countersensical judgment, refers backwards to an origin from experiential judgments" (187).

Husserl illustrates the backwards reference of judgment meanings to their genesis with an example of a "nominalized predicate" (184). He maintains that the "phenomenological *backwards reference*" is something that "a nominalized predicate (as expressible by such nouns as 'red' and 'the red') bears, in that it points back to a nominalizing activity, on the noetic side, and to the original predicate (as expressible by such predicates as 'red') on the noematic side."[133] This reference backwards holds, on Husserl's view, for every

133. Regarding 'nominalization,' see § 183 above.

nominalized meaning formation, the theory of forms, and any "procedure in *an analytics of consequence-relationships*." It provides for all of them "*a principle of genetic order*, which at the same time determines the specifically logical aim conferred on analytics with the concepts of laws of *truth*." Moreover, "with respect to the subjective," this "signifies that the predelineated *order of judgment-forms involves a* predelineated *order in the process of making materially evident* and in the different levels of *true materialities themselves.*" In other words, the predelineated meaning formations composing analytics contain "*hidden* meaning-moments" (185) referring backwards to the evidence that gives the true materialities themselves from which these moments ultimately originate, and this evidence itself is something that points back to the original judgments in which "the process of making logical principles evident" becomes "eidetically apprehensible" as "the essential form of this genesis." And, moreover, this process of making evident "applies not only to the syntactical implications but also to the deeper-lying genesis pertaining to the ultimate 'cores,'" the latter "referring backwards to their origination in experience."

§ 194. The Phenomenological Priority of Objectivities Giving Themselves over All Other Modes of Consciousness; Genetically Secondary Empty Consciousness

Husserl also utilizes "a proposition from the general theory of consciousness—more particularly, from the phenomenology of universal genesis in consciousness"—to establish the priority of objectivities "*giving themselves*" in "*non-predicative evidence* called *experience*" (186), an experience that, as we have seen, he emphasizes "gives individual objects" (189). All other modes belonging to the consciousness of objects are therefore "genetically secondary" (186), in the precise sense that every consciousness that does not possess the primary mode of giving objectivities themselves is an "empty consciousness," such that "Consciousness that gives us something is indeed always passing over, by way of retention and protention, into consciousness that does *not* give us something itself"—and therefore is "empty." Thus, for Husserl, "Even recollection, though it can be intuitive, is the awakening of an empty consciousness and points back to earlier original consciousness." In systematic judgment theory, then, "one must ascend from the experience that gives individual objects . . . , to the possible universalizations built on that experience, and ask how the underlying experience functions for the evidence of them" (189). It is at this point in his investigations that Husserl distinguishes between the material and formal apriori[134] and stresses that "the syntactical

134. See § 189 above.

cores, which seem to be functionless from the formal point of view" (194), are in fact "not wholly irrelevant" (191) for either "criticism of the principles of logic" or "even in the evidence of formal universalization."

§ 195. Husserl's Analyses Do Not Present the Phenomenology of the Constitution of the Ideal Existence of the Judgment Content as a Formal Universality

Our concern, of course, lies with formal universalization. Our claim is that Husserl does not show phenomenologically just *how* formal universalization has its genesis from the experience of individuals. In place of the critical-reflective exhibition of the evidence proper to formal universalization, Husserl in fact presents the following argument. The universalities that compose the formal apriori belonging to formal analytics and the material apriori belonging to the material sciences have an extension that applies to individuals. Therefore, for each apriori, albeit mediately in the case of the formal and immediately in the case of the material, individuals have a foundational role in the genesis of the judgments that yield the universal structures of either apriori. What exactly this role is in the case of the formal apriori belonging to formal analytics, however, Husserl does not specify. That is, Husserl's analyses do not elaborate precisely how, from experiential judgments about individuals, formalized meaning formations arise with the status of "*ideal 'existence*,'" which, as the content of judgment, "*is a presupposition for*, and enters into, *the ideal 'existence' of the judgment* (in the widest sense, that of a supposed categorial objectivity as supposed)" (193). Husserl himself is aware that the "accomplishment" (197) of "our investigation" "is no more than the exhibition of the necessity of *preliminary 'epistemological' work* that does justice to the essentially necessary relatedness of all judicative evidence to the spheres of experience." Indeed, he quite explicitly characterizes the results of his investigation as being "comprehended in principle, but not actually grounded in detail," and therefore not "actually shown."

Thus in the case of the formalization that, according to Husserl's preliminary epistemological work, should lead from the experience of individuals to the ideal existence of the judgment content as a formal universality, the details of the "evidence" that would support his claims made "in principle" (and thus without evidence) about this are not shown. Husserl does remark that the inquiry into the "'origin'" (193) of the "evidence" that gives the ideal existence of the judgment content refers "to the syntactical *cores*, which seem to be functionless from the formal point of view." And he also holds that they are not, because "the possibility of properly effectuating the possibility of

judgment (as meaning) is rooted *not only* in the syntactical *forms* but also in the syntactical *stuffs*" (194). Moreover, he maintains that this fact is missed by the formal logician, whose interest, on Husserl's view, is "one-sidedly directed to the syntactical—the manifold forms of which are all that enters into logical theory"—and who has algebraized "the cores as theoretical irrelevancies, as an empty anything that needs only be kept identical." However, Husserl's investigations neither present (nor purport to present) the evidence that would show how the formalizing "algebraization" mentioned here comes about nor show how the syntactical cores are relevant for the origin of the "formalized" ideal existence of the judgment content.

§ 196. Husserl's Phenomenological Theory of Judgment Does Not Address the Origin of the Process of Formalization Responsible for the "Algebraization" of the Cores of Formal Logical Judgment Meanings

Husserl's reference to the "algebraization" of the cores provides no more *evidence* of how formalization occurs than does his references to "formalizing universalization" as an emptying of individuals, such that they "become" anything whatever. This is to say, he does not provide any evidence that is capable of providing insight into the origin of the process of formalization responsible for the formalized meanings whose ideal existence is prior to and a condition for the ideal existence of the judgment itself. In accordance with his preliminary epistemological considerations, the clarification of the origin of these meaning formations should be sought in their intentional genesis, to which these formations would then be supposed to refer. But Husserl does not pursue the phenomenological inquiry into the genesis of formalized syntactical meaning, especially in connection with the "letter symbols" (195) that somehow function in connection with such meaning. What he does do is to claim, with respect to the objective focus of "[*f*]*ormal-logical considerations* and theory," that "every one of their logical forms, with their *S*'s and *p*'s, with all the letter symbols occurring in the unity of a formal nexus, *tacitly presuppose* that, in this nexus, *S*, *p*, and so forth, have '*something to do with each other*' *materially*." Support for this claim—or rather, its clarification—"is to be found in the *intentional genesis*" (194) of the judgment, which Husserl maintains is something that amounts to "its essentially necessary *motivational foundations*." The latter "include the necessity that the *syntactical stuffs* occurring in the unity of a judgment *have something to do with one another*." This necessity, in turn, "arises from the fact that the genetically most original manner of judging . . . is evidential judging and, at the lowest level,

judging on the basis of experience." The experiential basis "is always presupposed *as a harmonious unity of possible experience*," the harmony of which—because of the "*community of essence*"—exceeds even "a unity of experience" that has "become *discordant*" and is therefore composed of "*conflicting parts.*" That is, the experience presupposed by all judging "is still coherent," in the sense that "even in the mode of conflict" experience remains "a coherent experience," wherein "everything has 'to do' materially with everything else." The conclusion Husserl draws from these considerations is remarkable, considering that his investigations in *Formal and Transcendental Logic* do not ground or explore them in any detail: "Thus, *in respect of its content*, *every original judging* that proceeds coherently, has *coherence by virtue of the coherence of the matters in the synthetic unity of the experience*."

§ 197. Husserl's Thesis of the Independence of Formal Judgment and Ultimate Formal Structures from Algebraization and the Symbolic Indication of the Ultimate Members of Different Propositions

From the perspective of our concern with Husserl's account of formalization, what is most striking about his conclusion here is its "reduction" of formal logical and therefore "analytical" coherence to the coherence proper to the "synthetic" unity belonging to the things composing experience. This reduction, of course, does not imply the identity of formal and material coherence, but it does entail (phenomenologically) that they are genetically linked. We have already pointed out that Husserl's overall methodological aim is to extend the epistemological-critical scope of pure analytics beyond its quite proper disciplinary restriction to the formal universality of judgment-meanings and judgments, that is, to extend logic's scope from the formal to the transcendental treatment of judgment. To be consistent with this aim in the case of formalization, he would have to provide an evidential account of the genesis of the origin proper to the very process of formalization. That is because the presupposition behind Husserl's conviction that formal analytics can be "reduced" to its foundation in experience is that there is an evidential trail leading from 1) the experience of individuals and the synthetic unity belonging to the "things" composing them to 2) the ideal existence "in itself" of the formalized objectivity that is a necessary condition for analytic judgment.

Husserl's first appendix[135] to *Formal and Transcendental Logic* provides important clues as to why he does not recognize "in principle" the epistemo-

135. Gethmann ("Hermeneutic Phenomenology and Logical Intuitionism," 150) spec-

logical-critical need to provide a phenomenology of the genesis proper to the process of formalization and why he in fact does not investigate the evidence belonging to its origin. In his account of the articulation of predicative judgments, he holds that both the predicate and subject-members of the predication are "not detachable as something self-sufficient" (*FTL*, 259) and that therefore different propositions cannot have an identical member. This means that the letters employed in the symbolic indication of the ultimate members of different propositions are not, strictly speaking, what establishes the identity of the members of two different propositions, but that it is rather a non-symbolic judgment form that is responsible for this. Because with the "algebraization" of the members of the predication and therefore of the proposition itself the logician's focus is directed toward the letter symbols, Husserl's elision of the origin of the process (algebraization) that yields this symbolism becomes understandable. That is, it is understandable in light of his thesis of the independence of formal judgment (and, of course, all judgment) from the sense-perceptible signs and rules for their combination that compose the symbolic calculus that is brought into being with algebraization.

Husserl's thesis of the independence of formal judgment from its algebraization extends beyond the letters indicating the members of the proposition to the syntactical forms of the proposition itself, because he maintains that the "ultimate formal structures" (272) of analytic judgments possess *forms* that do not belong "to the *syntax* of the proposition itself" (271). These forms present "syntactical stuffs" (272) that are "*'stuffs' in the most ultimate sense of all*" (273), in that they "have no significational forms of any sort and underlie all formings of different sorts and at different levels." Yet they do have "forms," according to Husserl, because when different pure or ultimate syntactical stuffs are compared, "an essentially universal moment comes to the fore for us—in formalizing universality" (271). Husserl thus maintains that it is precisely this universal moment, which he designates the 'core formation', that enables penetration to the ultimate formal structures. This occurs with the comparison of the core formations themselves, such that what they have in common becomes apparent as an identity, which Husserl calls the "*core-stuff* of the particular core-formation or . . . the core-stuff of the syntactical stuff" (272). The correlate to the identical moment of the formal core-stuff—or "the *core*, for short—is the core-form."

ulates that the appendices to *Formal and Transcendental Logic* contain Husserl's responses to problems in the main text regarding its lack of specificity about the phenomenological foundation of formal logic, especially the foundation of "the instruments that precisely enable combination according to a calculus, namely propositional connectors and quantifiers."

Husserl uses the example of a comparison of "the core formations, *similarity* and *similar*, or *redness* and the adjectival *red*." He says, "we see that, in every such pair, *core-formations belonging to different categories* are contrasted," and also that "they have, in the stuff-aspects, *an essential moment in common*." Husserl concludes from this that "*Redness* and adjectival *red* have a community of 'content' within the different forms belonging to them respectively as core-formations, the forms that define the categories of substantivity and adjectivity."

According to Husserl the symbolic expression of the traditional judgment forms of formal logic disregards "the differences among the core-forms" (274) and thus designate "the *core-stuff*, which remains identical when the core-form changes." Moreover, analytics, "which, in respect of its theme, aims at the system of laws governing formal consequence-relationships and consistency, does not seek out ultimate cores." By disregarding the core-forms that belong to the judgment forms of traditional logic, their symbolic expression, Husserl maintains, is cut off from the ultimate formal structures of analytics. And because of the limitation of its theme to "formal consequence-relationships," analytics—whether it employs the symbolic calculus or not—is in principle incapable of investigating the ultimate *formal structure* belonging to and characteristic of the formal judgment-meanings and judgments that are presupposed by its theme.

§ 198. The Non-self-sufficiency of the Ultimate Members of a Proposition. The Non-algebraic Ground of the Identity of Propositional Contents

Husserl's articulation of the parts and formal structures of the predicative judgment, however, does not establish their judicative independence from the letter symbols employed in the proposition's symbolic expression but rather is guided by the very assumption of this independence. This is readily apparent in his attribution of non-self-sufficiency to the "ultimate members" that compose the whole of any proposition. Husserl illustrates the non-detachability of both the subject and predicate members of the proposition by considering these members as "indicated symbolically" (260), such as by the letters in the two propositions '*A* is *b*' and '*A* is *c*'. He maintains that "the two propositions do not have an identical member" because even though the "same object *A* is meant twice," it is "meant in a different How." The How concerns the opinion, in the sense of the proposition's characterization as "the meant as meant," and, according to Husserl, in each proposition "we have different things, each with a content, *A*, that is quite like that of the other." These contents are

"formed differently," however, which means that the identity relation between them is established *not* by the "identity" of the letter symbol '*A*' in each proposition but by the unexpressed "form, 'the same,'" which occurs in the second proposition. Thus, on Husserl's view, "we have here the unity of an unstated total proposition: *A is b*, *and the same A is c*," and this means: "The twice-occurring *A*-that-is-meant has a relational form in each clause," such that "The second *A* has identity-relatedness to the first," and thereby "the first receives a correlative identity-relatedness to the second."

Husserl's illustration here of the non-independence of the members of the proposition clearly supposes that the symbolic expression of these members is not what establishes the identity of the '*A*' as 'same' in each proposition. It is also clear that he does not discuss the symbolizing or algebraizing process that presumably yields, for instance, the letter '*A*' as the symbolic indication of an ultimate member of each different predicative judgment. Husserl simply says that these members are, at first, to be "indicated symbolically" by the letters in his example, which suggests that these members and the letters that indicate them "symbolically" are different. That is, he distinguishes the ultimate members of predicative judgments, which he articulates as the core stuffs and the core forms, from the letters that indicate them symbolically. However, in the same illustrating example, Husserl refers to the '*A*' as the non-self-sufficient "content" of the two different propositions, in the sense that "The same object *A* is meant twice, but in a different how" (260). The conclusion that this way of speaking about the '*A*' collapses it into the content of the proposition that Husserl originally characterized as indicating symbolically, in the sense that it is precisely the '*A*' that he is now treating as a member of the proposition, is neither difficult to avoid nor surprising.

This conclusion is not difficult to avoid because Husserl's way of speaking here leaves little room for doubting that by the letter sign '*A*' he understands anything other than the "representative" for any arbitrary substrate-member whatever, about which something is predicated. That is, Husserl understands the '*A*' here as the "symbol" for the materially empty and therefore formally universal category of 'anything whatever'.

On the other hand, this conclusion is not surprising because, as we have seen, neither Husserl's pre-phenomenological nor his phenomenological studies clarify the phenomenological "origin" of the symbol in the signitive sense in evidence here. Rather, he assumes that the "surrogate" logical significance of the "symbol" as a sense-perceptible sign, which is established by the "calculative rules of the game," is not, properly speaking, conceptual but merely "signitive." According to Husserl, only when the game's symbols are regarded as signs for "actual" objects of thinking, and the rules of the

game are regarded as "law-forms" applying to these objects, does the symbolic calculus acquire "actual" theoretical significance.

We want to stress, however, that Husserl nowhere provides—nor does he claim to provide—phenomenological justification for the symbolic way of understanding the letter '*A*' in evidence here: he investigates neither the genesis of the symbolization or algebraization that yields the sense-perceptible letter sign '*A*' as a representative of the formal category 'anything whatever' nor the genesis of the formalization itself that originally "constitutes" the unity of this category as a materially empty meaning structure. Nowhere, then, can we find in *Formal and Transcendental Logic* or elsewhere in Husserl's writings a phenomenology of judgment that unfolds the hierarchy of evidences that—according to his "preliminary 'epistemological' work"—lead, in a finite number of evidential steps, from the universality proper to formalized judgment meanings "to particulars with a material content that are themselves not universalities but individual." Moreover, as we have just seen, Husserl claims, in the absence of such a "theory of judgment," that the symbolic expression of the syntactical forms of analytic judgments cannot, in principle, express the ultimate formal structures of judgments, the non-syntactical "core forms," but can designate only the "core stuff," which maintains an identity despite the change in core-forms. Thus, at the very least, we are justified in saying that Husserl's analysis of formalization and symbolization in his most mature work on logic is incomplete.

Chapter Thirty-five

Klein and Husserl on the Origination of the Logic of Symbolic Numbers

§ 199. Overview

The results of the preceding discussion enable us to interpret Husserl's and Klein's accounts of the origin and mode of being of symbolic numbers in the light of their accounts of the origin and mode of being of non-symbolic numbers. This approach promises to clarify their thought on the origination of the logic of symbolic mathematics, because for both thinkers the origination of this logic is inseparable from the origin and mode of being of symbolic numbers.

By 'logic of symbolic mathematics' both thinkers understand the rule-governed method of designating and manipulating sense-perceptible signs to perform "calculations" with general mathematical objects. They agree that because the mere perceptual content of the signs involved in symbolic mathematics is insufficient to establish their mathematical significance, this significance must somehow be stipulated by the calculative method employed, only after which the sense-perceptible signs can become known as "symbols." The question about the origin of the logic of symbolic mathematics for both Husserl and Klein therefore involves the question about the origination of the stipulation of the significance proper to the symbols employed in this mathematics. Because chief among such symbols are the symbols for general magnitude, both thinkers regard the origination of the logic of symbolic mathematics as inseparable from the origin of the "concept" of symbolic number.[136]

Husserl's and Klein's investigations of the origin of the concept of symbolic numbers situate this origin in its relation to the non-symbolic concept of number. Notwithstanding this similarity, their accounts differ with regard

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136. For Husserl's account of this, see Part III, § 42; for Klein's, see §§ 97–122.

to the precise nature of this relation and to the precise mode of being of both symbolic and non-symbolic numbers. Our discussion of Husserl's account of these issues in *Philosophy of Arithmetic* and his subsequent critique of *Philosophy of Arithmetic*'s psychologism (and psychologism in general), together with his mature thought on the constitution of collections (and sets) and on the relationship between formal logic and formal mathematics, has shown that *Philosophy of Arithmetic*'s final formulation of the logical status of the symbolic calculus operative in universal arithmetic provides the framework for all his subsequent treatments of the mathematical and logical method of symbolic calculation. Thus, in Husserl's mature works, the signs belonging to the symbolic calculus are understood to refer neither to the same logical object as the authentic concept of cardinal number—namely, determinate pluralities of units—nor to the indirect ideal representation of this logical object accomplished by inauthentic systematic number concepts. Rather, the signs of the symbolic calculus are understood as the non-conceptual game's symbols, which are manipulated in accordance with the "rules of the game" of a conceptless calculational technique that acts as a surrogate for an actual theory of multiplicities. Moreover, we have seen that his mature account of the constitution of collections (and sets), together with that of the authentic cardinal number concepts, is quite close to the accounts in *Philosophy of Arithmetic*. This fact alone calls attention to several significant issues, chief among them being the distinction Husserl draws between the constitution of the categorial unity proper to formal logic and the collective unity proper to mathematics. We have seen that this distinction, while hitherto little appreciated, has important implications for both Husserl's formulation of the (modern) idea of the pure *mathesis universalis* as the rubric for the unification of formal logic and formal mathematics, and for the prospects of its realization along the lines delineated by his mature statement of the phenomenological project.[137] This statement projects the foundation for formal (or what he will come to call 'apophantic') logic and formal mathematics (understood as "formal ontology") to have its locus in the experience of individual objects.

Klein's account of the origin of symbolic number concepts articulates four strands belonging to their origination, each of which is documented in his successive reactivations and desedimentations of these concepts in the works of Vieta, Stevin, Descartes, and Wallis. Klein calibrates each of these strands in relation to the main features of the Greek ἀριθμός-concept: the delimitation of a definite amount of monads, whose mode of being is de-

137. Husserl attributes the modern idea to Leibniz, which, as Klein notes (see Part I, § 21), oddly passes over Descartes's role in its formulation. See Part III, § 100, where Klein desediments the origination of this idea itself.

cidedly non-conceptual. Using this calibration as his point of departure, Klein tracks the progressive emergence of symbolic number concepts by desedimenting and reactivating 1) Vieta's invention of the method of calculating with the species of magnitudes, 2) Stevin's reconfiguration of the concept of number on the basis of de facto numerical operations with the ciphers and positional system of the Arabs, 3) Descartes's algebraic reformulation of geometry as the *mathesis universalis*, and 4) Wallis's subordination of geometry to symbolic arithmetic in the *mathesis universalis*.

§ 200. Klein on Vieta's Identification of the Species of Number with Quantity in General

Vieta's calculation with the *species* (or εἶδος) of magnitudes, his *logistice speciosa* (understood as the 'general analytic' or 'analytical art'), designates these species with sense-perceptible letter signs that (once they are so designated) are *perceived* to have a numerical significance, albeit one that—in contrast to the ἀριθμός-concepts—is indeterminate. Precisely these characteristics, the apprehension of a sense-perceptible letter as having a numerical significance and the lack of this significance's reference to a determinate amount of things or units (monads), exhibit the salient qualities that define the modern concept of symbolic number—and that therefore justify crediting Vieta with its invention. By immediately meaning the general amount character that belongs to each definite amount (ἀριθμός) and only mediately meaning the items (things or units) that may be present in a given ἀριθμός, the "being" of the letters belonging to Vieta's calculational method have to be understood as *symbolic*. These letters, notwithstanding their lack of reference to definite things or units, assume a "numerical" significance because Vieta reinterprets the Greek ἀριθμός from the conceptual level of his *logistice speciosa*: the signs used for ἀριθμοί are identified with the numerical character of each number in general; the numerical sign '2', for example, no longer means a definite amount of things or units but rather the general concept of 'twoness' in general. The replacement of the numeral '2' with the letter 'A' is therefore only "logically required," for the relationship between the sign and what it designates has become "symbolic" in Vieta; in both cases, what the sign "designates" (respectively, the general numerical character of this one number or the general numerical character of each and every number) literally coalesces in the sign. The symbolic "being" of Vieta's numerals or letters therefore can be understood in neither the Platonic sense of the being of number as a generic unity that is independent of a delimited multitude of intelligible units (which are themselves independent of somatic units) nor in the Aris-

totelian sense, as the abstracted "being one" of the measure that delimits a likewise abstracted multitude of intelligible units. Rather, the being of Vieta's numerically significant signs is inseparable from the general concepts that they are stipulated as designating.[138]

Vieta's invention of the symbolic concept of number (and therefore this concept's origin) is made possible by three factors: 1) the transference of the "possible givenness" that belongs to the manner of treating as given the magnitudes that are sought (i.e., *unknown*) in traditional (pre-modern) geometrical analysis to the way in which ἀριθμοί are treated as *known* in arithmetical analysis, 2) the appropriation of the Diophantine εἶδος-concept (i.e., the representation of unknown definite amounts of units and their powers with signs), and 3) the understanding of "pure algebra" as simultaneously a general theory of equations and a general theory of proportions. The first factor led to a much sharper distinction between the calculation with determinate (known) and indeterminate (unknown) ἀριθμοί than heretofore had been possible in traditional (namely, Diophantine) arithmetical analysis; the second factor led to the "numerical" interpretation of the letter signs for known and unknown ἀριθμοί and their powers; and the third factor led to the "purity" of the equations belonging to the "analytical art" (in the sense that the magnitudes with which they deal are not identified with either geometrical figures or definite amounts of units).

Vieta's understanding of the "parallel" between analysis in traditional geometry and analysis in Diophantus's *Arithmetic* enables the separation of the latter's procedure of calculating with unknown (indeterminate) definite amounts (ἀριθμοί) from its procedure of computing "true" (determinate) definite amounts. The significant consequence of this separation is that for Vieta the locus of "analysis" shifts exclusively to the calculation with the unknown definite amounts—that is, to the indefinite "solution" of a given problem.

Traditional geometrical "analysis" distinguished a properly analytic aspect of its procedure, which treated the unknown magnitude as if it were known, from a synthetic aspect, which applied the results of the "analysis" either to the discovery of a theorem or to the solution (construction) of a "problem." In the case of a theorem, the analysis involved the search for truth, and was characterized as "theoretical," whereas in the case of the solution to a problem, the analysis supplied what was needed for the construction of a figure, and was called "problematical." The application of the analysis to either the theorem or problem, the "conversion" or "synthesis," is what led to the ἀπόδειξις, the "proof" or "demonstration" of the theorem or problem.

138. See Part III, §§ 98 and 106.

In the application of the analysis to a theorem, the ἀπόδειξις directly followed the synthetic derivation of the theorem from the "given" relations between the "given" magnitudes; in the application to (the solution of) a problem, the ἀπόδειξις was indirect, as the synthesis involved an additional step, the construction of the figure from precisely the dimensions granted as known in the analysis, only after which the ἀπόδειξις was complete.[139]

Analysis in Diophantus's *Arithmetic* also deals with the unknown (what is sought) as if it were something already given, albeit in a manner that reflects the difference between the objects of arithmetic and geometry. In either the theoretical or problematical analysis of geometry, the magnitudes of the unknown that are treated as something already given nevertheless remain *determinate* in a certain sense because even though they are not, properly speaking, determinate—that is, actually derived or constructed synthetically—they are still determinate in the equivocal sense of having the character of being (granted as) given with just these dimensions. In Diophantine analysis, the construction of an equation using signs for the species and powers of ἀριθμοί permits one to ignore the distinction between known and unknown ἀριθμοί while nevertheless dealing with them as with something already given. In both traditional geometry and Diophantine analysis, the analytical treatment is "general" because the object sought (the unknown) is treated in a manner that is not unequivocally determinate. However, owing to the differences in the determinate nature of the arithmetical and geometrical objects referred to in each type of analysis, the generality of their procedures is also different. In arithmetical analysis, it is already known in advance what the unknown (and therefore sought-for) object is, namely, a determinate amount of monads. In geometrical analysis, on the other hand, what the unknown is is something that cannot be known in a manner similar to this, because it must be either derived or constructed synthetically on the basis of both the magnitudes given from the beginning and those brought to light in the derivation or construction.

By treating geometrical and arithmetical analysis as completely parallel procedures, Vieta therefore disregards the difference in the generality of their objects. This enables him to identify the second aspect of Diophantine analysis, the computation of the "true" (that is, determinate) amounts of monads, with the synthetic construction (solution) in problematical geometrical analysis, and thereby to separate the arithmetically analytical calculation with unknown (indeterminate) ἀριθμοί from this computation. Instead of the calculation with the εἴδη (species) of the unknown and its powers re-

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139. See Part III, § 105, on the positions summarized in this and the following section.

maining only provisionally indeterminate (pending the "test proof" with determinate amounts of monads), as in Diophantus's *Arithmetic*, in Vieta such calculation becomes identified with the "analytical art." That is, calculation is now transferred entirely into the domain of the indeterminate. However, in addition to the *species* of the unknown and its powers that for Diophantus exhaust the units of calculation, for Vieta these units include as well the *species* of all known ἀριθμοί. This understanding is made possible by Vieta's transference of the "character of being given" that characterizes the "generality" of geometrical magnitudes, that is, the granting of the sought-after (and therefore unknown) dimensions as having a particular magnitude, to the definite amount that composes every ἀριθμός. Owing to the difference between geometrical and arithmetical objects, the transference cannot be totally isomorphic: that which has the character of being given in the case of geometrical objects is the magnitude of certain geometrical dimensions, which, after a successful demonstration, have the same mode of being as the dimensions themselves that were (initially unknown and therefore) sought after; in the case of determinate amounts of monads, the granting of every ἀριθμός as having the character of being given, which is responsible for their having the status of general magnitudes, does *not* lead to each resultant *species* of the known having the same mode of being as the determinate ἀριθμός itself that was initially sought after. That is because the mode of being of the ἀριθμός is composed of a multitude of units, while the character of the mode of being of the species consists in the "granting" that so many units have been given, or in the "possibility" of so many units being given.

When measured against the traditional distinction in geometry between theoretical and problematical analysis, that is, the distinction between analysis as a procedure that searches for truth and one that supplies what is needed for the construction (solution) of a problem, Vieta's procedure no longer distinguishes between the objects of these different procedures, theorems and problems. This is because he understands theorems to be problems whose solution is not so much the truths themselves about mathematical objects but the "art of finding or the finding of the finding" of the *general* solution to all mathematical problems.[140] Moreover, by understanding the equations comprising this "general art" to be proportions, in the traditional (Eudoxian) sense of theorems that are not limited to either geometrical or arithmetical objects,[141] Vieta extends the scope of the universality of the Diophantine species such that the generality of the indeterminate magnitudes

140. See Part III, § 103, regarding this last point and the position summarized in the preceding paragraph.

141. See Part III, § 104.

calculated with in the *logistice speciosa* enter a dimension that is identical with granting as given neither "unknown" geometrical nor arithmetical objects.[142]

For Klein, Vieta's interpretation of his *logistice speciosa* as an auxiliary method, one that remains in the service of arithmetic (*logistice numerosa*) and geometry, means that its symbolic number concepts nevertheless remain tied to the determinate beings that comprise the subject matters of these traditional disciplines. In the case of arithmetic, this means that the ultimate referent of Vieta's species signs—within the context of his methodology—remains that of the delimited amounts of units proper to the traditional ἀριθμός. In the case of geometry, it means that the species signs' ultimate referents remain the constructed figures that belong to traditional geometry. The link between Vieta's method and the determinate beings of traditional mathematics is reinforced, moreover, by the fact that the rules for its calculation with species are derived from the calculation with determinate amounts of monads. On the one hand, it is the reinterpretation of such numbers from the symbolic level of the *logistice speciosa* that permits the arrogation of numerical characteristics to the intrinsically non-numerical species concepts. On the other hand, it is Vieta's "law of homogeneity" that insures that the calculation with (arithmetically and geometrically indeterminate) species will solve problems in the realm of determinate mathematical objects. This law does do so because what it stipulates—that only magnitudes with a like "genus" can be added or subtracted in the calculation with species—has its basis in the results of calculations with determinate numbers and demonstrations with "traditional" geometrical magnitudes.[143]

§ 201. Klein on Stevin's Identification of the Mode of Being of Number with the Concept of Number

Stevin's assimilation of the *concept* of number to long-established *operations* on numbers using the ciphers and positional system of the Arabs gives rise to the understanding of the 'one' (or 'unit') as a number, '0' (the nought) as the *principium* (ἀρχή) of arithmetical numbers, number (no less than geometrical magnitude) as continuous quantity, and thus to the establishment of the ordinary understanding of the nature of number once and for all, according to which counting is equivalent to knowing how to deal with the ten Arabic ciphers (0, 1, 2, 3, 4, 5, 6, 7, 8, 9).

Whereas the symbolic number concepts invented by Vieta—the letter signs for the species of magnitudes—are still understood (within the context

142. See Part III, § 105.
143. See Part III, § 106.

of his methodology) to have as their ultimate reference definite amounts of units, Stevin identifies the mode of being of these units with the *concept* that is related to them. He arrives at this identification by, on the one hand, accepting the classical definition of number (*numerus*) as "a multitude consisting of units," while, on the other hand, interpreting the parts of this multitude as a whole—the units—as being "of the same material" as the whole. (Stevin, in fact, compares the material of number with the material of bread and water: just as a piece of bread is still bread or a drop of water is still water, likewise the unit, as part of number, is still number.) Because the "material" of the multitude as a whole is "number," it follows for Stevin that the unit, as a part of this whole, is "number," too. Moreover, Stevin identifies this material—number—with the ciphers that represent quantities, which means that he no longer takes numbers (together with the concept of number) to relate to definite amounts of units; instead, the unit is now understood, together with all other numbers, to have the mode of being of the "material" of number.[144]

Therefore Stevin understands the material of number to consist not in an actual multitude consisting of units but in the very *definition* of number as a multitude consisting of units; consequently, number—including 'one'—is now identified with its *conceptual* determination. The mode of being of the *actual* objectivity of multitudes of units is posited as identical to the *concept* (established by the classical definition of number) of a multitude of units. With this, both the 'multitude' and the 'units' (the 'one') assume the mode of being of a concept, which is to say, of a "general object," albeit one that is inseparable from the ciphers that express it. This understanding of numbers, which no longer knows definite amounts of units but only "numbers" expressed in ciphers, is a *symbolic* understanding, and in fact one that is a necessary condition for Stevin's characterization of arithmetic as the unlimited possibility of compounding such ciphers (symbols) in accordance with definite rules of calculation.[145]

Stevin's understanding of the material of number as akin to that of bread and water, together with his regard for the sign notation in his conception of quantity, are what is responsible for both the transfer of the ἀρχή of number from the unit to the nought and the assimilation of numbers to geometrical formations. With this transfer and assimilation, numbers—in contrast to the traditional understanding of the discreteness of ἀριθμοί—are understood to be both continuous and virtually identical with geometrical magnitude. The true interpretation of the sign that is called a 'point' ('·') is not

144. See Part III, § 108.
145. See Part III, § 109.

a unit, as the Greeks had thought, but the nought, which Stevin maintained is identical to the sign '0'. The nought's status as the true principle of number is further attested by its similarity to the geometrical point: just as the addition of a point does not increase the length to a line, likewise the addition of the nought does not increase the quantity of a number; moreover, neither the point nor the nought is a part of (respectively) a line or a number, and infinitely many of either will not form (respectively) a line or a number. The continuity of geometrical magnitude, exhibited by the continuous extension of a line, is matched by the continuous increase of number, by lining up ciphers (e.g., the number '6' increases to the number '60' by the juxtaposition of '0'). And, finally, the unit is a part of number just as a smaller line is part of a larger one, and the parts of the unit are also numbers, namely, "fractional numbers," which decrease infinitely. Stevin illustrates the breakdown of the ancient view of the discreteness of the definite amounts of objects that compose "numbers," and therewith the complete correspondence of geometrical magnitude and numbers, by comparing a continuous body of water and its corresponding continuous wetness with a continuous magnitude and its corresponding continuous number: as the continuous wetness of the body of water is subject to the same division and separation as the water, likewise the continuous number is subject to the same division and separation as its magnitude.[146]

§ 202. Klein on Descartes's Algebraic Reformulation of Geometry as the *Mathesis Universalis*

Descartes's algebraic reformulation of geometry as the *mathesis universalis* identifies the "general object" of this science with the substance of the corporeal world conceived of as "extension." As an indeterminate magnitude, the object of Descartes's *mathesis universalis* is not only something that must be symbolically represented and conceived but also something that, as a continuous magnitude, can be immediately understood as algebraic numbers. Descartes's reformulation of geometry and identification of its mathematical object with the "true" object of physics is attentive to the problem of how to reconcile 1) the indeterminacy belonging to the mind's concepts that compose the *mathesis universalis* and 2) the determinacy of their "real" referents, and it attempts to resolve this problem both mathematically and philosophically.

The "figures" in his *mathesis universalis*, being abstracted from both the figures with which geometers deal and every other determinate subject

146. See Part III, § 110.

matter, do not represent the extended figure of a real body. The rectilinear and rectangular plane surfaces (or straight lines) employed in his geometry are nevertheless called figures because he conceives of them as representing the "figurality" itself that is common to the figures belonging to the real extension of bodies. Descartes understands the proportions and equations with which these figures deal as a multitude—in the sense of the numbers with which algebra deals in setting up proportions between 'A', 'B', and 'C'—on the condition that the particular "unit" (the "common measure") in the comparison of the continuous and undivided magnitudes (lines and planes) of the figures in his geometry is known. Once that condition is met, these continuous magnitudes are immediately understood as the algebraic numbers that set up proportions (and thus equations), and therefore as numbers that no longer refer to the common measure of the lines and planes formed by the figures in Cartesian geometry. He grasps them in this way because what is now of concern in the algebraically conceived geometrical proportion is the "arrangement" (*ordo*) of the figures of Descartes's *mathesis universalis*, not the "measure" (*mensura*) of these lines' and planes' continuous magnitude.

His plane or linear figures therefore represent equally and identically "multitude or number" (*multitudinem sive numerum*) *and* continuous magnitude, which means that each figure does not represent a determinate amount of units of measurement (like Euclid's straight lines). Rather, it is the proper function of these figures "to image" the "true idea" (*vera idea*) of number (*numerus*), which, because it is an indeterminate multitude (algebraic quantity), might upon its encounter otherwise result in our being led astray by the pure intellect to misunderstand such a quantity as a "mere multitude" (*sola multitudo*). Such a formation, separated from all "enumerated things" (*res numeratae*), is precisely what the indeterminate multitude is not according to Descartes's *mathesis universalis*, because "methodical" considerations have led him to posit the identity of what it represents—extension with figure—with the substance of the corporeality belonging to things. Even though Descartes's figures, as figures, represent neither the determinate figures of traditional geometry nor the determinate extension with figure belonging to enumerated things, they are nevertheless able to image the true idea of number, because as "marks" (*notae*) they are graphic and therefore share with enumerated things measurability by many units. By presenting "some subject measurable by many units" (*subjectum aliquod per multas uniates mensurabile*), the figure prevents us from erroneously understanding the indeterminate multitude that it represents to be a "mere" multitude. It is precisely this representation (by determinate marks) of something that is indeterminate and yet nevertheless *not* separated from determinate things that permits the

characterization of Descartes's figures as "symbols" in exactly the same sense as the letter signs that occur in algebra, and especially in Vieta's general analytic. The originality of Descartes's achievement, however, is to accomplish the same reinterpretation of the traditional concept of ἀριθμός effected by Vieta and Stevin in the realm of traditional γεομετρία.

The reliance of Descartes's mathematics on the "mediating unit" (*mediante unitate*), understood as a unit of measurement, is a mathematical attempt to resolve the problem of the relation between the indeterminacy proper to algebraically numerical figures and the determinacy of the enumerated things to which these numbers must somehow—"in truth"—be related. Even though Descartes's application of the measure is limited to the symbolic formations of his *mathesis universalis*, his mathematical understanding of the measure remains basically that of the Aristotelian ἕν, a common measure (μέτρον). This is evident from his stipulations that 1) in the instance of the comparison of continuous and undivided magnitudes the "particular unit" must be known in order for their proportions to be understood as numbers, and 2) a subject measurable by many units must be employed to represent indeterminate magnitude, lest one err by conceiving of this magnitude as something that is separated from things.[147]

In Descartes's philosophy, however, the unit has a different status, that of a "simple thing" (*res simplex*), which is intuited by the "naked" or "pure" intellect when it beholds one of the ideas it has separated from existent things, an idea to which nothing truly in existence corresponds. The "unit" (*unitas*), as a "simple intellectual thing" (*res simplex intellectualis*), is therefore "not a quantity" (*non est quantitas*). However, because, as a simple thing, it has the distinction of "belonging" to both the spiritual and the bodily realms, it is a "simple common thing" (*res simplex communis*).[148] The question of how the unit, as a simple intellectual thing and therefore as an indeterminate concept to which nothing determinate corresponds, is nevertheless able to "belong" to the bodily realm touches on the insoluble problem of Cartesian philosophy, namely, how the relation between body and soul is to be conceived. Because Descartes encounters this problem originally in the mathematical realm—in the problem of reconciling the indeterminacy of the algebraic quantities with the traditional determinateness of number—the difficulty it poses for him is not crucial.[149] Indeed, Descartes's account of the pure intellect's use of the "power" of the imagination in the service of reconciling the

147. See Part III, § 114, regarding the positions summarized in this and the previous two paragraphs.

148. See Part III, § 113.

149. See Part III, § 114.

mathematical problem of the relationship between determinate and indeterminate quantity remains the first and to this day only philosophical attempt to fix the exact meaning of the abstraction that yields the new, algebraic number concepts that are employed by the *mathesis universalis*, which is to say, by the new "symbolic" mathematics.[150]

Descartes's account of the role of the imagination in addressing this problem focuses on the "pure" intellect's use of the imagination's power to make visible what otherwise would remain unvisualizable to the intellect, namely, the indeterminate "simple intellectual things" that it has separated from determinate things. The separation of these "pure" things occurs when the intellect refers only to itself and its own conceiving, which is "pure" in the precise sense that its "cognitive power" (*vis cognoscens*) "acts alone" (*sola agit*) in a manner that is entirely "divorced from the aid of any bodily image" (*absque ullius imaginis corporeae adjumento*). Referring solely to itself, the "pure" intellect is not only "*bare*" of any immediate reference to the world, but it is also unable to get hold of what it has abstracted by using the images the imagination offers it, because any connection between the determinacy of these images and indeterminacy of the "pure intellectual things" leads necessarily to contradictions. In the imagination, the "idea" of extension cannot be separated from the "idea" of body, nor the "idea" of number from the "idea" of the thing enumerated, or the "idea" of unity from the "idea" of quantity.[151]

To visualize and thus be able to grasp as "abstract beings" (*entia abstracta*) the "simple intellectual things" it has abstracted from these very same determinate "ideas," the pure intellect must make use of the alien imaginative power—but *not* its determinate images—to represent to itself the indeterminate and therefore unvisualizable content of what it has separated. The pure intellect is able to do this because even within the realm of the "alien" imaginative power, it retains the ability proper to it and foreign to this power, namely, that of separating indeterminate "simple intellectual things" from determinate images. So, for example, when the "pure" intellect separates 'fiveness' from five enumerated things and apprehends it as a "mere multitude"—and therefore as something that is separated from that to which it "in truth" belongs without being identical with it (i.e., the enumerated things)—the imaginative power, which ordinarily makes visible five units (perhaps as points), now makes visible something graphic, for instance (in the case at hand), the marks (*notae*) that compose an algebraic letter sign. The "pure" intellect, notwithstanding its involvement with the visible letter sign, retains its capacity to separate indeterminate "ideas" from determinate ones, which means

150. See Part III, § 112.
151. See Part III, § 113.

that it keeps distinct the determinacy of the letter sign from the indeterminacy of its "idea" of "mere multitude." This enables the visible letter sign to "represent" to the "pure" intellect the invisibility of some one of its "simple intellectual things," in the precise sense that the "pure" intellect is able to get hold of the thing separated through the sign's visibility, which it nevertheless keeps distinct from the "abstract being" of the "simple intellectual thing." Because what the letter sign represents is not the determinate thing (units in the case at hand) from which the indeterminate "idea" ("mere multitude" in the case at hand) has been separated, but this very "idea" itself, Klein characterizes its representative function as 'symbolic'.[152] It is precisely the "service" that the imagination's power provides to the "pure" intellect by making the determinate algebraic marks and geometrical figures visible to it as a symbolic representation of the indeterminate content separated by the intellect that Klein calls 'symbolic abstraction', in recognition of 1) its distinction from Aristotelian ἀφαίρεσις and 2) its role in generating the symbols of the *mathesis universalis*.

Descartes's account of abstraction, however, does not reconcile the indeterminacy of the quantity treated by the *mathesis universalis* and the determinacy of the quantity proper to both enumerated things and the real figure of extended bodies. He characterizes both kinds of quantity as that kind of thing that "accepts the more or less" and that can therefore be called 'magnitude'. The fact that Descartes stresses the non-identity of the indeterminacy of the "idea" of magnitude that is abstracted by the "pure" intellect and the determinacy of the "idea" of magnitude that is in the imagination raises the question of how he thinks "that nothing can be said of magnitudes in general which cannot also be ascribed to some specific form or other." Descartes's answer to this question appeals to both the "figurality" of the real extension of the magnitudes presented in the imagination and the fact that it is the *same* imagination that serves the "pure" intellect in its treatment of general magnitudes. He conceives of the "image-making organ itself, with the ideas existing in it," "as being nothing but a true body, really extended and having figure." Its extension and figure are received—literally—by the "impressions" it receives from the part of the world that makes them, and it is precisely the fact that the same corporeal nature of the world belongs to the imagination and all its ideas that allows the *mathesis universalis* to grasp the "true world." The *mathesis universalis* can do this because the "figurality" of the magnitude that is depicted in the imagination is the *real* extension of a body that has been abstracted from everything except that it has figure, and

152. See Part III, § 115.

it is precisely to this "figurality" that the *mathesis universalis* transfers what the "intellect allows us to say about magnitudes in general." The conclusion that Klein draws from Descartes's philosophical reconciliation of the indeterminacy of general quantity (magnitude) with that of the determinacy of specific quantity (magnitude) is that extension has a twofold character in Descartes's thought: as the object of "general algebra," it is "symbolic"; as the substance of the corporeal world, it is real.[153]

§ 203. Klein on Wallis's Subordination of Geometry to Symbolic Arithmetic in the *Mathesis Universalis*

Wallis's subordination of geometry to arithmetic in the *mathesis universalis* follows Stevin in characterizing 'one' as a number and the nought as the principle of number, but goes beyond Stevin (as well as beyond Vieta and Descartes) in his understanding of the "analytical art" as confined entirely within the bounds of symbolic arithmetic. Wallis's reflections on number 1) make apparent the difference between 'thinking' (*cogitare*) and 'saying' (*dicere*) that comes to the fore in the conceptuality of the modern (symbolic) understanding of number, 2) establish the identity of the universality of the symbols (or species) with which the *logistice speciosa* calculates as *both* general magnitudes and thus precisely also numbers, and 3) formulate the object of the algebraic expansion of logistic and arithmetic as number, symbolically conceived as ratio. This means that the conception of the object of arithmetic and logistic is now completely in accord with algebra as a general theory of proportions and ratios, and that the "material" of the universal and fundamental science of algebra is no longer connected in any way to the pure units of the ancient tradition, as it had been for both Vieta and Descartes. The "material" of the analytical art for Wallis is numbers, conceived of in a manner that equates their "being" with their symbolic character, in the precise sense that the inseparability of 1) the composition of this material as signs (that represent general magnitudes) together with 2) the status of these signs as mathematical objects permits their mode of being to be grasped in an immediate (and therefore non-problematic) manner in the notation.

Wallis establishes the 'one' as a number by drawing a distinction between it and the 'unit'. What makes this distinction possible is the difference that emerges when he formulates what is meant when a number is thought and when it is spoken. For example, when Wallis *thinks* that 'four' has "the same force" as 'four units', units for him are neither a number nor its parts, but

153. See Part III, § 113.

either its denomination or the numbered thing. However, when he *says* 'four' in answer to the question 'How many units are there?', 'four' is precisely their number. It is the same in the case of 'one': when *thought*, 'one' has the "same force" as one unit (*unam unitatem*), one monad (μίαν μονάδα), and the unit is either the denomination of the number or thing numbered; when 'one' is *said*, however, it is a number, since it articulates precisely how many of something there are, namely, a single one. Thus, even though what is meant by 'four' or 'one' are four or one units, what is expressed and addressed is the number, understood without the counted things, and therefore conceived of in this sense as something that is "objectless." "Saying" number in this sense is possible only if number (and therefore 'one') is understood as a "symbol," and not as a definite amount of units.[154]

Wallis's account of the nought as the principle of number permits him to establish the priority of arithmetic over geometry, on the basis of the higher and more abstract nature of its objects in comparison with geometry, aprioricity that results in his characterization of "universal algebra" as being in truth arithmetic, not geometry. He establishes the nought as the principle of number (following Stevin) on the basis of its analogy with the point in geometry, and observes that by overlooking this, the ancients' "algebra" was limited to heterogeneous geometrical magnitudes, such as lines and planes compared with solids.[155] Wallis goes beyond Stevin, however, by interpreting the meaning of the ratio (λόγος) in Euclid's general theory of proportions as arithmetical (rather than geometrical), albeit arithmetical in the sense of symbolic arithmetic.[156] What enables Wallis to do so is his understanding of the various "algebraic powers" as being nothing but mutually homogeneous quantities in continuous proportion, an understanding that is possible because he regards numbers not as mere reference signs but as symbols that not only "represent" but also *are* mathematical objects. By contrast, homogeneity for Vieta was a requisite *condition* for operating with algebraic magnitudes because he still understood such magnitudes, notwithstanding their universality, in terms of their relation to operations on multitudes of units. As a consequence of this relation, Vieta's law of homogeneity stipulated that only algebraic magnitudes with a like "genus" (i.e., "degree" or "rung designation") could be added or subtracted in calculations with species. For Wallis, this requirement is no longer necessary because numbers, as "indices" of ratios, no longer mean a definite amount of objects (or units), but rather a homogeneous magnitude, the homogeneity of which is identical with their dimensionlessness.

154. See Part III, § 118.
155. See Part III, § 119.
156. See Part III, § 121.

Wallis explicitly states that in arithmetic equations are frequently made among various powers that are not of the same degree (e.g., $2a^2 = 6a$ and $2a^3 = 6a^2$), but that these equations nevertheless are homogeneous, because he conceives of the "arithmetic degrees" (*Gradus Arithmetici*) themselves as being "numbers in continuous proportion" (*numeri continue proportionales*). For Vieta these degrees precisely are *not* numbers, because as the genera of the "powers" of the magnitude of unknown numbers they are not, strictly speaking, themselves numerical in the sense that the ultimate reference of the unknown numbers are for him numerical; that is, these genera are not composed of multitudes of units. For Wallis, by contrast, the genera *are* precisely numbers, because for him numbers no longer consist in multitudes of units but, in effect, in the "number genus," that is, in the commonality of having the character of signs. Both "ordinary" and "algebraic" numbers are therefore essentially homogeneous and for this reason "universal," where these qualities are identical to their intrinsic symbolic character. Moreover, because all symbolic numbers are now understood to belong to the same genus, the genus of algebraic magnitudes no longer changes with changes in their "height" or "altitude," which eliminates the need for Vieta's special "rung" (or "degree") designations for each algebraic number.[157]

Wallis's characterization of the essential homogeneity of numbers is grounded in his understanding that the numbers of arithmetic are "nothing but the 'indices' of all the possible ratios whose common consequent is 1, the unit."[158] He understands the ratios, in turn, as "fragments of the unit" (*Unitatis fragmenta*), which are in between the 'one' and the 'nought'. The 'unit' presupposed by Wallis here, as "something continuous and divisible into as many (equal) parts as you please," he explicitly identifies as that of the "moderns," because he was acutely aware of its indivisible status for the ancients. In addition, he conceives the "fraction" as being "not so much a number as an index or ratio numbers have to one another," which means, however, that "fractions," too, are "numbers," because they have "numbers" as their terms. Wallis grasps the ratio indicated by number (and between numbers) as a comparison between two magnitudes in which it is determined how many times one magnitude is contained in another. Because the ratio is the result of division, it can be established directly (without further comparisons) from the "quotient" resulting from the division. If the magnitudes of the dividend and divisor are homogeneous (have the same dimensions), the quotient indicates how many times one magnitude is contained in another, with the quotient itself being a "number" that has no dimensions, that is, a dimensionless quan-

157. Ibid.
158. Ibid.

tity. If the dividend and divisor are heterogeneous, it follows that they do not have the same amount of dimensions and that, strictly speaking, division is not involved, because one magnitude is not contained in the other. Instead of division, Wallis considers what is involved in this case to be an "application" (*applicatio*), although he nevertheless understands the quantities involved in it also to produce quotients that are a dimensionless number. On the basis of these considerations, Wallis concludes that because the quotients that result from the comparison (and therefore relation) between any kind of magnitude are dimensionless, they are of the same kind (homogeneous) and therefore such as to be comparable to one another. In other words, the dimensionlessness of these numbers is identical with their role as the indices of ratios, and, because of this, all ratios of whatever quantities are homogeneous with one another and therefore can be compared with one another.

The dimensionlessness of numbers for Wallis is identical with their symbolic character; their role as "indices of ratios" is immediately visible in the notation, which is what leads Wallis to call the 'one' or 'unit' a denominator and to "assert that the whole theory of ratios belongs more to arithmetical than to geometrical investigations." Wallis therefore represents "the final act" in the symbolic reinterpretation of the ancient ἀριθμός insofar as his understanding of the universality of arithmetic as a general theory of ratios permits the objects of the algebraic expansion of arithmetic and logistic to be determined as a symbolically determined ratio. This conception of number both accords with that of algebra as a general theory of ratios and proportions and does away with the ancient controversy over the proper "material" of the numbers themselves. This "material" is now immediately graspable in the notation, thereby eliminating the problem of whether the mode of being proper to numbers should be conceived in terms of formations that are independent of somatic beings or obtained from them by abstraction.[159]

§ 204. Klein's Account of the Origination of the Logic of Symbolic Mathematics

Klein's account of the origination of the logic of symbolic mathematics employs the Scholastic distinction between first and second intentions, or, more properly, the distinction between the objects of first intentions and those of second intentions, to "express" the origin of symbolic numbers. Because for Klein the origin of such numbers is inseparable from the origination of the logic proper to symbolic mathematics, an account of their origin is the *sine*

159. See Part III, § 122, regarding the position summarized in this and the previous paragraph.

qua non for an account of the origination of this logic. He uses this Scholastic language to 1) describe both the shift from ancient to modern numbers that becomes evident in his desedimentation and reactivation of the status of the number concepts in Vieta, Stevin, Descartes, and Wallis, and 2) delineate the corresponding shift in the paradigm characteristic of the ancient and the modern "concepts" of number, the latter shift being characterized by Klein as the transformation of the ancient concept's "conceptuality."

First intentions concern the existence and quiddity of an object, its being in its own right; second intentions concern an object insofar as it has being in being known, in apprehension. Hence, the state of being of an object in cognition is *second*, while the state of being of an object in itself is *first*. Because the Greek ἀριθμός is inseparable from the direct reference to a multitude of definite things, the status of its referents lends itself to being designated as first intentional. Because the concept of 'indeterminate or general quantity' concerns an object insofar as it is known, the status of its referent lends itself to being designated as second intentional. The sense-perceptible mark that belongs to the symbolic number is, like any other sense-perceptible thing, the object of a first intention, and, because of this, Klein maintains that the "conceptuality" characteristic of the mode of being belonging to the modern concept of number is tantamount to the apprehension of the object of a second intention as having the being of the object of a first intention. Moreover, he maintains that the modern "conceptuality" of number is only manifest in its contrast with the ancient Greek "conceptuality," which is characterized by the first intentional status of the objects to which it refers and is therefore related.

Klein also appeals to the distinction between first and second intentions to clarify Descartes's attempt to understand the origin of the novel mode of being that belongs to the "true idea" of number as "mere multitude" (*sola multitudo*)—an idea that, being separated from all "enumerated things" (*re numeratate*), can be grasped only by the "pure" intellect with the assistance of the "power" of the imagination. Descartes's appeal to the power of the imagination to assist the pure intellect in making visible to itself the indeterminate object that it has already abstracted from its own power of knowing determinate numbers is an attempt to characterize the peculiar mode of being belonging to the symbolic numbers with which algebra deals. Abstraction in Aristotle presupposes definite beings that are intelligible in terms of common qualities, the latter being "lifted off" the former in accordance with a process that is more logical than psychological;[160] abstraction in Descartes presupposes definite beings but not their intelligibility—in the case at hand, their

160. See Part III, §§ 86–87.

"intelligibility" as so many beings. Rather, his abstraction works on the mind's act of knowing a multitude of units, separating out the mind's own conceiving of that multitude, which it immediately makes objective. The mind turns and reflects on its own knowing when it is directed to the idea of number as a multitude of units,[161] and in so doing, it no longer apprehends the multitude of units directly, in the "performed act" (*actus exercitus*) and thus as an object of its first intention, but rather indirectly, in the "signified act" (*actus signatus*), as an object of its second intention. Thus, notwithstanding the fact that what the intellect is conceiving is a multitude of units, the intellect's immediate apprehension of its own conceiving of this multitude, an apprehension that grasps it *as* something, *as* one, and therefore *as* a being, has the effect of transforming the *apprehension* of the multitude belonging to the number into a seemingly independent being, albeit a being that is only a "rational being" (*ens rationis*).[162] This "rational being," therefore, is the result of the intellect, which, secondarily (in reflection) intends a thing already conceived before, and intends it insofar as it has been conceived. When the rational being is then "grasped *with the aid of the imagination* in such a way that the intellect can, in turn, take it up as an object in the mode of a 'first intention,' we are dealing with a *symbol*, either with an 'algebraic' letter-sign or with a 'geometric' figure as understood by Descartes" (*GMTOA*, 222/208).

Klein therefore characterizes abstraction for Descartes as symbolic because the concept it yields is manifestly not something that is lifted off the intelligible qualities of things but rather something whose very mode of being is inseparable from: 1) the intellect's pure—by 'pure' is meant completely separate from the things it apprehends—grasping of its own power to apprehend these qualities themselves, and 2) this power's being apprehended as itself an object whose mode of being is nevertheless akin to the very things from which its mode of being separates itself. Klein stresses that the "kinship" between the power of apprehension proper to the "pure" intellect and that which is effectively foreign to it (i.e., the things possessing the intelligible qualities that are apprehended by the "pure" intellect's power) is established by making this power "visible." The algebraic letter "signs" of Vieta or the "geometric" figures of Descartes are what accomplish this. They are what—in the language of the Schools—allow the object of a second intention to be apprehended as the object of a first intention, and are therefore "symbols." The indeterminate or general object yielded in "symbolic abstraction" is neither purely a concept nor purely a "sign," but precisely the unimaginable and

161. That is, to the *idea* of either just so many units (e.g., 'fiveness'), or the *idea* of each and every number, of number (quantity) in general.

162. See Part III, § 115.

unintelligible identification of the object of a second intention with the object of a first. This identification is "unimaginable" because "images" properly—both for the ancient Greeks and for Descartes—refer either to particular objects of first intentions or to their particular "common qualities."[163] The identification of second and first intentional objects is "unintelligible" because, for "natural" predication, to say that a concept is both general and particular "at the same time" is nonsensical.

The origination of the logic proper to symbolic mathematics comes about for Klein when the "analytic art" is understood to be coextensive with symbolic arithmetic. This occurs when the symbolic numbers constitutive of "algebraic magnitudes" lose 1) their sense of being *representatives* of the genera of the "powers" of quantities of units and 2) the geometrical "ratio" (λόγος) in Euclid's general theory of proportions (which refers to heterogeneous geometrical magnitudes) is interpreted as a ratio in the sense of symbolic arithmetic. Algebraic numbers lose their function of representing the genera of the powers of known and unknown quantities when the "homogeneity" of these genera ceases to be a "condition" for the analytical art's logistical operation with algebraic magnitudes. The Euclidian sense of ratio gives way to that of symbolic arithmetic's when the meaning of ratio is formulated as a dimensionless "index" that results from the comparison between two magnitudes—a comparison that determines how many times one magnitude is contained in another whose "quotient" is directly determined. The loss of the algebraic number's reference to the genera of the powers of quantities of units is tied to the symbolic formulation of the meaning proper to ratio, because this formulation understands numbers to be precisely the indices of all possible ratios with a common consequent of 1, the unit. In this formulation, algebraic numbers are symbolically determined ratios, in the exact sense that the comparison of any genus of magnitude is understood to yield a number (quotient) that has no dimensions. With this, algebraic magnitudes are characterized by their essential homogeneity as dimensionless quantities, and hence the generic homogeneity of the powers proper to

163. Indeed, it is for this reason that Descartes, on Klein's view, stresses that the "power" of imagination, and not the imagination's "images," assists the pure intellect in grasping the completely indeterminate concepts it has separated from the ideas that the imagination offers it, because these ideas are precisely "determinate images"—and therefore intrinsically unsuitable for representing to the intellect its indeterminate concepts. However, being indeterminate insofar as it is not limited to any particular one of its images, the imagination's power is able to use its own indeterminacy to enter into the "service" of the pure intellect and make visible a "symbolic representation" of what is otherwise invisible to the intellect by facilitating, so to speak, the identification of the objects of first and second intentions in the symbol's peculiar mode of being. Here the imagination's facilitation involves, as it were, a bestowal of its "power" of visibility on the concept's invisibility.

quantities of units is no longer a requirement for the *logistice speciosa*. The genera of the powers of algebraic magnitudes no longer relate (even indirectly) to multitudes of units because such "arithmetical degrees" are themselves taken to be homogeneous "numbers in continuous proportion."

When numbers no longer consist in multitudes of units but in the "number genus," that is, in the commonality of a signitive character that makes visible in the notation their role as "indices of ratios," both "ordinary" and "algebraic" numbers become "universal" by virtue of their essential dimensionlessness. So understood, numbers become identical with their intrinsic symbolic character, and the ancient controversy over whether their proper mode of being *and therefore the logic of numbers* is something that is independent of somatic beings or abstracted from them is eliminated once and for all. What numbers are is now immediately graspable in their symbolic notation, whose "universality" is tied not to multitudes of units but to the notion of the unit as a '1'. When it is understood as a symbol that "expresses" a *res simplex communis* and therefore a *res simplex intellectus*, the '1' is grasped in a manner that renders it *independent* of the ancient logical controversy about the proper mode of being of numbers. Apprehended in this way, the '1' is what permits the "analytical art" to be understood as coextensive with symbolic arithmetic. No longer a "unit" of measure, the '1' to which the dimensionless numbers of symbolic arithmetic are the indices as possible ratios with a common consequent is, together with the numbers it makes possible, the *sine qua non* of the origination of the logic of symbolic mathematics. In Klein's words, "The object of arithmetic and logistic in their algebraic expansion is now defined as 'number,' and this means as a symbolically conceived ratio—a conception consonant with that of algebra as a general theory of proportions and ratios" (235/223).

§ 205. Husserl's Account of the Origination of the Logic of Symbolic Mathematics

Husserl's account of the origination of the logic of symbolic mathematics cannot be separated from a historical account of the development of his thinking. His early, pre-phenomenological research (in *Philosophy of Arithmetic*) into the logic of symbolic mathematics and the symbolic calculus generally is, in an important respect, definitive for his later, explicitly phenomenological research into the experiential basis of both formal mathematics (formal ontology) and the mathematization of the life-world. Important here is his characterization of the *non-conceptual* nature of the symbolic algorithms employed by "formal" mathematics. This view of the matter initially emerged

in the wake of his recognition of the need to abandon both Weierstrass's thesis regarding the foundational role of the concept of cardinal number for universal analysis (*arithmetica universalis*) and, in connection with this, Brentano's thesis regarding the logical equivalence of the conceptual contents proper to authentic and symbolic presentations.[164] As a consequence, Husserl came to understand the symbolic algorithms operative in formal mathematics as a calculational technique, composed of the signs and "rules of the game" that act as surrogates for genuine deductive thinking. As he puts it in his Schröder review, "calculation is no deduction. Rather, it is an external surrogate for deduction."[165]

Husserl initially (1890) defined formal logic itself "as a symbolic technique"[166] and understood the symbolic calculus determinative of universal analysis to be part of formal logic so defined. However, he eventually came to regard formal logic as a theoretical discipline directed toward conceptual contents that are more fundamental than the symbolism associated with any logical or arithmetical calculus. He characterizes formal logic in this sense as "pure" logic and as having as its conceptual content materially empty "manifolds" conceived as the correlates of the pure theory forms "constructed" by formal deductive systems. Husserl therefore comes to understand universal analysis as part of pure logic, as an instance of manifold theory, and, as such, he no longer strictly identifies it with a symbolic algorithm. The symbolic calculus that composes the algebra of universal analysis thus remains identified with a concept-less calculational technique in Husserl's mature work. And, on his view, this also holds for the symbolic calculus associated with the pure (completely formalized) *mathesis universalis.* Guided by Leibniz's idea of the *mathesis universalis* as a universal and formal science that has rational priority over any "material" mathematical discipline and any "material" logic, Husserl distinguishes the mathematical investigation of the theory forms of "determinate manifolds" from the essentially correlated but nevertheless distinct formal logical "theory of the theories" of manifolds.[167]

First developed in the last part of *Philosophy of Arithmetic*, Husserl's characterization of the symbolic calculus as a "mere" calculational technique informs his critique of Hilbert's program in § 33 of *Formal and Transcendental Logic*. There he distinguishes "an actual theory of manifolds" from "a *discipline comprising deductive games with symbols*." The *sine qua non* for the latter's becoming the former is clearly set forth as regarding "the game-symbols

164. See Part III, §§ 45 and 50.
165. See § 147 above.
166. *Stumpf Letter*, 161/17.
167. See § 181 above.

as signs for actual objects of thinking—units, sets, multiplicities"—together with the bestowal "on the rules of the game the significance of *law-forms* applying to these multiplicities." Likewise, in the *Crisis*, Husserl expresses the same view in his critique of the logically derivative status of the "symbolic calculating technique" responsible for the "completely universal formalization" of meaning in mathematized natural science. Thus, in connection with the calculative method operative in the "'theory of manifolds' in the special sense" of "the universal science of the *definite* manifolds," he writes: "Here the *original* thinking that genuinely gives meaning to this technical process and truth to the correct results (even the 'formal truth' peculiar to the formal *mathesis universalis*) is excluded." Moreover, Husserl also maintains that "in this manner it is also excluded in the formal theory of manifolds itself."

Above all, what is noteworthy in Husserl's discussions of the logically derivative status of the symbolic calculus in his mature work is the absence of phenomenologically descriptive analyses in support of their underlying claim, namely, that the original thinking and "actual" or "true" concepts proper to the pure *mathesis universalis* are excluded by its technical method of symbolic calculation. It is likely that this absence is due to the fact that Husserl considered his pre-phenomenological analysis of the claim's substance sufficiently definitive to obviate the need for proper phenomenological substantiation.[168] Whatever its source, however, the absence of such analyses raises the question of the phenomenological status of the "formalization" of knowledge accomplished by the *mathesis universalis*: does it owe its origination to the technical method of symbolic calculation, or to the original thinking and concepts that are putatively excluded by this method, or somehow to their "relationship"?

Husserl's phenomenological analyses, from the *Logical Investigations* onward, do address the origin of the formal object "constituted" by formalization, the *Etwas-überhaupt*, attributing it to a "formalizing abstraction" that is distinguished from both the "generalizing abstraction" that places "emphasis on a non-independent 'moment' of a content" and "the corresponding ideation under the title of 'ideating abstraction'" that brings the generalized universal itself to givenness.[169] Moreover, he terminologically fixes the distinction between general and formal essences in *Ideas I*, where the relationships between "generalization" and "formalization" are characterized as "essentially heterogeneous."[170] However, Husserl's analyses in *Ideas I* do not address the phenomenological origination of these processes, but instead ap-

168. See § 155 above.
169. See § 165 above.
170. See n. 77.

peal to "eidetic intuition" as the legitimating source of their essential difference, and one will search Husserl's works in vain for analyses that provide a precise account of the phenomenological origin of either the formal concept 'anything whatever' or the formalizing abstraction (formalization) in which it putatively originates. In the *Logical Investigations*, he says that "purely categorial" thinking is substituted for material thinking in formalization, together with the replacement of "indefinite" expressions for names with content.[171] In *Experience and Judgment*, formalization is characterized as an "emptying" of the objective material content of categories, while in *Formal and Transcendental Logic* it is "each *individual*" that is said to be "*emptied*."[172] And, it bears repeating, none of Husserl's major works subsequent to *Philosophy of Arithmetic* discuss the origination of the symbolic calculus in relation to either the logical structure of the formal category of 'anything whatever' or the genesis of the process of formalization itself.

Husserl's analyses of formalization, therefore, are not in accord with his own requirements in *Formal and Transcendental Logic* for a proper (phenomenological) theory of judgment. There he sees such a theory as charged with the task of providing an account of the hierarchical, finite, step-by-step evidential "genesis" from the experience of individuals to which the formal category of the 'anything whatever', as the elemental judgment meaning of pure analytics (i.e., of the pure *mathesis universalis*), "refers backwards." Because Husserl characterizes this evidence as two-sided and he includes in its hierarchy both judgment meanings and the judging process itself, the lack of a proper phenomenological theory of judgment informing his analyses also means that they do not address the "origin" of the formalizing process itself.[173] In the absence of the presentation and descriptive analysis of such evidence, Husserl's answer to the question of the relationship of the method of symbolic calculation to the "true" nature of formal objectivity, as well as to the logical formalization he maintains yields this objectivity, necessarily remains phenomenologically unclarified.

Indeed, even if one were to grant Husserl's guiding thesis, that the method of symbolic calculation is a non-conceptual technique that excludes both genuine formal logical thinking and its actual conceptual objects, his phenomenology would nevertheless have not addressed the origin of the surrogate "logical" function accomplished by the signs and rules of its calculus. That is to say, it is one thing to *claim*, as Husserl does in his most mature discussion of logical judgment, that neither genuine logical deduction

171. See § 167 above.
172. See §§ 176 and 189 above.
173. See § 196 above.

nor the ultimate formal structures of the "syntactical stuffs" presupposed by formal logical syntax are expressed by the algebraic signs and the rules for their combination that characterize the *mathesis universalis*'s method of symbolic algebraization.[174] It is quite another to provide phenomenologically descriptive analyses that demonstrate the truth of this claim.

Because Husserl's early phenomenological investigations of formalization do not explore what his mature investigation (in *Formal and Transcendental Logic*) characterizes as its "genesis," and because the programmatic nature of the latter investigation marks, at best, only a "methodological" advance over the imprecision of the earlier investigations, the distinction he makes between 1) the formally derivative status of the method of the symbolic calculus and 2) the conceptual formality proper to the "pure" logic that this method somehow serves has *not* been established phenomenologically. This means, among other things, that the guiding problem of Husserl's first philosophical work, the origin of the logic of symbolic mathematics, remains strangely unresolved in his mature phenomenological works. In place of its resolution in a completed phenomenological theory of judgment that would present the "foundation" of the logic constitutive of the pure *mathesis universalis*, Husserl's penultimate statement (found in *Formal and Transcendental Logic*) on the origination of the formalization coincident with this foundation characterizes it as follows: 1) the fundamental conceptual element of formalized (apophantic) logic and formalized mathematics (formal ontology) is the materially empty category 'anything whatever'; 2) because of this, each of these disciplines is part of the highest formalized discipline, the pure (because completely formalized) *mathesis universalis*; 3) actual (i.e., conceptual) formal logical relations do *not* originate in the symbolically algebraized syntax of the *mathesis universalis*'s method, and its letter symbols generally do not express the ultimate formal elements of the syntactical cores proper to the logical judgment; 4) all *logical form* tacitly presupposes ultimate individual contents that have *something to do with each other materially*; and, finally, 5) only the as yet unrealized phenomenological program of "transcendental logic," which is devoted to the evidential explication of the peculiarly sedimented individual and material sense implications of formalized meaning elements and syntactical connections, is capable of providing an account of the origination of the formalized concepts and truth values of the *mathesis universalis*.

Husserl's final statement, in the *Crisis*, on symbolic mathematics characterizes "[m]athematics and mathematical science, as a {well-fitting} garb

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174. See §§ 197–98 above.

of ideas, or the *garb of symbols* of the symbolic mathematical theories" (*Crisis*, 52/51), which "encompasses everything which, for scientists and the educated generally, *represents* the life-world, *dresses it up* as 'objectively actual and true' nature." Husserl maintains the "garb of ideas" is recognized neither by its innovators nor by contemporary thinkers for what it is, namely, "a displaced '*symbolic*' meaning" (44/51) wherein "what is actually a *method*" (52/51) is taken "for *true being*." The consequence of this, he contends, is that "the true meaning of the method, the formulae, the 'theories,' remained *unintelligible*" (52/52) for its innovators and remains such also for us. What "was lacking" for them and "what is still lacking" for us

> is the actual self-evidence through which he who knows and accomplishes can give himself an account, not only of what he does that is new and what he works with, but also of the implications of meaning which are closed off through sedimentation or traditionalization, i.e., of the constant presuppositions of his constructions, concepts, propositions, theories.

The fragmentary investigations in the *Crisis* do not provide or otherwise investigate the self-evidence lacking in the case of symbolic mathematics, and its discussion of the *mathesis universalis* contains a footnote referring, "for a more exact exposition" (45/46), to the accounts of it in the *Logical Investigations* and *Formal and Transcendental Logic*, which we have shown to fall short of providing the evidence proper to the origination of the basic concept of the *mathesis universalis* or the formalization in which it originates. Husserl's phenomenological answer to the question of the origin of the logic of symbolic mathematics in his final work remains, as in his others, general and therefore incomplete. Succinctly stated, his answer reads: sedimented in the symbolic formulae and theories of "the universal, self-enclosed idea of a highest form of algebraic thinking, a *mathesis universalis*" (44–45/45), is a reference back to the individual objects of the life-world, together with the indication of "ascending orders of *intuitions*" originating in the life-world indicated by these formulae, which mark the progressive formalization that makes possible the life-world's mathematization.[175] The logic of symbolic mathematics therefore owes its origin to the individual objects of the perceptual life-world, which, in the terminology of *Formal and Transcendental Logic*, have something to do with each other materially.

175. See Part II, § 34.

Chapter Thirty-six

Conclusion

§ 206. Klein's de facto Completion of Husserl's *Crisis*

By situating the historical-epistemological project of recovering the origin of the mathematical sciences proposed in Husserl's *Crisis* within his own recovery of its origin in *Origin of Algebra*, Klein makes the strong case that Husserl's phenomenology nevertheless does indeed possess the methodological resources for investigating the guiding "presupposition" of its account of the origin of the logic of symbolic mathematics. Husserl's presupposition, of course, is that the true object of the formal *mathesis universalis*—the formalized *concept* of the 'anything whatever'—is irreducible to its method of calculating symbolically. These resources are found both in Husserl's programmatic articulation of the phenomenological task of investigating the genesis of formalized meaning in *Formal and Transcendental Logic* and in his fragmentary de-sedimentation of the formalizing mathematization of the life-world in the *Crisis*. We have shown that, as a result of the programmatic nature of the former and the fragmentary character of the latter, Husserl's own employment of these resources does not sufficiently address the presupposition in question. His analyses in *Formal and Transcendental Logic* do not take up the phenomenological problem of the genesis of the method of symbolic calculating, and those in the *Crisis* do not desediment the phenomenological meaning of the historical origin of this method in François Vieta's establishment of algebra.

In what follows, we shall show that Klein's account of the origin of algebra resolves the issue of the constitution proper to the fundamental concept of the formal *mathesis universalis*, the *Etwas-überhaupt*. We shall do so by tracing "the genuine discovery of the formal" (*FTL*, 84), which Klein and Husserl agree "was first made, at the beginning of the modern age, by way of Vieta's establishment of algebra," back to its "intentional-historical" roots in the life-world. In concert with this, we shall reactivate the intentional "anticipation" and "accomplishment" that yielded the origin of what is now the pregiven tra-

dition of symbolic calculation. The "historical" dimension of what we want to show requires that Vieta's establishment of algebra be situated in terms of its relationship to the traditional "pre-formalized" mathematical treatment of geometrical and arithmetical "objects" that are pregiven in the pre-scientific life-world. Crucial for this is Klein's account of Vieta's transformation of the Diophantine method of calculating with the species (εἶδος) Diophantus applied to unknown numbers and their powers, which documents his (Vieta's) application of the ancient geometrical method of analysis and synthesis (found, e.g., in Pappus) to the arithmetic of Diophantus.

The "genuine discovery of the formal," for Husserl as for Klein, coincides with the anticipation and accomplishment of the equations composing Vieta's "pure algebra," which initiates the process of "formalization" in mathematics and logic and which, in Husserl's words, is "the process whereby material mathematics is put into formal-logical form, where expanded formal logic is made self-sufficient as pure analysis or theory of manifolds" (*Crisis*, 46/47). The reactivation of the "primal establishment" of this process in Vieta will provide the basis for the investigation of the answer to Husserl's phenomenological question of the origin and therefore true locus of the foundation of the process of formalization that eventually yields the object of the pure *mathesis universalis*. In this way, we shall explore the warrant for Husserl's pre-phenomenological presupposition that this object has a conceptual status that is more fundamental and therefore, in some sense, independent of the symbolic calculus. In short, we shall pursue the answer to the question of whether the formalization that yields the *Etwas-überhaupt* is a non-symbolic or symbolic process, and therefore the answer to the question of whether this "concept" is itself symbolic or whether it is independent of the signs and syntax belonging to the algebra proper to the symbolic calculus, in view of the results of Klein's research. In advance of our considerations, however, we note that the answers to these questions will hinge on whether the formalization that is coincident with the genuine discovery of the formal is dependent on the representation by algebraic letter signs of the indeterminate object proper to pure algebra.

§ 207. Desedimentation of the Formalization Constitutive of the Formal Concept 'Magnitude in General'

Klein shows Vieta's "discovery" of the formal to be grounded in the transformation of the concept of magnitude from a concept that is necessarily determinate—in the precise sense that its meaning is inseparable from a reference to either multitudes of units or geometrical figures—into a concept that is "general," in the exact sense that its meaning does not necessarily refer to either

multitudes of units or geometrical figures. What makes this discovery possible is, on the one hand, Vieta's innovation in how mathematical science deals with the unknown in the case of numbers and, on the other, his identification of the letter signs that he uses to represent both unknown and known multitudes of units with the general concept of multitude itself. Vieta's innovation was to attribute to *known* amounts of multitudes of units, that is, to known numbers, the status of being "granted as given" that, until then, had been reserved (in Diophantine analysis) for *unknown* amounts of units (numbers) alone. This innovation permitted Vieta 1) to use the letter signs representing the species of numbers and their powers for both known and unknown amounts of units and 2) to shift the solution to the operations with these letters signs entirely into the domain of the indeterminate (the granted as given). Because this shift yields for the first time concepts that are indeterminate—namely, the general concepts of numbers (i.e., twoness, threeness, etc.), the concept of number in general (i.e., any quantity whatever), and the concept of magnitude in general (i.e., magnitude that is identical with neither multitudes of units nor geometrical figures)—it is coincident with the origination of the process of formalization and, therewith, of the formalized concept.

By using the results of Klein's research to reactivate the intentional anticipation that led to Vieta's accomplishment of the origin of what now has the status of something pregiven in the tradition of symbolic calculation, namely, the sense-perceptible letter sign as the universal symbol for the materially empty formal category of any object whatever, we will be able to "test" the truth of Husserl's pre-phenomenological presupposition that this category and therefore the object of the pure *mathesis universalis* is a concept that is more fundamental than, and hence logically independent of, the symbolic calculus. The intentional anticipation behind Vieta's accomplishment involves the transformation of the objectivity of the ideal objects proper to the general (theoretical) mathematical treatment, and the generality of this treatment itself, that characterizes an already existing mathematical tradition. Prior to this transformation, the objectivity of the ideal objects of mathematics was divided along the lines of discrete multitudes or continuous magnitudes; the "generality" of mathematics characterized the method with which these fundamentally different ideal objects were treated, but *not* the method's object. The constitution of the object of mathematics as a general object that is identical with neither multitudes of discrete units nor continuous magnitudes is initially anticipated on the basis of Vieta's "noetic" intention,[176] which disregards the difference in the determinacy of the modes of givenness proper to

176. By 'noetic' here is meant the manner and direction of the cognitive regard, as distinct from the related object toward which it is directed.

the *unknown* continuous (geometrical) magnitudes and *unknown* discrete (arithmetical) multitudes that are treated as known by the (necessarily "preformalized") method of mathematical analysis.

According to the traditional method of geometrical "analysis," the mode of givenness belonging to the "*granting* as given" and therefore granting as "known" of an unknown continuous (geometrical) magnitude is equivalent to the mode of givenness proper to a continuous magnitude that is actually known: in both modes, manifestations of magnitudes with determinate geometrical dimensions are given. In the case of the mode of givenness proper to the granting as given and therefore as known of an unknown discrete (arithmetical) multitude, however, its mode of givenness is not equivalent to that of a discrete multitude that is actually known: the mode of givenness of a known discrete multitude manifests a determinate multitude of units, whereas the mode of givenness of the unknown discrete multitude that has been granted, qua its "species," as given, manifests not a determinate multitude of units but only the "supposition" that so many units have been given.

By disregarding this difference and therefore treating as analogous the modes of givenness of *unknown* discrete multitudes and unknown continuous magnitudes, Vieta is able to anticipate an entirely "analytic" and therefore general treatment of arithmetical "problems." He is able to do so, because once *the mode of givenness* of the unknown discrete multitude is assumed—contrary to what, in "fact," is the case—to be equivalent to that of the unknown discrete multitude *itself*, the "synthetic" necessity of performing determinate calculations in order to solve arithmetical "problems" is eliminated. In a manner akin to theoretical "analysis" (of "problems") in geometry, the analytical treatment of the unknown in Vieta's *logistice speciosa* is able to terminate its derivation of the "truth" directly on the basis of what is granted as known, without the additional step of the synthetic construction, which heretofore (in Diophantus's logistical arithmetic) had been necessary in order to demonstrate—by producing a determinate multitude—that the problem had been actually resolved.

Hence the transformation of the object of arithmetical-mathematical analysis that enables Vieta's anticipation of a "pure" analysis is tied to his "noetic" transformation of the cognitive intention characteristic of mathematical analysis in general. This transformation may be characterized as 1) a turning away from the distinct modes of givenness proper to both determinate multitudes and continuous magnitudes that 2) simultaneously grasps the granted as given unknown discrete multitudes as "something" whose mode of givenness is equivalent to, what, in *fact*, it is not equivalent to, namely, to that of unknown continuous magnitudes granted as given. That

is because unknown continuous magnitudes, unlike unknown discrete multitudes, have a mode of givenness that is similar to the givenness of the mathematical object in question when it is known: each presents a geometrical figure with determinate dimensions. With discrete multitudes, matters are otherwise, for while the mode of givenness of the known discrete multitude presents a determinate amount of units, the mode of givenness of the unknown discrete multitude does not present anything determinate. In other words, the transformation initiated by Vieta in the cognitive regard proper to mathematical analysis involves the grasping of the unknown and therefore "possible" multitude of a determinate amount of discrete units in terms of *the mode of being given* of a granted as known *unknown* magnitude—the continuous magnitude of a geometrical figure—whose mode of being given "actually" differs from the mode of being given of the discrete units that necessarily compose any known or unknown discrete multitude.

What makes this accomplishment possible is Vieta's redirection of his cognitive regard, from the direct apprehension of the determinate multitude of units that composes an actually given discrete multitude to the direct apprehension of its own apprehension of this multitude. The apprehension by Vieta's cognitive regard of its own apprehension of a given discrete multitude 1) not only turns away from the discrete units that belong to its apprehension of this multitude but 2) also treats the "unity," in the sense of the being "one" of its apprehension of its own apprehension, as something whose mode of givenness is akin to the discrete units that it has turned away from (in order to grasp its own apprehension of the units). In other words, the entirely "analytical" treatment of arithmetical problems is made possible by the cognitive intention's directedness to its own apprehension of determinate amounts of units in a manner that yields this apprehension itself as a mathematical "object." And because this object is at once indeterminate (i.e., it does not refer to a known amount of discrete units) and unitary (i.e., its mode of being is nevertheless treated as something, as "one"), it represents a "general" object or, equivalently, a "formalized" concept.

The phenomenological question of what it is that makes this indeterminate and therefore general concept itself possible is, of course, what is driving our investigation. More precisely, the question is whether such a concept can be thought apart from its connection with the letter signs that Vieta's "analytic art" uses to represent the species of known and unknown magnitudes, or whether letters signs, however arbitrary, are a necessary condition for the constitution of a formalized concept and thus for the "analytic" cognitive intention to be directed to such a concept. At issue, then, is the phenomenological status of the novel mode of being that belongs to

what, for Vieta and modern mathematics, is the "true concept" of number, namely, number "formalized" as "mere multitude" (*sola multitudo*), rather than a definite amount of definite units. As an object of the analyst's cognitive intention, this concept is separated from all "enumerated things" (*re numeratate*). How, then, can this intention grasp it, given the fundamental phenomenological correlation between the regard of intentionality and the intentional object to which this regard must, with "eidetic" necessity, be directed? How, in other words, does the "abstract," formalized concept of a general mathematical object appear as an intentional object?

Arithmetical abstraction (ἀφαίρεσις) in Aristotle presupposes definite beings that are intelligible in terms of common qualities, the latter being "lifted off" the former in accordance with a process that is more logical than psychological. Its logical dimension is determined by the fact that his account of arithmetical abstraction and abstraction generally represents an answer to the question of the origin of the unity and indivisibility supposed by Greek mathematics to characterize the noetic beings investigated by arithmetic and geometry. In the case of arithmetic, abstraction addresses the *true* mode of being of the unity of a determinate amount of determinate items that are intelligible and indivisible, that is, the true mode of being proper to a noetic ("pure") ἀριθμός. Aristotle's rejection of the Platonic claim that what is responsible for the being "one" (ἕν) of a "pure" ἀριθμός is a γένος that is "outside" or "alongside" of the multitude of intelligible and indivisible units that compose it provides the indispensable context for his claim that this being "one" arises ἐξ ἀφαιρέσεως (from abstraction). Aristotle's account of arithmetical abstraction therefore presupposes both an already established traditional mathematical understanding of the being of a noetic ἀριθμός and a philosophical controversy about the "unity" of this being—namely, whether or not it has a mode of being that is properly characterized in "separation" (χωρισμός) from the being of the items that compose an arithmetical multitude.

Aristotle's answer, of course, is that it does not have such a mode, but that the unity of an ἀριθμός arises from abstraction and is therefore an inseparable "piece" of the sensuously perceivable beings (αἰσθητά) that compose an ἀριθμός. He characterizes abstraction as the mathematician's isolation, from among all the sensible qualities belonging to the bodily items in a multitude, of the common quality that permits the answer to the question 'how many?' to be given with respect to these items. This isolation occurs on the basis of the mathematician's "positing" of what is *not* separate in the perception of sensible bodies as separate insofar as all their sensuously perceivable aspects are disregarded, save for the quality that permits them to be just so many. Initially, this quality is their species (εἶδος) (e.g., apple, soldier, body, etc.), which, even

though it is incapable of existing in separation from the being proper to the items of which it is the kind, is nevertheless treated "as if" it were separable, thereby permitting their mathematical investigation. The "unity" of the "heap" (σωρός) of sensible beings that compose an ἀριθμός[177] is therefore their species,

177. Aristotle's references to a "heap" in connection with the question of the being of number are embedded in discussions that do not explicitly claim that number is like a heap. Rather, in each case number being "like a heap" is presented as the conclusion that follows if number is "not one" (*Metaph.* 1044a 5), or not "a whole that is something over and above the parts" (a 10), or "not some one thing (ἕν τι)" (1084b 21). Aristotle, however, is clearly intent on establishing that the manner of being of numbers, as a mathematical thing, is "derivative" (1077a 19–20) in the sense that its being does not "take precedence over sensible things" (a 17–18), and, therefore, that "they are not capable of being somewhere as separate" (1077b 14–15). That from numbers' incapacity to have being in this regard it follows for Aristotle that "each is not one but is like a heap" (1044a 4) can be seen as the positive result of his polemic against those people who speak of numbers as being one in the sense that thinghood (οὐσία) is one. Thinghood is not one "in the way they say it is, as though it were a unit or point, but each independent thing (οὐσία) is a complete being-at-work-staying-itself (ἐντελέχεια), and a particular nature (φύσις)" (a 7–9); and number for Aristotle is manifestly not one in this sense of an independent thing. Thus, when Aristotle asserts that "it is necessary to a number that there be something by means of which it is one" (a 2–3), he immediately qualifies this assertion (following Ross's and not Jaeger's text of the *Metaphysics*) by adding, "that is, if it is one" (a 4); and he goes on to claim that those who make the claim that number is one are unable to say by what means it is so. Aristotle's point, then, is that those (Plato and other members of the Academy) who claim that number is one 1) are wrong because their claim is based on the mistaken supposition that number is one as thinghood is one and that therefore the unity of number is capable of being separate from sensible things, 2) deserve criticism because they cannot say by what means number is one, and then 3), because of (1) and (2), number is not some one thing but rather is like a heap.

A syllable is the opposite of a heap according to Aristotle, in that a syllable is something else than the letters (elements) out of which it is composed, the implication clearly being that the heap is not something else than the elements that compose it. In both the case of a heap and a syllable, Aristotle maintains that "the whole is one" (*Metaph.* 1041b 11–12), although the something else than the elements in the instance of a syllable is likened to "the thinghood (οὐσία) of each thing (for that is what is primarily responsible for the being of it)" (1041a 27–28), which is "not an element but a source (ἀρχή)" (a 31) of its being an independent thing. In the case of the heap, Aristotle's point is that the whole is not one in this sense. Thus he argues that "if number is separate" (1084b 2–3), and, therefore, "insofar as a number is composite" (b 4), "the one is prior," but "insofar as the universal and form are prior, the number is; for each of the units is part of the number as its material, but number is in the manner of εἶδος." Because the εἶδος is indivisible, the Platonists say that it is also one, so that both the units, as the parts that compose the material (ὕλη), and the εἶδος and thinghood (οὐσία) of number are one, and, as such, are sources (ἀρχαί). Aristotle holds, however, that "this is impossible" (b 19): "for if the number is some one thing and not like a heap," then its εἶδος and material are not only one in different senses, but "in truth each unit has being as a potency (δύναμις)" (b 21) and "not as fully at-work (ἐντελέχεια)" (b 22–23). Number, then, cannot be separate as the Platonists say, that is, in accord with the two sources of its being one thing that they identify, the indivisibility of the εἶδος and the units proper to the material. Not being separate, it follows that number is not some one thing, but like a heap, that is, like a whole whose being one and therefore thinghood is derived, in this case, from the multitude of units as such that compose the whole's elements, in the precise sense of the "how many" indicated by this non-independent whole.

which, in its isolation from the other qualities of these beings, is “abstracted from,” “lifted off” of these beings in a manner that allows it to function as their common “measure.” Moreover, even though the sensible beings that are unified in their ἀριθμός are subject to partition, their natural kind, in its function as a measure, is not subject to partition and thus is indivisible.

From among the “abstracted” “species” that, in their function as measures, provide the unity of ἀριθμοί of apples, soldiers, bodies, etc., the mathematician can further abstract their common quality of being one and being indivisible. This he accomplishes by disregarding the different natures of the species, which isolates their common quality as measures—namely, being “one” and “indivisible”—and then treating this “as if” it were separable from what it, in truth, is not: both the natural species and the multitude of somatic beings to which these species inseparably belong. In this manner, the mathematical μονάς (unit) originates as something whose mode of being is nothing other than the character of being a measure, that is, “one” and “indivisible,” which has been “lifted off” somatic beings, even though, in truth, its mode of being remains that of “pieces” of sensibly perceived bodies that have been transformed into “species-neutral” monads by abstraction.

Does the formalizing abstraction in modern mathematics that originates with Vieta’s “analytic art” function similarly? Even on the basis of our incomplete account of it so far, it should be clear that it does not. This needs to be stressed in the strongest possible terms. Aristotelian abstraction yields “units” that are species-neutral but hardly indeterminate in the sense in which Vieta’s letter signs representing the species (εἶδος) of known and unknown *numeri* are indeterminate. The mode of being of Aristotelian abstracted units remains wholly determinate because of their status as “pieces” of sensuously perceivable beings. They are, to be sure, no longer subject to the senses—that is, as “abstracted beings,” they are apprehended by thought and given “in advance” of mathematical science—but this does not mean that they achieve any independent being “alongside” of αἰσθητά: the mode of being of a species-neutral unit remains entirely dependent on the “particular” being of sensuously perceivable bodies.

To call an Aristotelian unit—or any unit whose mode of being is characterized as arising “from abstraction” in the Aristotelian manner—‘formal’ is, subsequent to Vieta’s invention of the *formalized* concept, to risk equivocation:[178] The “conceptuality” of the “formal” in Aristotle’s sense is inseparable from, and therefore presupposes, the being of determinate, sensuously perceivable bodes; the “conceptuality” of the “formal” in Vieta’s sense not only

178. See n. 91 above, where Husserl’s equivocal use of the term “formal” is noted along precisely these lines.

makes no such presupposition, but neither can it be properly grasped insofar as its "formality" is conceived as relating to any determinate being—whether bodily or intelligible. The conceptuality of the formalized concept invented by Vieta not only is not dependent on any such beings, but this independence is inseparable from its very meaning. The concepts composing Vieta's "analytical art" (*logistice speciosa*) *mean* precisely any arbitrary (possible) number (in the case of the formal concepts of numbers in general, e.g., twoness, threeness, etc.), any arbitrary quantity (in the case of the formal concept of number in general), and any arbitrary magnitude (in the case of the formal concept of magnitude in general). Husserl clearly saw this distinction in the conceptuality of the formal when he remarked, in *Formal and Transcendental Logic*, that Aristotle "lacked formal ontology, and therefore lacked also the cognition that formal ontology is intrinsically prior to the ontology of realities" (*FTL*, 70), because "Aristotle relates his analytics to the real world and, in so doing, has not yet excluded from his analytics the categories of reality."

However, Husserl's insight into the logical limits of Aristotelian conceptuality does not extend to his insight into the "abstraction" to which Aristotle appealed so as to account for the mode of being of the unity of intelligible beings. From his earliest to his final works, Husserl clearly characterizes the cognitive origination of the generically empty formalized concepts of the 'anything' or 'anything whatever' and the 'one' in the manner of Aristotelian "abstraction." This is most obvious in *Philosophy of Arithmetic*, where Husserl accounts for the origin of both of these concepts on the basis of "inattention" to individual contents "as contents determined thus and so," from which he maintains the abstractive passage to the general concept follows, because these contents "are considered and attended to only as some contents in general, each one a *certain something*, a *certain one*" (*PA*, 79). Frege's critique of the inability of Husserl's appeal to "inattention" to yield the general (formalized) concept of number is therefore on the mark insofar as it highlights the limits of Aristotelian abstraction to account for the origin of formalized concepts.[179] And while Husserl's later works do not appeal to inattention in order to account for the abstractive origin of formalized universality, they nevertheless ap-

179. In reference to Husserl's account of the abstraction, Frege writes ironically: "Inattention is an exceedingly effective logical power; whence, presumably, the absentmindedness of scholars." He goes on to say, however, in a manner that is quite to the point with respect to the inability of abstraction in the Aristotelian manner to generate a formalized concept: "For example, let us suppose that in front of us there are sitting side by side a black and a white cat. We disregard their color: they become colorless but are still sitting side by side. We disregard their posture: they are no longer sitting, without, however, having assumed a different posture; but each one is still at its place. We disregard their location: they are without location, but still remain quite distinct. Thus from each one we have perhaps derived a general concept of a cat" (Frege, 319).

peal to an "abstractive emptying" of material contents as its formalizing origin, and they do so in a manner that we have shown not only to lack precision but also to fall short of his own most mature stipulation regarding the evidence of a concept's genesis necessary to account most fully for its origin.

The inappropriateness of Husserl's appeal to Aristotelian abstraction to account for the origin of formalized concepts can be highlighted most forcefully by way of a consideration of the difference between the manner of universal applicability of an Aristotelian species-neutral monad and Husserl's own account of the concept of the 'anything whatever'. The "universality" of an Aristotelian monad is grounded in its indivisibility as a measure, which renders it as a "completely exact" (ἀκριβέστατον) measure and allows the arithmetician, having posited it as detached, to apply its unity as a measure (its "being one") to the species that function as the indivisible arithmetical measure of any determinate beings whatever. The universality of the Aristotelian "one," in other words, presupposes both individual beings and their "intelligibility" as species. By contrast, in Husserl's account of the "universality" of the 'anything whatever', it does not function as the exact measure of the particular species of beings, does not, in its "unity," measure their "being one," but rather *means* precisely any arbitrary thing that happens to "fall" under the "unity" of its "materially" indeterminate (i.e., species-*independent*) concept. In other words, on Husserl's view, the "universality" of the 'anything whatever' presupposes neither the "intelligibility" as a species nor the measurability as an individual "one" of that which "falls under" its concept.

Returning to Vieta's method, Klein has shown that while it presupposes definite beings, it most definitely does not presuppose their "intelligibility" as species or species-neutral monads, which permits their measurement by one and therefore enumeration. Rather, Vieta's formalizing abstraction works on the cognitive intention's "act" of knowing a multitude of units, separating out its own conceiving of that multitude, which it immediately makes objective. The cognitive intention turns and "reflects" on its own knowing when it is directed to the concept of number as a multitude of units,[180] and in so doing, it no longer apprehends the multitude of units directly (in its straightforward intention and thus as the straightforward object of this intention) but rather indirectly (in the apprehension of its own knowing intention) as the intentional object of this intention. Thus, notwithstanding the fact that what is being presented to the cognitive intention is a multitude of units, its immediate apprehension of its own conceiving of this multitude, an apprehension that grasps it *as* something, *as* one, and therefore *as* an in-

180. That is, to the *concept* of either just so many units (e.g., 'fiveness'), or the *concept* of each and every number, of number (quantity) in general.

tentional object, transforms the *apprehension* of the multitude belonging to the "number" into a seemingly independent object, albeit an object that is only an "intentional" object. This "intentional object," therefore, is the "constitutive" result of an act of intentionality, which, secondarily, intends a thing already conceived before, and intends it insofar as it has been conceived.

In order to grasp the intentional unity of the formalized "intentional object" constituted in formalizing abstraction as a "concept" that is distinct from the determinate content of the same "concept" (in this case, multitudes of units), the cognitive regard's "pure" intention must somehow make it "visible." That is because the indeterminacy of the formalized intentional object renders it "invisible" to the pure intentional regard that has separated it from the determinate modes of givenness of (in the case at hand) discrete units. It is invisible because notwithstanding its status as an intentional "object," the very mode of its givenness as *indeterminate* does not present an object with the mode of givenness belonging to that of straightforwardly intended mathematical objects, that is, discrete or continuous magnitudes. Properly speaking, the formalized intentional object does not present an object at all but rather the apprehension of the apprehension of a multitude of objects. Neither this secondary apprehension nor the initial apprehension of a multitude of objects has itself an *objective* mode of givenness; objects, to be sure, are given to the initial straightforward intentional regard, but neither its noetic intentional directedness nor its secondary apprehension is itself given *as* an object. Not being "objectively" given, the apprehension of apprehension that composes the indeterminacy of the formalized "concept" is invisible to the *formalizing* (and therefore "pure") intention that constitutes it as a "concept" that is separate from the concepts that render intelligible the straightforwardly apprehended mathematical objects.

In order to grasp its formalized concept, then, the formalizing regard of the pure cognitive intention must somehow render it visible. Reflection on the manner it does so reveals that this is accomplished by the pure cognitive intention's use of the "imagination," specifically of the impure "modification" of consciousness that is inseparable from the presentation of images—but *not* these determinate images themselves. In this manner, the pure cognitive intention is able to represent to itself the indeterminate and therefore unvisualizable content of what it has constituted.[181] The pure cognitive intention characteristic of Vieta's "analytic art" is able to do this, be-

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181. Phenomenological reflection on the essential correlation between the modification of consciousness that *intends* imagistic phenomena and these phenomena themselves uncovers 1) the "impurity" of this modification and 2) its non-coincidence with the essentially determinate character of any one of the images that appear as the "object" of its intention. The

cause even within the realm of the imaginative modification, which is "alien" to the purity of its intention, it retains the ability proper to it and foreign to this modification, namely, of separating—and therefore "constituting"—indeterminate "formalized concepts" from the determinate images presented by the imagination. Thus, for example, when the "pure" intention of the cognitive regard separates 'fiveness' from 'five' enumerated things and apprehends it as a "mere multitude" (and therefore as something that is separated from that which it "in truth" belongs to without being identical with, that is, the enumerated things), the imaginative modification, which ordinarily would make visible five units (perhaps as points), now makes visible something graphic—such as, in this case, the mark (*nota*) that comprises an algebraic letter sign. The cognitive regard of the "pure" intention, notwithstanding its involvement with the visible letter sign, retains its capacity to separate indeterminate "concepts" from determinate ones, which means that it keeps distinct the determinacy of the letter sign from the indeterminacy of its concept of 'mere multitude'. This enables the visible letter sign to "represent" to the "pure" intention's cognitive regard the invisibility of a "general concept," in the precise sense that its "pure" regard is able to get hold of the thing separated through the sign's visibility, which it nevertheless keeps distinct from the "intentional unity" of the "formalized concept." Because what the letter sign *represents* is *not* the determinate thing (units in the case at hand) from which the indeterminate "concept" ('mere multitude' in this case) has been separated but this very "concept" itself, its representative function is "symbolic." It is precisely the "service" that the imaginative modification provides to the "pure" intention proper to the cognitive regard involved in formalizing abstraction—by making determinate algebraic marks visible to it as a symbolic representation—that renders the genuine discovery of the formal dependent on the representation by letter signs of the indeterminate object of pure algebra.

In addition to the necessity of a visible mark's being made available to the cognitive regard in order for it to represent to itself the formalized concept that it has constituted via its apprehension of its own apprehension of a

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modification's "impurity" is a consequence of the non-conceptual essence of the image-phenomenon itself and of the essential possibility proper to the imaginative modification of consciousness to *exceed* the scope of any given manifold of image-phenomena. It is precisely the *difference* manifest in the said non-coincidence of the modification, between the imaginative modification of consciousness and the essentially determinate images that are presented in this modification, that permits the pure intentionality operative in formalizing abstraction to enlist the modification of consciousness characteristic of the "imagination" into the service of rendering visible the indeterminate concepts that are yielded in this abstraction.

multitude, there are two other characteristics of Vieta's method of formalization that distinguish it decisively from Husserlian "categorial intuition."

First, the cognitive regard's "apprehension of its own apprehension"—namely, its granting as given an unknown multitude of units and treating the "possibility" coincident with this mode of "supposed" givenness as an actual "object" and therefore as a "conceptual unity" in its own right—does not concern its cognition of the qualities belonging to an individual object. What concerns the granting as given of the unknown that is objectified in Vieta's "analytical art" are not the unknown qualities of a substrate object that are "cognitively" related to this object via the copula, the doxic thematization of which manifests for Husserl the intuition of categoriality. Rather, the unknown that is granted as given in Vieta's analytic treatment of the unknown as known is a determinate amount of determinate units, that is, the determinate unity of a multitude.

Secondly, the proper mode of being of this determinate unity—which, for the ancients, is in dispute because of the recognition that it differs from the mode of being of the "units" that compose it—ceases to be an issue and therefore matter of dispute for the "analytic art" because the art understands this mode of being to be "conceptual." That is, the practitioners of the analytical art understand the mode of being of the unity of a multitude to be precisely the *concept of a multitude*. As we have shown, Husserl's thought originates in the unsuccessful attempt to provide an account of psychological genesis of the *content* of this concept. Also, from the *Logical Investigations* on, he consistently distinguishes the unity of its collective whole–part "categorial" structure from the unity of the "categorial" whole–part structure of individual objects, and he distinguishes their corresponding judgment forms as well. Moreover, as we have also shown, Husserl himself does not understand the paradigm that governs the categoriality of intuition with respect to logical substrates—namely, "empty intentions" being fulfilled in doxic acts that "posit" "pregiven" and "pre-predicative" categorial "states of affairs"—to be applicable in the case of the "retrospective apprehension," as a "set," of the "noetic unity" that pre-predicatively apprehends a plurality.

These distinctions prevent either the formalized unity of the concept of a multitude or the unity of its content (i.e., its "extension" in a multitude of objects) from being characterized as the unity apprehended in the "categorial intuition" of the whole–part structure of a logical substrate. Husserl's account of the retrospective grasping of the noetic apprehension of a plurality as a set must also be distinguished from Vieta's formalization of a multitude, because, as we have shown, in this case Husserl articulates the unity as being that of a "concrete" plurality. That is, he understands the "object"

constituted in this manner to be that of the collection itself, namely, the "unity," which, as a "whole," encompasses the straightforwardly apprehended individual objects that compose a plurality and that, as its "parts," do not enter into the content of their collective unity as such. In other words, at issue here is not the formalized concept of multitude but what, in *Philosophy of Arithmetic*, is termed the *content* of the concept of multiplicity.

These same two characteristics that prevent the formalization accomplished by Vieta's "analytical art" from being subsumed under the meaning structures and judgment process articulated by Husserlian "categorial" intuition also prevent that formalization from being subsumed under the heading of what Husserl characterized as 'nominalization'. We have shown that Husserl understands nominalization to be a law "evinced in logic," according to which something nominal corresponds to every proposition and propositional content. In the case at hand, the plural that appears in judging, upon being "nominalized," is said to be transformed into a singular substrate object, the "set," which becomes an "object about which" additional judgments can be made. Given the nominalized object's status as a "form of judgment" for Husserl, we have also shown that, rather than accounting for the non-psychological and therefore logically objective collective unity of a multitude, his account presupposes the prior availability of "formations" that possess just such unity. Moreover, precisely its status as a "form of judgment" presupposes that the formalizing abstraction that came into being with Vieta's "analytical art" has *already occurred*. Thus, rather than being a phenomenological account of the genesis of either the collective logical unity of a multitude or its formalization, Husserl's exposition of the logical law of "nominalization" presupposes—without accounting for this presupposition—the already constituted phenomena of collective logical unity and its formalization.

The answer, then, to the phenomenological question of whether the formalization that is coincident with the genuine discovery of the formal is dependent upon the representation by letter signs of the indeterminate object proper to pure algebra is 'yes'. The intentional structure of this dependency, as it functions in a symbolic calculus, is most aptly formulated as follows: the algebraic letter sign's representation of the formalized concept (or, object—it makes no difference) that is inseparable from this concept's intentional content is, qua its sense-perceptible status, grasped by the regard of the cognitive intention operative in symbolic calculation in accordance with its straightforward mode of givenness. That is, in symbolic calculation, the formalized concept/object is grasped *not* according to its intentional genesis in formalization as a secondarily intended intentional unity but as a "unity" whose mode of being is akin to the unity of straightforwardly intended objects.

As we have seen, Klein distinguishes the new mode of abstraction made possible by Vieta's "analytical art" from Aristotelian ἀφαίρεσις, and terms it 'symbolic abstraction'. We have used Klein's account of its "primal institution" in Vieta's invention of the "analytical art" (the *logistice speciosa*) as the guiding clue for fulfilling a decisive aspect of the project that Husserl announced in his *Crisis*. Specifically, Klein's account of Vieta's transformation of the ancient ἀριθμός into the modern symbolic concept of number has been used to desediment the origin of the algebra and its "anticipation" of the process of formalization in mathematics and logic that culminates in both the idea and the "reality" of a completely formalized science, a "pure" *mathesis universalis*. With this, we conclude, the phenomenological condition of possibility proper to the constitution of a symbolic mathematical object is accounted for, namely, of an indeterminate and therefore general object that possesses a "possible" mode of being that becomes, via its visible "expression" in signs, an "actual" mathematical object.

§ 208. Desedimentation of the Historicity of the Origin of Formalization from the Pregiven Life-world

In light of the preceding discussion, Husserl's "presupposition" that formalization has already occurred, even as he investigates the origination of the formalized logic belonging to the "pure" *mathesis universalis*, may be unpacked by revisiting his account of the logical independence of the proposition's content from the non-syntactical employment of algebraic signs in apophantic logic.

On the one hand, Husserl maintains that the propositional content indicated by the "symbolic letter" is independent of the letter itself that functions to indicate it. Thus, the content of the proposition members indicated by the "symbol" 'A' in the propositions 'A is b' and 'A is c' are independent of the "identity" of the letter 'A' that appears in both propositions, because, according to Husserl, the proposition, the opinion that forms the "meant as meant," is formed differently in each proposition. Husserl takes this to illustrate that the "ultimate members" that compose the whole of any proposition are "non-self-sufficient" and thus that the subject and predicate members of a proposition are "non-detachable." In these two propositions, the same 'A' is meant twice, but it is "meant in a different How." Husserl maintains that the identity relation between these two propositions is established by something other than the "As'"—i.e., plurality of the letter 'A' in the propositions—"symbolic" (algebraic) function in these propositions, namely, by an "unstated total proposition" that is composed of the "unex-

pressed" propositional form 'the same' in the second proposition that relates back to the first occurring 'A': that is, *A is b, and the same A is c*.

On the other hand, it is clear that what Husserl understands by the letter sign 'A', in its function to indicate symbolically a content belonging to these two propositions, is *not* literally the letter 'A' itself, as a sense-perceptible mark, but, rather, the concept of any arbitrary propositional content whatever, with absolutely no individual or material restrictions. The 'A' letter*s* in this example thus signify, for Husserl—*at the same time*—1) the non-detachable "ultimate members" of the whole of two different propositions, each of which is propositionally *different* due to its givenness in different Hows, and 2) the formalized *identity* proper to concept of the 'anything whatever', which, because of its function to "indicate" propositional contents that are individually and materially indeterminate, is inseparable from the letter sign 'A' appearing in both propositions.

What Husserl understands "at the same time" here is, at once, a determinate letter with an indeterminate conceptual meaning *and* a propositional content that is *not* identical with the indeterminate meaning that is inseparable from this letter. In other words, Husserl understands the letter as a symbol that stands for the universal concept of being a proposition just as he understands the "actual" contents of any proposition to be determinate propositional members. The object of Husserl's understanding here, therefore, is isomorphic with precisely what is meant by a symbol in Vieta's *logistice speciosa*, because the latter means, at once, the universal concept of quantity *and* the actual contents of any quantity, a multitude of units. However, Husserl's way of speaking about the 'A' as 'the same object A', as if the 'A' itself were the ultimate propositional member itself, even after having previously "conceptually" distinguished the 'A' (as an algebraic letter symbol) from the non-algebraic ultimate members of a proposition that it indicates, highlights the fact that the structure of the 'A' as a symbol is not transparent in his thinking. Husserl's manner of talking about the object of the proposition member as if it were the 'A' itself, in the sense that the 'A' *represents* its indeterminate *logical content*, just as he talks in a manner that distinguishes the 'A' from this logical content as an object, indicates, moreover, that the formalized structure of the symbol is "sedimented" in his thinking. As a consequence, the symbolic abstraction instituted by Vieta's *logistice speciosa* is likewise "sedimented" in Husserl's articulation of the algebraic sign's function (in the apophantic proposition) to indicate non-syntactically ultimate propositional members that are *not* algebraic.

This way of talking has its basis in Husserl's conviction, which we have shown to be unfounded, that the symbolic calculus is *logically* independent

of the *actual* concepts of formalized logic. Thus Husserl's investigations of formalization seek to establish its foundation in the experience of the determinate (individual) objects that he posits as the source of the actual concepts of formalized logic—despite his acute awareness that the meaning of the indeterminate concept of the 'anything whatever', because it is inseparable from any *arbitrary* object whatever, is therefore "analytically" irreducible to precisely the experience of individual objects. Husserl, of course, is aware of this irreducibility, but, as we have also demonstrated, because his understanding of abstraction is basically Aristotelian, his theory of judgment is, in principle, incapable of accounting for the origin of a materially indeterminate concept. Husserl's basis for distinguishing between the formalized structure of the 'anything' and the 'one' is instructive in this regard. The 'one' belongs, he contends, to the concept of 'multiplicity', while the 'anything' does not belong to it, and, consequently, the content of the concept of multiplicity is a concrete multitude of 'ones' or 'units', whereas the "content" of the concept of 'anything' is decidedly *not* multitudinous in this or any other respect. Yet this difference does not mean that the 'anything' or 'anything whatever' is "singular," if by this is meant that it is 'one' in the sense of the 'ones' belonging to a multitude. We have seen that, as an "indeterminate" concept, the meaning of the 'anything whatever' belongs to a fundamentally different conceptual dimension than that of the arbitrary objects that "fall under it." It is meaningful to say of the latter that they are "singular," since each member of the multitude to which they necessarily belong is precisely "one"; but it is not meaningful to say of the 'anything whatever' that it is "one" in this sense: its meaning, as a formalized "unity," is beyond the opposition of "one and many."

Because its meaning is "beyond" that opposition, it both makes the "mode of being" of Vieta's symbol possible and is also responsible for the sedimentation—in the symbolic calculation made possible by the symbol—of the problem of the mode of being belonging to the "unity" of a multitude. The problem of what, subsequent to Vieta's invention of the symbol, is now properly the "pre-formal" (in the sense of formalization) "mode of being" of the unity of both limited and unlimited multitudes, drops totally out of the picture with Vieta and the formalized science his "analytical art" anticipates. That is, the problem that informs the two ancient Greek paradigms for the conceptuality of concepts—the Platonic thesis of a "separate" γένος as being responsible for such unity and the Aristotelian thesis of an abstracted species-neutral monad—is seemingly eliminated by the formalization of unity engendered by the symbolic calculus. As in Galileo's geometrical mathematization of the life-world, wherein the "true" being of the "empirical" shapes of

sensuously perceived objects is posited as having the mode of being of the ideal objects investigated by "pure" geometry, Vieta's *numerical* formulation of the ideal species of the determinate amounts of determinate units likewise leads to the thesis that the "true" being of objects that belong to the life-word have a being that properly belongs to the concepts of a mathematical science. Only, in the case of the mathematization that is initiated by the "analytical art," instead of sensuously perceived objects belonging to the "pre-scientific" life-world being "mathematized," it is the unities of the determinate multitudes of "empirical" and "intelligible" objects that "order" these objects (i.e., "numbers") that are posited (with their mathematization) as having the mode of being belonging to a mathematical science. In this case, of course, the science at issue is the incipiently formalized mathematical science of the *logistice speciosa*, and it is the mode of being of its indeterminate concepts that is posited as the "true" being of the *order*—or, more precisely, the "laws" governing the order—of the objects in the life-world. The problem of the proper mode of being belonging to the unity of determinate multitudes of determinate units is therefore eliminated, and, consequently, the problem of the "unity" of the "one and the many" is also eliminated, for this "unity" is now conceived in terms of the mode of being belonging to the algebraic concept of 'number', that is, the general concepts of twoness, threeness, etc., which, because they are identified with their "signs," become what to this day are self-evidently identified with the "true being" of "natural" numbers.

Because Vieta's anticipation of a formalized science originates not from a transformation of the perceptual life-world but from the transformation of a pre-existent mathematical science, we must look for the ultimate sources of the origin of this anticipation elsewhere than in Husserl's attempt to desediment the origin of the formalizing impulse in mathematics from the perceptually oriented praxis of measurement and its transformation of the life-world. The apprehension of the apprehension of the granted-as-given unknown and known determinate multitudes of determinate units, in which originates the symbolic abstraction that yields the formalized concepts of both numbers and number itself in general, manifestly does not involve the perception of individual objects in the life-world. This is to say, the grasping of the "possible" mode of being that is characteristic of the "supposition" that so and so many units are given, as an "actual" mode of being that is brought about in symbolic abstraction, does not involve what Husserl calls a 'modalization' of the straightforward sense perception of objects. Again, to use Husserl's terminology, the apprehension of the "supposed meaning, as supposed," that is characteristic of the formalized "proposition"

coincident with Vieta's symbols, does not arise on the ultimate basis of the disappointment of a cognitive intention previously directed toward individual objects. The perception of the sense-perceptible signs that "symbolize" the indeterminate concepts of multitude generated by symbolic abstraction that occurs in the symbolic cognition of Vieta's "analytical art" owes its origin to the transformation of a pre-existing scientific "logistic."

The pre-existing "logistic" that Vieta transforms likewise does not owe its origin to any kind of "modalization" based in the straightforward perception of individual objects. Indeed, as Klein has definitively shown, the "logistic" that Vieta transforms is a method of exact mathematical calculation that discovers, in advance of the perceptual acquaintance with determinate amounts of things and their relations, both their amounts and relations on the basis of "concepts" whose unity is irreducible to the "being one" of sensuously perceived individual objects. The originating impulse of the formalization instituted in Vieta's "analytical art" therefore ultimately leads back to Plato's distinction between 'arithmetic' and 'logistic' and his demand for a theoretical logistic.

Klein's account of this original impulse traces it back, as it were, to the "life-world"[182] praxis of dealing with determinate amounts of determinate items, that is, with delimited multitudes and their relations and the praxis of arriving at their delimitations and relations. In Plato's numerous references to the *τέχνη* of dealing with *ἀριθμοί* that characterize it indiscriminately as 'arithmetic' or 'logistic', Klein finds evidence of an original, pre-theoretical praxis of counting and calculating that does not discriminate between the delimitation of the things in the life-world into determinate amounts by counting them and the relating of these amounts to one another by partitioning and multiplying them. That is, because partitioning and multiplying determinate things have their ultimate basis in addition and subtraction, that is, in counting, Klein takes the distinction between disciplines that are devoted to counting and calculating to be a "theoretical" distinction. He finds evidence for this in Plato's strangely involved "definitions" of arithmetic and logistic, which characterize the former as studying the "odd" and the "even" with respect to the multitude they make in relation to themselves and each other, and the latter as studying "what" multitude the "odd" and the "even" make, again in relation to themselves and each other.

182. Klein, of course, uses the terminology of the "life-world" retrospectively in his "Phenomenology and the History of Science," and therefore subsequent to his own investigations of the origin of algebra, to describe, following Husserl, the source of "the original arithmetical evidences" and the "original 'evidence' and the original experience of things" concealed by the natural science made possible by "the method of symbolic abstraction."

The significance of these definitions for locating the life-world origin of the impulse toward the formalization that Vieta realizes is threefold, according to Klein. First, the avoidance of any mention of ἀριθμός indicates that arithmetic is not originally "number theory" but rather the τέχνη of correct counting. Second, the τέχνη of "logistic" is originally subordinate to "arithmetic" because, in order to establish the relations between multitudes in all calculations involving multitudes, knowledge is required beforehand of both how different determinate amounts of multitudes are related to each other and the characteristics of these determinate multitudes themselves. And, third, *knowledge* of "numbers," of their undeniable character of uniting many in one and one in many, is of no *direct* concern to the *arts* of counting and calculating. This "gift of the gods" is not the subject of study proper to the mathematical *arts*, and, insofar as these arts arise on the basis of a praxis that is part of the *perceptual* life-world, the original impulse toward mathematical formalization cannot, properly speaking, be located either in them or in *this* life-world.

Rather, Klein locates the original impulse toward mathematical formalization that transcends the perceptual life-world in the *question* that asks about the availability to the soul of the ἀριθμοί employed by the τέχνη of arithmetic before an actual count of sensuously perceivable things is undertaken. Plato's answer to this question—the positing of *noetic* ἀριθμοί with a mode of being that is separate from and therefore irreducible to the perceptual objects of the life-world—not only inaugurates *theoretical* arithmetic, as the *episteme* of the "pure" (non-sensuously perceived) principles responsible for the delimitation of multitudes of likewise "pure" units, but also leads to the *idea* of an equally theoretical logistic. Hence, Klein's provocative thesis that the Neoplatonic articulation of the difference between arithmetic and logistic on the basis of the opposition between arithmetic's theoretical nature and logistic's practical nature is a distortion of their original status in Plato. Thus, in addition to the theoretical study of "pure" principles responsible for the delimitation of multitudes of "pure" units, Plato also envisions a theoretical logistic devoted to the study of the "pure" relations between delimited multitudes that are available to the soul before it undertakes an actual calculation. Given the subordinate role of the art of logistic to the art of arithmetic, *theoretical* logistic Plato likewise holds to be subordinate to *theoretical* arithmetic.

His ontological doctrine of the χωρισμός of the ἀρχή responsible for the indivisibility of the 'one', of course, prevented the realization of the vision of a theoretical logistic in strictly Platonic terms, because it is irreconcilable with the need to partition the "unit" of calculation that occurs in order to execute many calculative operations. Aristotle's account of the one's mode of being arising "from abstraction," and thus its function as a measure, enabled the par-

tial realization of the Platonic vision of a theoretical logistic in Diophantus's *Arithmetic* insofar as part of the latter's method involved the calculation with the *species* of ἀριθμοί whose units were routinely partitioned as a matter of course. However, the fact that the divisibility of the Aristotelian unit, as a principle, is ultimately tied to the divisibility of the sensuously perceivable beings that are inseparable from the being of the species violates the Platonic stipulation that a theoretically "pure" logistic be capable of executing exact calculations independent of sensible beings. Thus, it is Vieta's transformation of the Diophantine method, the crucial moment of which is the "conceptualization" of the apprehension of the apprehension that grants as given both unknown and known determinate multitudes of units, that brings about the methodical realization of the Platonic vision of a "pure" theoretical logistic, though in conceptual terms that are completely un-Platonic.

It must be noted, however, that even Vieta's invention of the formalized concept does not fully realize the Platonic ideal of a purely noetic science. While the species calculation of Vieta's "analytic art" does indeed permit exact calculations to be made without any recourse to either individual objects or the (Aristotelian) species that renders such objects intelligible, the auxiliary function of these calculations for the solving of problems raised by traditional arithmetic and geometry limits the "purity" of his art, because its ultimate goal lies in its "deformalization" in order to solve problems rooted in traditional mathematical objects, that is, discrete and continuous magnitudes. As a result, the "analytical art" as a whole in Vieta remains distinct from the innovative method of symbolic calculation realized in its *logistice speciosa*. Likewise, even in Descartes's elevation of the "analytical art" to the *mathesis universalis*, the true science of the substance of the world understood as "extension," its realization of the status proper to the Platonic ideal of a theoretically "pure" logistic remains elusive. The dual status of "unity" in Descartes's "analytical geometry," as both a "simple idea" (and therefore a formalized concept) and a common unit of measure (and therefore a determinate concept), together with his twofold understanding of 'extension' as—again, both—a "simple idea" (and therefore a formalized concept) and the "material" substance of the world (and therefore a determinate being), prevent his vision of the *mathesis universalis* from realizing the Platonic ideal.

In both Vieta and Descartes, the "analytical art" is not understood to be identical with its method of symbolic calculation, because each operates with the basic stipulation that the "true" referents of its calculus are homogeneous objects in the world. Vieta's "law of homogeneity" makes this explicit, while it is implicit in Descartes's equivocation regarding the nature belonging to the mode of being of "unity." It is thus only in Wallis's *mathesis*

universalis, which establishes the identity of the "analytical art" with "symbolic calculation," that Plato's vision of a "theoretical" logistic that is "pure," that is to say, in which exact mathematical calculations are made without cognitive recourse to sensuously perceived beings in the life-world, is realized. Wallis's symbolic reformulation of the "homogeneity" of the object of mathematics both establishes this identity and thereby "constitutes," for the first time, the realm of "abstract" mathematical objects whose mode of being is transparently discernible in their sense-perceptible manifestation as symbols.

Klein's tersely stated observation that the modern (symbolic) solution to the traditional problem of the "one and many" being of "numbers" suffers the same fate as Plato's eidetically numerical solution to it—namely, that by attributing a numerical being to concepts, it transcends the limits of intelligibility established, for all time, by the *logos*—is, as we have tried to show, the problem that drives Husserl's investigations into the origin of the logic of symbolic mathematics from the first to the last of his works. We have also tried to show not only that Husserl's noble attempt to account for this logic on the basis of the being of objects whose unity is given in the perceptual life-world is unsuccessful, and is so on purely systematic grounds, but also that the purity of these grounds is conditioned by a strand of the historicity of the conceptuality of the most basic concept of arithmetic, the number, which is disclosed in one of the three greatest philosophical works of the twentieth century (and which, nevertheless, remains to this day an all-but-unknown and unstudied work): *Greek Mathematical Thought and the Origin of Algebra*.

Coda: Husserl's "Platonism" in the Context of Plato's Own Platonism

Philosophers of mathematics who attach the term 'Platonism' to the thesis that ideal mathematical objects are independent of the mind are usually quick to point out that no historical claim about Plato's philosophy is "necessarily" attached to this thesis, and with good reason. Husserl's own embrace of the term is connected with his "separation" from psychologism, and it is intended to signal not the independence of the ideal, or, more properly, formalized truths of "pure" logic from a metaphysical entity called the 'mind', but rather the independence of the cognitive content of such truths from the "psychological" experience in which Husserl is convinced they must necessarily appear in order to be grasped. This independence is established on the basis of the recollective representation of the "numerical" identity that composes the unity proper to the *irreality* of the logical content of cognitive meaning as it is accessed across manifold, psychologically individuated acts.

Both of these versions of "Platonism" depart radically from the "Platonism" of Plato himself that emerges through Klein's 1) desedimentation of the Greek mathematical context of Plato's philosophy and 2) reactivation of the "genuinely Platonic" doctrine of ἀριθμοὶ εἰδητικοί disclosed by Klein's location of Aristotle's polemic against this doctrine within its proper non-symbolic mathematical context. Plato's own "Platonism" in this case stems from neither a theory of mind and mind-independent "ideal" objects nor a theory of the "constitution" of such objects in temporally discrete acts of a "subject" (regardless of whether it is conceived of psychologically or transcendentally). On the contrary, the "Platonism" of Plato concerns the mode of being of the "one over many" unity characteristic of the mathematical numbers—and all else—with which human thinking (διάνοια) is cognitively concerned. The specifically "Platonic" concern is to provide an account of that which is responsible for "delimiting" the unity of a given multitude. The account is itself rooted in the "supposition" (ὑπόθεσις) that best accounts for the undeniable characteristic of any such unity, namely, its being "one" in a manner that excludes it from its being also its opposite—being "many"—and its delimiting capacity, in the exact sense of rendering "determinate" a given multitude from among the "indeterminate" multitude that composes the condition of not being a unity.

That the best account supposes a unity that is accessible only to thought, and therefore is a "*noetic*" unity, follows from the unlimited divisibility of the unity of sensuously perceived things. That such a unity is non-mathematical follows from the "impurity" of the "one over many" unity that is characteristic of numbers, a unity that includes its opposite insofar as it is at once "one *and* many." That this non-mathematical unity is therefore "eidetic," and that it must be supposed in order to account for the unity of any delimited multitude, follows from the recognition that delimited multitudes, despite their being many, are also precisely "one" and that they are such in a manner that exceeds their being many and therefore the "unity" for which the mathematical "monad" (as a one among many ones) can account.

Viewed in the light of Klein's reactivation of the "genuine Platonic doctrine" of unity, Husserl's "Platonism" assumes a radically different character. Its locus shifts from the recognition of the need to separate the unity of the formal logical content of cognition from the unity of cognitive lived experience, which informs Husserl's understanding of his own "Platonism," to that of the "numerical" mode of being that he posits as constituting the cognitive re-presentation of the "psychically *irreal*" logical content of all cognition. As we have shown, Husserl characterizes the mode of being of this logical content as "numerically one," though without providing a constitutive analysis of

its properly "arithmological" being as a unity. That is, he does not say how the "unity" in question is able to encompass a manifold of temporally discrete acts without its unity being "distributed" in the content of these acts; rather, he simply asserts that the manifold of these acts "posits" the said unity as "numerically identical" (i.e., as "one").

On the basis of these considerations, we conclude with the following observations. Aristotle's denial that, properly speaking, there is any "unity" in a delimited multitude of things does not address the problem of concern to both Plato's and Husserl's "Platonism": the irreducibility of the "common thing" (κοινὸν) responsible for the "collective unity" in such a multitude to the logical content or relations of its "members." That is because, for Aristotle, this unity is nothing other than the unit of measure that is the subject of the counting that delimits a given multitude. Thus, for him, the "unity" of three philosophers is 'philosopher'. Plato's *eidetic* solution to the problem of unity that both he and Husserl address, namely, the philosophical need to provide an account of the collective unity responsible for the delimitation of given multitudes, does not resolve the "logical" problem of the formalized unity that Husserl sought to separate from psychologism with his self-conscious appeal to "Platonic unity." This problem—namely, the phenomenological condition of possibility of a concept that is *indeterminate* while also being understood to refer, "at the same time," to *individual* objects—was first addressed in the psychological investigations in *Philosophy of Arithmetic*. That work formulated the authentic concept of cardinal number as the collective unity of a multiplicity of arbitrary objects that fall under the generically empty (and, therefore, formalized) concept of the 'anything', and the non-psychological and therefore "logical" nature of its unity remains, following Husserl's simultaneous self-critique of psychologism and embrace of "Platonism," unresolved in his subsequent thought. Nor can Plato's eidetic unity solve this problem, for, insofar as it has not been formalized, its content is unsuited to serve as the condition of possibility proper to formalized unity. Whether modern set theory, which Klein noted in 1932 "first tries to separate these two constituents"—namely, 1) the indeterminate and therefore formalized concept and 2) individual objects—"to clarify what 'at the same time' means," has managed to resolve or to make "progress" in the direction of a resolution of this problem is the subject matter for another study. Likewise, the subject matter for another study is whether Klein's desedimentation of the Greek mathematical context of Plato's "unwritten doctrine" of ἀριθμοὶ εἰδητικοί and his subsequent reactivation of this doctrine itself can contribute anything to this theory.

Glossary of Greek and German Terms

Greek-English

ἀριθμός, pl. ἀριθμοί: number (Klein translates this word with the German word *Anzahl*, which means a number of things, rather than with *Zahl*, which is the German word for number in the modern sense. See also Part I, n. 9.)

ἀριθμὸς αἰσθητός: sensible number

ἀριθμοὶ εἰδητικοί: eidetic numbers

ἀριθμὸς μαθηματικός: mathematical number

αἴσθησις: sense perception

αἰσθητόν, pl. αἰσθητά: sensible object

ἀναλογία: proportion

αόριστος δυάς: indeterminate dyad

ἀπόδειξις: demonstration

ἀπορία: impasse (*not* perplexity)

ἀρχή, pl. ἀρχαί: ruling beginning, source

ἁρμονία: harmony

ἀφαίρεσις: abstraction

γένος, pl. γένη: genus, family; often: the highest εἴδη

διάνοια: reckoning, thinking something through

δύναμις: power

ἕν: one

εἶδος, pl. εἴδη: kind

ἐπιστήμη: knowledge

ἐξ ἀφαιρέσεως: from abstraction

θάτερον: the other

ἰδέα = εἶδος

καθ᾽ αὑτό: by itself

καθόλου: over the whole, universal

κίνησις: motion, change

κοινόν: common thing

κοινωνία: community

κοινωνία τῶν εἰδῶν: community of εἴδη

λογιστική: logistical art

λογιστικὴ θεωρία: theoretical logistic

λόγος: speech

μέγιστα γένη: greatest kinds

μέθεξις: participation

μάθημα: learning matter

μέτρον: measure

μὴ ὄν: non-being

μέρη: parts

μὴ ὄντα: non-beings

μίμησις: imitation

μονάς, pl. μονάδας: monad, unit

νοητόν, pl. νοητά: intelligible object, object of thought

ὄν: being

οὐσία: being (Plato), thinghood (Aristotle)

πάθη: characteristic, property

πλῆθος: multitude

ποσόν: answer to the question “how many”

πρὸς ἄλλο: in relation to another
πρός τι: in relation to
πρώτη φιλοσοφία: first philosophy
σωρός: heap
στάσις: rest
τάξις: order
ταὐτόν: the same
τέχνη: art, skill
ὕλη: material
ὑπόθεσις: supposition
φύσις: nature
χωρισμός: separation

German-English

Anzahl: cardinal number (defined as the number used in simple counting to indicate how many items there are in an assemblage. See also Part III, n. 2.)
Anzahlbegriff: cardinal number concept
beliebiges Etwas: arbitrary anything
Einzelfall: instance
Etwas: anything
Etwas überhaupt: anything whatever
faktisch: factical, actual (see Part I, n. 19)
Gesamtheit: totality
Inbegriff: assemblage
irgendein: some or other
kollektive Verbindung: collective combination
kollektives Zumsammengreifen: collective grasping together
Leistung: accomplishment
Mannigfaltigkeit: multiplicity
Mannigfaltigkeitslehre: theory of manifolds
Mehrheit: plurality
Mehrheitserfassung: grasping of plurality
Mehrheitszeichen: sign(s) of plurality
Menge: multitude, set (after 1895; see Part IV, n. 60)
Mengenvorstellung: presentation of multitudes
real: real
Repräsentant: representative
rückgreifendes Erfassen: retrospective grasping
sachliche: material
Sachverhalt: state of affairs
Sinn: meaning, sense
Sinngebilde: meaning formation, significant formation
Sinnesgeschichte: meaning-history
Speilregeln: rules of the game
Vielheit: multiplicity
Vielheitsvorstellung: presentation of multiplicity
Verbindung: combination
Vorstellung: presentation
wirklich: actual (see Part I, n. 19)
Zahl: number
Zuwendung: regard of advertence

English-German

accomplishment: *Leistung*
actual: *faktisch*, *wirklich*
anything whatever: *Etwas überhaupt*
anything: *Etwas*
arbitrary anything: *beliebiges Etwas*
assemblage: *Inbegriff*
cardinal number: *Anzahl* (see note under *Anzahl*)
cardinal number concept: *Anzahlbegriff*
collective combination: *kollektive Verbindung*
collective grasping together: *kollektives Zumsammengreifen*
combination: *Verbindung*
factical: *faktisch*
grasping of plurality: *Mehrheitserfassung*
instance: *Einzelfall*
material: *sachliche*
meaning formation: *Sinngebilde* (*also*: significant formation)
meaning-history: *Sinnesgeschichte*
meaning: *Sinn* (*also*: sense)
multiplicity: *Mannigfaltigkeit*
multiplicity: *Vielheit*
multitude: *Menge* (*also*: set)
number: *Zahl*
plurality: *Mehrheit*
presentation of multiplicity: *Vielheitsvorstellung*
presentation of multitudes: *Mengenvorstellung*
presentation: *Vorstellung*
real: *real*
regard of advertence: *Zuwendung*
representative: *Repräsentant*
retrospective grasping: *rückgreifendes Erfassen*
rules of the game: *Speilregeln*
sense: *Sinn* (*also*: meaning)
set (after 1895): *Menge* (*also*: multitude; see Part IV, n. 60)
sign(s) of plurality: *Mehrheitszeichen*
significant formation: *Sinngebilde* (*also*: meaning formation)
some or other: *irgendein*
state of affairs: *Sachverhalt*
theory of manifolds: *Mannigfaltigkeitslehre*
totality: *Gesamtheit*

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Conventions of Citation

When a work is cited repeatedly and in uninterrupted succession within the body of the text, its abbreviation is cited only in the first instance. Page numbers are given without the abbreviation of the word 'page', except where confusion might occur.

Wherever a work is cited in original and in translation, the page reference to the text in the original language always precedes that to the work in translation; both page references are separated by an oblique stroke. In those instances in which a translation includes the pagination of the original in its margins, only the latter is cited; such exceptions are indicated in the relevant bibliographic references in the notes and below. Series of in-text references are self-contained, as it were; footnotes that occur in the course of those series are autonomous and thus do not require that the abbreviation of the text cited within the body of the text be provided again.

Abbreviations of frequently cited works are indicated both in the note in which the work is first cited and in the list of abbreviations, which precedes the Introduction. (For the sake of brevity and in keeping with practices in Husserl scholarship, volumes from Husserl's collected works in German, Husserliana, are cited, after the first full reference, as *Hua* following by roman volume number.)

Translations cited have been consulted but have been modified wherever it has been deemed necessary and without notice. See the Glossary for key terms and their translations. While the author's interventions in quotations are given in square brackets, those by Jacob Klein and Edmund Husserl, in their respective texts, are indicated by braces. Eva Brann's additions in her translation of Klein's *Greek Mathematical Thought and the Origin of Algebra* are enclosed in angle brackets, '⟨ ⟩'.

Unless otherwise indicated, all cross-references are to the Part in which they are made. Note numbers are specific to a given Part, whereas section numbers run consecutively, from the beginning to the end of this study.

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